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BALANCE FUNCTION GENERATED BY LIMITING CONDITIONS

This article conducts an analysis of the inherent constraints governing the formation of the price function that describes the interaction between two markets. The research not only identifies these constraints but also obtains an explicit form of the specified function.

The key factors considered in constructing the price function are defined in the article. Through analyzing these constraints and their impact on market interaction, a formula for the price function is provided. This approach not only reveals the essence of natural constraints in forming the price function but also provides a contextual foundation for negotiations shaping a fair exchange price for the interaction process between two markets. This offers a theoretical basis for modeling and solving similar problems arising during practical economic activities.

Two economies, Economy 1 and Economy 2, producing goods X and Y with linear Production Possibility Curve (PPC) graphs, are under consideration. The cost of producing one unit of good X relative to Y is denoted as R_1 for Economy 1 and R_2 for Economy 2. Exchange between economies occurs in a market, where the possible exchange is Δx units of X for $\Delta y = R_{market} \cdot \Delta x$ units of Y, and vice versa.

If R_1 is less than R_2, Economy 1 specializes in the production of X, and Economy 2 specializes in Y, fostering mutually beneficial trade. For mutually beneficial exchange on the market with a price R_{market}, it is necessary and sufficient that $R_1 \leq R_{market} \leq R_2$.

The article also explores the concept of a fair exchange price, specifying conditions for symmetry, reciprocity, and scale invariance. Notably, it indicates that the unique solution satisfying these conditions is $f(R_1, R_2) = \sqrt{R_1 \cdot R_2}$.

In the context of balanced exchange, where economies gain equal profit per unit of the acquired good, the balanced exchange price $R_{market}[balance]$ is determined as $R_{market} = \sqrt{R_1 \cdot R_2}$. This serves as a fair price, meeting the aforementioned conditions of symmetry, reciprocity, and scale invariance.

In the provided example with $R_1 = 2$ and $R_2 = 8$, the article examines the mutually beneficial interval for R_{market} and computes the balanced and fair exchange price.

Keywords: fair price, market modeling, limiting conditions.

Introduction

In this article, we will consider natural restrictions on the forming price's function that describes the interaction between two markets. Based on the described restrictions, this function will be obtained in an explicit form. This article also continues to study the special properties of functions in various applied areas [2], [3], [4].

Definitions

We have two economies, Economy 1 and Economy 2, producing goods X and Y with linear PPC [1] (Production Possibility Curve) graphs.

Let R_1 be the cost of good X relative to good Y in Economy 1 (i.e., to produce an additional 1 unit of good X in Economy 1, you need to forgo producing R_1 units of good Y), and let R_2 be the cost of good X relative to good Y in Economy 2 (i.e., to produce an additional 1 unit of good X in Economy 2, you need to forgo producing R_2 units of good Y).

There is also a market where you can exchange Δx units of good X for

$$\Delta y = R_{\text{market}} \cdot \Delta x$$

units of good Y, and vice versa, Δy units of good Y for

$$\Delta x = \frac{1}{R_{\text{market}}} \cdot \Delta y$$

units of good X.

It is evident that if the cost R_1 of good X relative to good Y in Economy 1 is lower than the cost R_2 of good X relative to good Y in Economy 2, then it is preferable for Economy 1 to specialize in the production of good X, and for Economy 2 to specialize in the production of good Y. Indeed, the relative costs of increasing the production of good X are lower for Economy 1, and for good Y, respectively, for Economy 2.

With such specialization, mutually beneficial trade becomes possible between the two economies using the surplus of goods produced as a result of their respective specialization. The details of this exchange will be discussed below.

Formalization of the market's exchange

Let us consider the case when $R_1 < R_2$. Under such conditions, it is preferable for Economy 1 to specialize in the production of good X and for Economy 2 to specialize in the production of good Y.

Let's consider the desired situation in Economy 1:

$$(x_1, y_1)$$

Moving along the PPC, instead of (x_1, y_1), we produce:

$$(x_1 + \Delta x, y_1 - R_1 \cdot \Delta x)$$

which, with the condition $\Delta x > 0$, corresponds to a local specialization of Economy 1 in producing good X.

We then exchange the surplus Δx units of good X, obtained from specialization, for

$$\Delta y = R_{\text{market}} \cdot \Delta x$$

units of good Y produced in Economy 2:

$$(x_1, y_1 - R_1 \cdot \Delta x + R_{\text{market}} \cdot \Delta x) = (x_1, y_1 + \frac{R_{\text{market}} - R_1}{R_{\text{market}}} \cdot \Delta y)$$

Now, consider the desired situation in Economy 2:

$$(x_2, y_2)$$

Moving along the PPC, instead of (x_2, y_2), we produce:

$$(x_2 - \frac{R_{\text{market}}}{R_2} \cdot \Delta x, y_2 + R_{\text{market}} \cdot \Delta x)$$

which, with the condition $\Delta y = R_{\text{market}} \cdot \Delta x > 0$, corresponds to a local specialization of Economy 2 in producing good Y.

After exchanging on the market with Economy 1

$$\Delta y = R_{\text{market}} \cdot \Delta x$$

units of good Y for Δx units of good X, the situation in Economy 2 becomes:

$$(x_2 - \frac{R_{\text{market}}}{R_2} \cdot \Delta x + \Delta x, y_2) = (x_2 + \frac{R_2 - R_{\text{market}}}{R_2} \cdot \Delta x, y_2)$$

Thus, economies 1 and 2 can interact, specializing in the production of certain goods and exchanging surpluses on the market, which promotes more efficient resource utilization and overall prosperity.

Mutually Beneficial Market's Rate Interval

It is evident that for the exchange to be mutually beneficial for both economies, it is necessary and sufficient to satisfy the following system of inequalities:

$$\begin{cases} \frac{R_{\text{market}} - R_1}{R_{\text{market}}} \geq 0 \\ \frac{R_2 - R_{\text{market}}}{R_2} \geq 0 \end{cases}$$

From this system, we derive the necessary and sufficient condition for a mutually beneficial exchange between economies 1 and 2:

$$R_1 \leq R_{\text{market}} \leq R_2$$

which can be reformulated as:

$$R_{\text{market}} \in [R_1, R_2]$$

Fair Exchange Price. In order for the price of exchange $R_m = f(R_1, R_2)$ on the market could be considered fair, it is necessary to fulfill three natural conditions.

1. $f(R_1, R_2) = f(R_2, R_1)$
2. $f(1/R_1, 1/R_2) = 1/f(R_2, R_1)$
3. $f(a \cdot R_1, a \cdot R_2) = a \cdot f(R_1, R_2)$

Conditions' explanation. Indeed, let's delve into the explanation for the natural conditions that the fair exchange price function $f(R_1, R_2)$ should satisfy:

1. **Symmetry**: The first condition, $f(R_1, R_2) = f(R_2, R_1)$, ensures that the exchange price remains the same regardless of which economy is producing good X and which is producing good Y. In other words, the exchange rate should be symmetric with respect to the two economies' roles.

2.**Reciprocity**: The second condition, $f\left(\frac{1}{R_1}, \frac{1}{R_2}\right) = \frac{1}{f(R_2, R_1)}$, reflects the reciprocity in trading goods. If it's fair to exchange good X for good Y at a certain rate, it should also be fair to exchange good Y for good X at the inverse rate.

3.**Scale Invariance**: The third condition, $f(a \cdot R_1, a \cdot R_2) = a \cdot f(R_1, R_2)$, ensures that if the costs of producing goods X and Y in both economies are scaled up or down by a factor of "a," the fair exchange rate should also scale accordingly. This ensures that any change in the scale of production costs retains the consistent relationship between these variables, even though the numerical values may change due to unit adjustments.

Fair price's theorem

Theorem 1. *The only function that satisfies these 3 conditions is:*

$$f(R_1, R_2) = \sqrt{R_1 \cdot R_2}$$

Proof. We aim to prove that the only function satisfying the given conditions is $f(R_1, R_2) = \sqrt{R_1 \cdot R_2}$.

We start by considering the reciprocity condition:

$f(a, 1/a) = 1/f(1/a, a)$

Using (1) and the symmetry condition, we get: $f(a, 1/a)^2 = 1 \Rightarrow f(a, 1/a) = 1$

Next, we use the scale invariance property: $f(a, 1/a) = af(1, 1/a^2) \Rightarrow f(1, 1/a^2) = 1/a$

Now, we use the scale invariance property and (3) to analyze $f(a, b)$:

$f(a, b) = af(1, b/a) = a \cdot \sqrt{b/a} = \sqrt{a \cdot b}$

In summary, we have demonstrated that under the given conditions, the function $f(a, b) = \sqrt{a \cdot b}$ satisfies all criteria. Furthermore, we have shown that any function satisfying the given conditions must be of the form $f(a, b) = \sqrt{a \cdot b}$. Therefore, the proof is complete.

This theorem establishes that among all possible functions that adhere to the principles of symmetry, reciprocity, and scale invariance, the square root of the product of the relative production costs of goods X and Y, i.e., $\sqrt{R_1 \cdot R_2}$, stands as the unique solution. This mathematical result provides a concrete expression for the fair exchange price that maintains a harmonious balance between the economic principles driving mutual trade. In essence, it underlines the inherent connection between the relative costs of production and the equitable exchange rate, offering a precise formula for economies to adopt when engaging in mutually beneficial trade.

Balanced Exchange Price

Under these exchange conditions, Economy 1 gains a profit of $\frac{R_{\text{market}} - R_1}{R_{\text{market}}}$ units of good Y for each purchased unit of good Y on the market, while Economy 2 gains a profit of $\frac{R_2 - R_{\text{market}}}{R_2}$ units of good X for each purchased unit of good X on the market.

Let's call the exchange balanced if the profit per unit of the acquired good is the same for both economies:

$$\frac{R_{\text{market}} - R_1}{R_{\text{market}}} = \frac{R_2 - R_{\text{market}}}{R_2}$$

From which we get:

$$\frac{R_1}{R_{\text{market}}} = \frac{R_{\text{market}}}{R_2}$$

$$R_{\text{market}}^2 = R_1 \cdot R_2$$

As a result, under the condition:

$$R_1 \leq R_{\text{market}} \leq R_2$$

we obtain the necessary and sufficient condition for a mutually beneficial balanced exchange:

$$R_{\text{market}} = \sqrt{R_1 \cdot R_2}$$

This condition ensures that both economies can engage in trade in a way that benefits both of them, leading to an efficient allocation of resources and economic growth.

As we see from theorem 3, the *balanced price* is also fair price according to the conditions of *Symmetry*, *Reciprocity* and *Scale Invariance*.

Example

Problem's Formulation. Let's provide an example with real values of R_1 and R_2 to calculate the mutually beneficial range for R_{market} and balanced exchange's price $R_{\text{market}}[balance]$.

Suppose we have two economies, Economy 1 and Economy 2, producing goods X and Y.

Let $R_1 = 2$ be the cost of producing an additional 1 unit of good Y in Economy 1 (i.e., to produce an extra unit of good Y, you need to forgo producing R_1 units of good X), and let $R_2 = 8$ be the cost of producing an additional 1 unit of good Y in Economy 2 (i.e., to produce an extra unit of good Y, you need to forgo producing 2 units of good X).

Mutually Beneficial Market's Rate Interval. For the exchange rate R_{market} of good X to good Y under these conditions, the exchange will be mutually beneficial for both economies for any value of R_{market} in the interval $[2, 8]$.

For example, if 1 unit of good X is exchanged for 5 units of good Y in the market, the exchange between the economies will remain mutually beneficial.

Balanced Exchange's and Fair Price. Now we can calculate the balanced $R_{\text{market}}[balance]$ using the formula:

$$R_{\text{market}} = \sqrt{R_1 \cdot R_2}$$

$$R_{\text{market}} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$$

Thus, in this example, the value of $R_{\text{market}}[balance]$ will be 4. This value indicates how many units of good Y can be obtained on the market for each unit of good X. And the value of $\frac{1}{R_{\text{market}}[balance]}$ indicates how many units of good X can be obtained for each unit of good Y.

Please note that in real situations, the values of R_1 and R_2 may vary and depend on specific economic conditions and trade relationships between countries or regions.

Список літератури

1. Sickles Robin C., Zelenyuk Valentin. Measurement of productivity and efficiency. Cambridge University Press, 2019.
2. Морозов Д. І. Поліноміальне представлення двійкових дерев ентропійних бінарних кодів.*Могилянський математичний журнал.* 2022. Т. 4. С. 20–23.
3. Морозов Д. І. Ізометрії та стискаючі функції кільця Z_2. *Вісник Запорізького національного університету. Фізико-математичні науки.* 2014. Т. 1. С. 90–98.
4. Морозов Д. І. Ізометричність поліномів над кільцем цілих 2-адичних чисел. *Наукові записки НаУКМА. Фізико-математичні науки.* 2011. Т. 113. С. 13–15.

References

1. Robin C Sickles and Valentin Zelenyuk, *Measurement of productivity and efficiency* (Cambridge University Press, 2019).
2. D. I. Morozov, "Polynomial representation of binary trees of entropy binary codes," Mohyla Mathematical Journal. **4**, 20–23 (2022).
3. D. I. Morozov, "Izometrii ta styskaiuchi funkcii kilcia Z_2," Visnik Zaporizskogo Nacionalnogo Universytetu. Fizyko-matematychni nauky. **1**, 90–98 (2014).
4. D. I. Morozov, "Izometrychnist' polinomiv nad kilcem cilyh 2-adychnyh chysel," Naukovi Zapysky NaUKMA. Fizyko-matematychni nauky. **113**, 13–15 (2011).

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ФУНКЦІЯ БАЛАНСУ, ЩО ГЕНЕРУЄТЬСЯ ОБМЕЖУВАЛЬНИМИ УМОВАМИ

У цій статті проаналізовано природні обмеження, що визначають формування цінової функції, яка описує взаємодію між двома ринками. Предметом дослідження є не лише виявлення цих обмежень, а й отримання явного вигляду вказаної функції.

У статті визначено ключові фактори, які враховують при побудові цінової функції. В результаті аналізу цих обмежень та їх впливу на ринкову взаємодію надано формулу цінової функції.

Такий підхід не лише дає змогу розкрити сутність природних обмежень у формуванні цінової функції, а й надає контекстну базу для перемовин, що формують справедливу ціну обміну для процесу взаємодії двох ринків. Це надає теоретичне обґрунтування для моделювання та розв'язку подібних задач, що виникають під час практичної економічної діяльності.

Розглянуто дві економіки, Економіку 1 та Економіку 2, які виробляють товари X та Y із лінійними графіками виробничих можливостей. Вартість виробництва одиниці товару X відносно Y позначається R_1 для Економіки 1 і R_2 для Економіки 2. Обмін між економіками відбувається на ринку, де можливий обмін Δx одиниць X на $\Delta y = R_{\text{market}} \cdot \Delta x$ одиниць Y та навпаки.

Якщо R_1 менше ніж R_2, то Економіка 1 спеціалізується на виробництві X, а Економіка 2 — Y, що сприяє взаємовигідній торгівлі залишками. Для взаємовигідного обміну ціною на ринку R_{market} необхідно та достатньо $R_1 \leq R_{\text{market}} \leq R_2$.

У статті також розглянуто концепцію справедливої ціни обміну, вказано на умови симетрії, взаємності та інваріантності масштабу для її визнання. Зокрема, зазначено, що єдиним розв'язком, який відповідає цим умовам, є $f(R_1, R_2) = \sqrt{R_1 \cdot R_2}$.

У контексті збалансованого обміну економіки одержують рівний прибуток за одиницю отриманого товару, і збалансована ціна обміну $R_{\text{market}}[balance]$ визначається як $R_{\text{market}} = \sqrt{R_1 \cdot R_2}$, що є справедливою ціною, для якої виконуються вищезгадані умови симетрії, взаємності та інваріантності масштабу.

У наведеному у статті прикладі з $R_1 = 2$ та $R_2 = 8$ розглянуто взаємовигідний інтервал для R_{market} та обчислено збалансовану та справедливу ціну обміну.

Ключові слова: справедлива ціна, моделювання ринку, обмежувальні умови.

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