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EMPIRICAL ANALYSIS OF MOON'S GRAVITATIONAL WAVE AND EARTH'S GLOBAL WARMING

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Abstract. This research examines a possibility of a disturbance by Moon's gravitational wave to the Earth's global warming process in comparison with the increase of global volume of carbon dioxide. Because the general theory of relativity that predicts the gravitational wave of a planet has a dimension of 1/(distance)2, we analyzed the data sets of global temperature and global carbon dioxide, with this dimension of gravitational wave using Least Squares Estimation of Linear Classical Regression Model, Generalized Classical Regression Model, and Nonlinear Regression Model. The results suggest that there is a disturbance to the process of global warming by the Moon's gravitational wave. However, there is uncertainty for this conclusion because the Moon's rotational movement around Earth gives different type of distributions of its sample data, while global temperature and carbon dioxide increase proportionally accordingly to available time-series.

Keywords: global warming, Moon and Earth, global carbon dioxide, gravitational wave.

INTRODUCTION

Einstein's theory of gravitational wave predicts that it contains a factor of λ, which has the dimension of $\frac{1}{r^2}$, where r is the distance (kilometers), to where the gravitational wave reaches from a planet. Therefore, in this research, $\frac{1}{r^2}$ is considered as a surrogate of the intensity of the gravitational wave, and its relation to the global temperature is analyzed, together with the global carbon dioxide, in time-series.

THEORY

In the general theory of relativity [1], gravity is described by the derivatives, $g_{\mu\nu}$, of the scalar potential, $V = -m/r$, where μ and ν are 0, 1, 2, 3, which indicate the coordinates of the empty curved-space in 4 dimensions, where x^0 is for time, x^1, x^2 and x^3 for space, and m is mass of a planet, and r is the distance from the cen-

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ter of the planet. The gravitational field in the empty space is described by Ricci tensor:

$$R_{\mu\nu} = 0\,, \qquad (1)$$

while gravitational wave is described by the solutions of $g_{\rho\sigma,\mu\nu} = 0$ in harmonic coordinates, where the condition of harmonic coordinates is $g^{\mu\nu}\left(g_{\rho\mu},_{\nu} - \frac{1}{2}g_{\mu\nu},_{\rho}\right) = 0$, where each of $g_{\rho\sigma}$ satisfies the d'Alambert equation, $g^{\mu\nu}(V,_{\mu\nu} - \Gamma^{\alpha}_{\mu\nu}V,_{\alpha}) = 0$, where each of α, μ, ν, ρ and σ indicates each of the coordinates, x^0, x^1, x^2 and x^3. Here, $g_{\mu\nu},_{\rho} = \frac{d}{dx^{\rho}}g_{\mu\nu}$, and $g_{\mu\nu},_{\rho\sigma} = \frac{d^2}{dx^{\rho}dx^{\sigma}}g_{\mu\nu}$.

When the gravitational waves are all moving in the same direction, for example x^3, $g_{\mu\nu}$ are functions of only one variable, x^3 in time-series. And, in more general case, $g_{\mu\nu},_{\sigma} = u_{\mu\nu}l_{\sigma}$ when $g_{\mu\nu}$ are all functions of the single variable $l_{\sigma}x^{\sigma}$, while l_{σ} are the constants that satisfy $g^{\rho\sigma}l_{\rho}l_{\sigma} = 0$, and $u_{\mu\nu}$ is the derivative, $g_{\mu\nu}$, of the function $l_{\sigma}x^{\sigma}$. And, then, after the transformation of the tensors, we get:

$$u^{\nu}_{\rho}l_{\nu} = \frac{1}{2}ul_{\rho}\,. \qquad (2)$$

And, then, $\Gamma^{\sigma}_{\alpha\sigma} = \frac{1}{2}ul_{\alpha}$. Now, the gravitational wave moves in the direction l_{σ} of the form, $x^{\mu'} = x^{\mu} + b^{\mu}$, where b^{μ} is a function only of $l_{\sigma}x^{\sigma}$ with the restriction that wave moves only in one direction. And, then, the equation (2) indicates the flow of the energy in the direction of x^3:

$$16\pi t^0_0 = \frac{1}{4}(u_{11} - u_{22})^2 + u_{12}{}^2\,.$$

Here, $t^{\nu}{}_{\mu}$ is a pseudo-tensor, which means a quantity, given by $t^{\nu}{}_{\mu}\sqrt{\;} = \frac{\partial\tilde{L}}{\partial g_{\alpha\beta},_{\nu}}g_{\alpha\beta},_{\mu} - g^{\nu}{}_{\mu}\tilde{L}$, while $L = g^{\mu\nu}(\Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho} - \Gamma^{\rho}_{\mu\sigma}\Gamma^{\rho}_{\nu\rho})$, $R = g^{\mu\nu}R_{\mu\nu} = R^* - L$, $R^* = g^{\mu\nu}(\Gamma^{\sigma}_{\mu\sigma},_{\nu} - \Gamma^{\sigma}_{\mu\nu},_{\sigma})$, $\tilde{L} = L\sqrt{\;}$, $\sqrt{\;} = \sqrt{-g}$, and $g = g_{00}$.

The gravitational field equation of the empty space (1) is generalized to a tensor equation:

$$R_{\mu\nu} = \lambda g_{\mu\nu}\,, \qquad (3)$$

where λ is a constant. The values of $R_{\mu\nu}$ contain second derivatives of the $g_{\mu\nu}$, because $R_{\mu\nu} = \Gamma^{\alpha}_{\nu\alpha},_{\nu} - \Gamma^{\alpha}_{\mu\nu},_{\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}$; so, λ must have the dimension of (distance)$^{-2}$. Where the planet exists, this constant coefficient λ must be small enough, so that the flow of energy does not disturb the coordinate that the planet makes, as shown in the tensor equation (1).

There is a comprehensive action principle:

$$\delta(I_g + I') = 0\,, \qquad (4)$$

where I_g is the gravitational action, and I' is the action of sum of all the other fields; while, $\delta I_g = -\int (R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R)\sqrt{-g}\, \delta g_{\alpha\beta} d^4 x$, and $I = \int R\sqrt{-g}\, d^4 x$.

For the cosmological theory, an extra term is added, such as: $I_c = c\int \sqrt{-g}\, d^4 x$, where c is a suitable constant. And, then, $\delta I_c = = c\int \frac{1}{2} g^{\mu\nu} \delta g_{\mu\nu} \sqrt{-g}\, d^4 x$, and the action principle (equation (4)) gives:

$$(16\pi)\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) + \frac{1}{2} c g^{\mu\nu} = 0 . \qquad (5)$$

Then, the equation (3) gives $R = 4\lambda$, and hence: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\lambda g_{\mu\nu}$. And, if $8\pi c = -\lambda$, it satisfies the equation (5).

This theory suggests that Moon emits gravitational wave to Earth, which is a flow of energy; and its intensity includes a dimension, related to $R = 4\lambda = -8\pi c \propto \frac{1}{r^2}$, where r is the distance between two planets. In this research, we use $\frac{1}{r^2}$ as an indicator of the gravitational wave. In the following sections, we report the methods and the results of the analysis. It is assumed that the global temperature is an indicator of the energy, which is given by the indicator of Moon's gravitational wave. The global carbon-dioxide is analyzed together in the mathematical models for the data analysis, in order to evaluate the importance of Moon's gravitational wave.

METHOD OF THE RESEARCH

The descriptive statistics of the data, from 1987 till 2009, of the global temperature (increased degree Celsius since 1978) [2], the global carbon dioxide (million metric tons) [3], the distance between Moon and Earth (r : kilometers) [4], and calculated $\frac{1}{r^2}$ ((kilometers)^2), are shown in Table 1.

Table 1. Descriptive statistics

Variable	Global Temperature (°C) *	Carbon-gas (million metric tons) **	Distance between Moon and Earth (r: kilometers)	$1/r^2$ ((kilometers)$^{-2}$)
Mean	0,29130	$1{,}25165 \cdot 10^3$	$3{,}62618 \cdot 10^5$	$7{,}60509 \cdot 10^{-12}$
Standard deviation	0,12125	$2{,}14245 \cdot 10^2$	$5{,}98411 \cdot 10^2$	$2{,}51097 \cdot 10^{-14}$
Minimum	0,10000	$8{,}92000 \cdot 10^2$	$3{,}61583 \cdot 10^5$	$7{,}56999 \cdot 10^{-12}$
Maximum	0,43000	$1{,}62600 \cdot 10^3$	$3{,}63483 \cdot 10^5$	$7{,}64865 \cdot 10^{-12}$
Skewness	–0,21063	0,14292	–0,15249	0,15787
Kurtosis	1,29401	1,82491	1,67498	1,67879
Valid number of observations	23	23	23	23

* Increased degree Celsius since 1978

** To convert these estimates to units of carbon dioxide (CO_2), simply multiply these estimates by 3,667 [2]

Regression analysis is made on the global temperature, the global carbon-dioxide and $\frac{1}{r^2}$, with the models, considered below.

Least Squares Estimation of Linear Classical Regression Model

The global temperature $Y = \{y_1, \ldots\ldots y_n\}$, the constant value 1, x_1, the measured global carbon-dioxide, x_2, and the inverse of the squared distance between Moon and Earth, x_3, are transformed into the forms of $n \times 1$ vectors, y, x_1, x_2, x_3, where n is the number of observation, 23. Then $n \times k$ matrix $X = \{x_1, x_2, x_3\}$ is defined, where $k = rank(X)$.

We assume that: $E(y) = X\beta$, $V(Y) = \sigma^2 I$, X non-stochastic, and rank$(X) = k$ =3, where $E(y)$ is the mean of y, $V(y)$ is the variance of Y, and $\sigma^2 = V(y)$. I is $n \times n$ matrix, in which all diagonal elements are 1, and other elements are 0. In the Classical Regression Model, it is assumed that the diagonal elements of $V(Y) = \sigma^2 I$ are all of the same value, σ^2. And, all covariances are assumed to be zero. With the following algebra, b (estimated coefficient β from the sampled data) and σ^2 are calculated:

$Q = X'X$, where X' is a transposed matrix of the matrix X;

$b = Q^{-1}X'Y$, where Q^{-1} is an inversed matrix of the matrix Q;

$\hat{Y} = Xb$: expected global temperature Y;

$e = Y - \hat{Y}$;

$$V(b) = \frac{e'e}{n-k} Q^{-1}.$$

And square-root of the diagonal elements of $V(b)$ are the standard errors of elements of the estimated coefficient-vector b.

Time Series

After applying the Classical Regression Model to this problem, we examine the time-series of the sampled data of global temperature, carbon-dioxide and $\frac{1}{r^2}$, in order to estimate the independency (or dependency) and the distribution patterns of these variables. For this purpose, we calculate the autocorrelation of each of the sampled data of these three variables, with the following algebra:

From n consecutive observations, $y_1, \ldots, y_n$, we make a vector $y = (y_1, \ldots, y_n)^{\mathrm{T}}$, where '$\mathrm{T}$' transposes a vector. And then we calculate: sample mean: $m = \sum_{t=1}^{n} y_t \big/ n$, sample autovariance: $c_0 = \sum_{i=1}^{n} (y_i - m)^2 \Big/ n$, the first sample autocovariance: $c_1 = \sum_{i=2}^{n} (y_i - m)(y_{i-1} - m) \Big/ (n-1)$, and then similarly, the second sample autocovariance: $c_2 = \sum_{i=3}^{n} (y_i - m)(y_{i-2} - m) \Big/ (n-2)$, and so forth.

Then we calculate the sample autocorrelations: $r_j = c_j / c_0$.

Generalized Classical Regression

In general, the autocorrelation suggests whether, or not, changes in time-series of each of the variables are related to its own past; or it suggests whether or not, the variable in the past is independent from the present time with the same pattern of the distribution of the variable as it currently has. By comparing three autocorrelations for three variables of global temperature, carbon-dioxide and $\frac{1}{r^2}$, we will be able to estimate the distribution pattern of the standard deviations of the $n \times k$ matrix $X = \{x_1, x_2, x_3\}$, to see if the diagonal elements of the assumed matrix of variances (square-root of standard deviation) $\Sigma = V(Y) = \sigma^2 I$ are all equal and/or if the covariances are zero, or not. If not, the Classical Regression Model is not applicable; but, instead, we need Generalized Classical Regression Model, in which $\Sigma = V(Y) \neq \sigma^2 I$, and/or the matrix Σ contains non-zero covariances.

In this research, we examine two possibilities:

a) Pure Heteroskedasticity, in which diagonal elements of Σ are all different; and,

b) First-Order Autoregressive Process, in which the first-order autocovariance, $c_1 = \sum_{i=2}^{n} (y_i - m)(y_{i-1} - m) \Big/ (n-1)$ is not zero.

In case of Pure Heteroskedascity, the y_i's are uncorrelated, but have different variances: the matrix Σ is diagonal, with diagonal elements $\sigma_1^2, \ldots, \sigma_i^2, \ldots, \sigma_n^2$. Here we assume an $n \times n$ matrix H that makes $H \Sigma H' = I$. H is the diagonal matrix that has the $1/\sigma_i$'s on its diagonal. If the σ_i's are known, then we can transform the data by dividing all variables at the i th observation by σ_i to get

$$y_i^* = y_i \big/ \sigma_i, \quad x_{ij}^* = x_{ij} \big/ \sigma_i, \quad \text{where } i = 1, 2, \ldots, 23;\ j = 1, 2, 3.$$

Then Classical Regression Model will apply to the new data and the regression of Y^* on X^* will produce the Least-Squares Estimation of Generalized Classical Regression Model of b^* with the same procedure shown in the analysis of the Least Squares Estimation of Linear Classical Regression Model.

When we assume that the time-series of the global temperature is First-Order Autoregressive Process, a common practice is: at first, run Least Squares Estimation of Linear Classical Regression Model of y on X to get the residuals $e = Y - \hat{Y}$.

Then regress e_i on e_{i-1} (across $i = 2,\ldots,23$ in the time-series) to estimate $r_1 = c_1/c_0$: as $\hat{\rho} = \sum_{i=2}^{23} e_i e_{i-1} \Big/ \sum_{i=2}^{23} e_{i-1}^2$ **.** No intercept is required when the sum of the residuals $\sum_{i=1}^{23} e_i$ is zero. And then transform the data as bellow, using $\hat{\rho}$: $y_i{*} = y_i - \hat{\rho} y_{i-1}$ and $X{*} = X_i - \hat{\rho} X_{i-1}$, where $X_i = \{x_{1,i}, x_{2,i}, x_{3,i}\}$ and $X_{i-1} = \{x_{1,i-1}, x_{2,i-1}, x_{3,i-1}\}$; and then run Least Squares Estimations of Linear Classical Regression Model of $y{*}$ over $X{*}$.

Nonlinear Regression Model

In this research, we try to analyze the database also with Nonlinear Regression Model, with Cobb-Douglas function, $y = b_1 x_2^{b_2} x_3^{b_3}$. Not like as Least Squares Estimation of Classical Regression Model, we cannot calculate the coefficients, b_1, b_2, b_3, algebraically; but, we can calculate them only numerically:

Now $$h = h(b_1, b_2, b_3, x_2, x_3);$$

$$z = \partial h/\partial b_1 \, \partial h/\partial b_2 \, \partial h/\partial b_3 = z(x_2, x_3, b_1, b_2, b_3);$$

$$u = y - h = u(y, x_2, x_3, b_1, b_2, b_3).$$

We seek the values of b_1, b_2, b_3 that make $z'u = 0$. We assume that b_1^0, b_2^0, b_3^0 are the initial guessed values for b_1, b_2, b_3. Then, $h^0 = h(b_1^0, b_2^0, b_3^0, x_2, x_3)$, $z^0 = z(x_2, x_3, b_1^0, b_2^0, b_3^0)$, $u^0 = y - h^0 = u(y, x_2, x_3, b_1^0, b_2^0, b_3^0)$. The linear approximation to h at the point (b_1^0, b_2^0, b_3^0) is $h = h^0 + z^0(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0)$, so that order of approximation,

$$u = y - h = y - [h^0 + z^0(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0)] =$$

$$= u^0 - z^0(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0);$$

$$\varphi(b_1, b_2, b_3) = u'u = u^0{}'u^0 + (b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0) z^0{}' z^0 -$$

$$- 2(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0) z^0{}' u^0 ;$$

$$\varphi'(b_1, b_2, b_3) = \partial\varphi/\partial b_1 \, \partial\varphi/\partial b_2 \, \partial\varphi/\partial b_3 = 2(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0) z^0{}' z^0 - 2 z^0{}' u^0 .$$

Set $\varphi'(b_1, b_2, b_3) = 0$, and solve for $(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0) = z^0{}'u^0 / z^0{}'z^0$.

And then take the resulting $(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0)$ as the new b_1^0, b_2^0, b_3^0 and restart the calculation. Continue until the result converge, that is until $(b_1 - b_1^0)(b_2 - b_2^0)(b_3 - b_3^0) \cong 0$.

In practice, the derivative $z = \partial h / \partial b_1^0 \, \partial h / \partial b_2^0 \, \partial h / \partial b_3^0$ can be approximated numerically as

$$z^0 = b_1^0 (x_2^{(b_2+p_2)(b_3+p_3)} - b_1^0 x_2^{(b_2-p_2)(b_3-p_3)}) / [2p_1 2p_2 2p_3],$$

where p_1, p_2, and p_3 are small steps.

RESULT

The results of Least Squares Estimation of Classical Regression Model are shown from Table 2 to Table 6.

Table 2. Matrix $Q = X'X$ in Classical Regression Model

23,00000	$2{,}87880 \cdot 10^{4}$	$1{,}74917 \cdot 10^{-10}$
$2{,}8788010^{4}$	$3{,}70424 \cdot 10^{7}$	$2{,}18930 \cdot 10^{-7}$
$1{,}74917 \cdot 10^{-10}$	$2{,}18930 \cdot 10^{-7}$	$1{,}33027 \cdot 10^{-21}$

Table 3. Matrix $X'Y$ in Classical Regression Model

6,70000
$8{,}92389 \cdot 10^{3}$
$5{,}09527 \cdot 10^{-11}$

Table 4. Matrix $b = Q^{-1}X'Y$ in * Classical Regression Model

for 1 (x_1)	-1,17863
for Carbon dioxide (x_2)	$5{,}33150 \cdot 10^{-4}$
for $(1/r^2)$ (x_3)	$1{,}05537 \cdot 10^{11}$

* With this model, $R^2 = 0{,}88602$.

Table 5. Matrix $V(b) = \dfrac{e'e}{n-k} Q^{-1}$ in Classical Regression Model

7,71895	$-7{,}67170 \cdot 10^{-6}$	$-1{,}01370 \cdot 10^{12}$
$-7{,}67170 \cdot 10^{-6}$	$1{,}82931 \cdot 10^{-9}$	$7{,}07689 \cdot 10^{5}$
$-1{,}01370 \cdot 10^{12}$	$7{,}07689 \cdot 10^{5}$	$1{,}33176 \cdot 10^{23}$

Table 6. Coefficients and standard errors of the coefficients in Classical Regression Model

Variable	Coefficient	Standard error
for 1 (x_1)	–1,17863	2,77830
for Carbon dioxide (x_2)	$5{,}33150 \cdot 10^{-4}$	$4{,}27704 \cdot 10^{-5}$
for $(1/r^2)$ (x_3)	$1{,}05537 \cdot 10^{11}$	$3{,}64933 \cdot 10^{11}$

The coefficients of Table 6, which are calculated by Classical Regression Model, show that $x_3 \left(\dfrac{1}{r^2}\right)$ influences y (global temperature) more than x_2 (carbon-dioxide) does; however, the standard error of the estimated coefficient of x_3 is larger than x_2's. In order to investigate this large size of the standard error of the coefficient of x_3, we analyzed the patterns of the changes of y, x_2, and x_3, in time-series, by calculating their autocorrelations. Fig. 1 shows the auto-correlation of y, Fig. 2 of x_2 and Fig. 3 of x_3.

LAG	AUTO. CORR.	STAND. ERR.
1	0.90515	0.20851
2	0.80416	0.33871
3	0.69154	0.41347
4	0.56876	0.46102
5	0.42716	0.49058
6	0.27787	0.50649
7	0.12488	0.51308
8	-0.02070	0.51440
9	-0.15047	0.51443
10	-0.25543	0.51634
11	-0.33871	0.52181
12	-0.39937	0.53128
13	-0.43875	0.54418
14	-0.45745	0.55935
15	-0.45119	0.57538

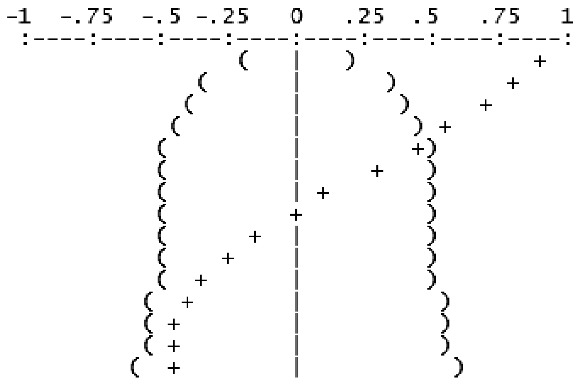

Fig. 1. Calculated autocorrelation of global temperature (y)

LAG	AUTO. CORR.	STAND. ERR.
1	0.86665	0.20851
2	0.71362	0.32983
3	0.57735	0.39125
4	0.44782	0.42668
5	0.32807	0.44665
6	0.21974	0.45701
7	0.12497	0.46158
8	0.03791	0.46305
9	-0.04374	0.46318
10	-0.10986	0.46336
11	-0.18050	0.46449
12	-0.23879	0.46753
13	-0.28814	0.47281
14	-0.33789	0.48038
15	-0.37183	0.49060

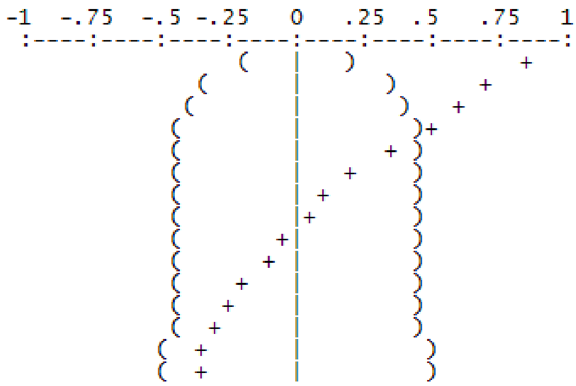

Fig. 2. Calculated autocorrelation of global carbon dioxide (x_2)

LAG	AUTO. CORR.	STAND. ERR.
1	0.07631	0.20851
2	-0.78648	0.20973
3	-0.35533	0.31269
4	0.65964	0.32978
5	0.43470	0.38287
6	-0.40140	0.40376
7	-0.54880	0.42075
8	0.21791	0.45080
9	0.51983	0.45536
10	0.00044	0.48047
11	-0.47905	0.48047
12	-0.13385	0.50080
13	0.36961	0.50236
14	0.22436	0.51404
15	-0.26133	0.51828

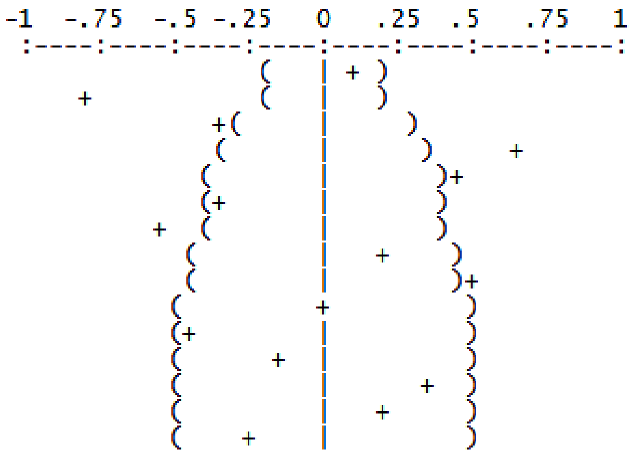

Fig. 3. Calculated autocorrelation of $1/r^2$, x_3

The autocorrelation of y (sample data of global temperature) in Fig. 1 suggests that all sample autovariance c_0 of y are same over different i s, where $i = 2,\ldots,23$; and, sample autocovariances c_i s are becoming smaller when i becomes larger; so, this sample data of y suggests possibilities of both Heteroskedasticity and Autoregressive Process.

The autocorrelation of x_2 (sample data of carbon dioxide) in Fig. 2 also suggests possibilities of both Heteroskedasticity and Autoregressive Process. However, the autocorrelation of x_3, $1/r^2$, in Fig. 3 shows a different pattern of its distribution, in comparison with Fig. 1 and Fig. 2. And, then, because of these observations of autocorrelations, we further tested Generalized Classical Regression Model by regressing y over x_2 and x_3, assuming the following: Pure Heteroskedacity, where the diagonal elements of Σ have different variances, $\sigma_1^2,\ldots,\sigma_2^2,\ldots,\sigma_{23}^2$, and First Order Autoregressive Process, in which the first-order autocovariance, $\hat{\rho} = c_1/c_0$ is not zero, but the same value.

The results of the analysis with Generalized Classical Regression Model in the assumed Pure Heteroskedasticity and the assumed First-Order Autoregressive Process are shown from Table 7 to Table 11.

Here, it is noted that for b) First-Order Autoregressive Process, at first, we calculated the residuals $e = Y - \hat{Y}$, and $\sum_{i=1}^{23} e_i$ to see if the sum of the residuals is zero. And, then, we knew $\sum_{i=1}^{23} e_i = 4.52104 \times 10^{-11}$, which is small enough to assume as it is zero. And, then, we regressed e_i on e_{i-1} (across $i = 2,\ldots,23$ in time-series), and then, we got $\hat{\rho} = 0.90997$.

Table 7. Matrix $Q = X'X$ in Generalized Classical Regression Model

Pure Heteroskedasticity			First-Order Autoregressive Process		
23,00000	$1,34369\cdot10^{2}$	$6,96610\cdot10^{3}$	0,17832	$2,82472\cdot10^{2}$	$1,35613\cdot10^{-12}$
$1,34369\cdot10^{2}$	$8,07005\cdot10^{2}$	$4,06960\cdot10^{4}$	$2,82472\cdot10^{2}$	$4,62382\cdot10^{5}$	$2,14549\cdot10^{-9}$
$6,96610\cdot10^{3}$	$4,06960\cdot10^{4}$	$2,10987\cdot10^{6}$	$1,35613\cdot10^{-12}$	$2,14549\cdot10^{-9}$	$1,03352\cdot10^{-23}$

Table 8. Matrix $X'Y$ in Generalized Classical Regression Model

Pure Heteroskedasticity	First-Order Autoregressive Process
55,25547	$7,97115\cdot10^{-2}$
$3,43513\cdot10^{2}$	$1,26456\cdot10^{2}$
$1,67350\cdot10^{4}$	$6,06319\cdot10^{-13}$

Table 9. Matrix $b^* = Q^{-1}X'Y$ in Generalized Classical Regression Model

Pure Heteroskedasticity*		First-Order Autoregressive Process
for 1, x_1	-9,72055	0,37507
for Carbon dioxide, x_2	0,94202	$1,36503\cdot10^{-5}$
for $(1/r^2)$, x_3	$2,18557\cdot10^{-2}$	$6,61708\cdot10^{9}$

* With this model, R^2 = 0,88602, which is as same as R^2 of the classical regression model in Table 4.

Table 10. Matrix $V(b^*) = \frac{e'e}{n-k} Q^{-1}$ in Generalized Classical Regression Model

Pure Heteroskedasticity			First-Order Autoregressive Process		
$5{,}24998 \cdot 10^{2}$	-0,11178	-1,73122	0,62342	$-3{,}49710 \cdot 10^{-5}$	$-7{,}45413 \cdot 10^{10}$
-0,11178	$5{,}71097 \cdot 10^{-3}$	$2{,}58920 \cdot 10^{-4}$	$-3{,}49710 \cdot 10^{-5}$	$1{,}37856 \cdot 10^{-8}$	$1{,}72693 \cdot 10^{6}$
-1,73122	$2{,}58920 \cdot 10^{-4}$	$5{,}71096 \cdot 10^{-3}$	$-7{,}45413 \cdot 10^{10}$	$1{,}72693 \cdot 10^{6}$	$9{,}44181 \cdot 10^{21}$

Table 11. Coefficients and standard errors of the coefficients in Generalized Classical Regression Model

Variable	Pure Heteroskedasticity		First-Order Autoregressive Process	
	Coefficient	Standard error	Coefficient	Standard error
for 1 (x_1)	-9,72055	22,91283	0,37507	0,78957
for Carbon dioxide (x_2)	0,94202	$7{,}55710 \cdot 10^{-2}$	$1{,}36503 \cdot 10^{-5}$	$1{,}17412 \cdot 10^{-4}$
for $(1/r^2)$ (x_3)	$2{,}18557 \cdot 10^{-2}$	$7{,}55709 \cdot 10^{-3}$	$6{,}61708 \cdot 10^{9}$	$9{,}71690 \cdot 10^{10}$

The adjusted coefficients of Table 11, which were calculated by the assumption of Pure Heteroskedasticity in Generalized Classical Regression Model, suggest that x_3 $\left(\frac{1}{r^2}\right)$ influenced y (global temperature) less than x_2 (carbon-dioxide) did; while, the standard error of the estimated coefficient of x_3 is almost equal to x_2's. On the other hand the assumption of First-Order Autoregressive Process suggests that x_3 $\left(\frac{1}{r^2}\right)$ influenced y (global temperature) more than x_2 (carbon-dioxide) did; while, the standard error of the estimated coefficient of x_3 is larger than x_2's.

In order to further investigate the relation between x_3 $\left(\frac{1}{r^2}\right)$ and y (global temperature), we also analyzed the same data set by Nonlinear Regression Model of Cobb-Douglas function. The result is shown in Table 12.

Table 12. Coefficients of Cobb-Douglas model, $y = b_1 x_2^{\,b_2} x_3^{\,b3}$

Coefficient	Estimated coefficient	Standard error
b_1 coefficient of 1	0,000103	0,02761
b_2 coefficient of x_2	2,126546	0,23431
b_3 coefficient of x_3	0,283107	10,62035

The estimated coefficients of nonlinear Cobb-Douglas function show that the coefficient of x_2 is larger than the coefficient of x_3. This result suggests that the carbon dioxide is more influential to the global warming, than $\frac{1}{r^2}$, if the global temperature is to be described by the Cobb-Douglas function.

ANALYSIS OF THE CALCULATED RESULTS

We cannot measure Moon's gravitational wave; while the general theory of relativity only suggests that it includes dimension of $\frac{1}{r^2}$, where r is a distance between Moon and Earth in kilometers. The result of the Least Squares Estimation of Linear Classical Regression Model suggests that the influence of Moon's gravitational wave to the global warming is large; however, the standard error of the estimated coefficient is also large. On the other hand, the autocorrelations of the global temperature, in time-series, suggests that the process of the global warming could be explained by its own history, which could be also influenced by carbon dioxide and gravitational wave from Moon. However, as shown in Fig. 4, the distribution of $\frac{1}{r^2}$ is cyclic in time-series because Moon rotates on oval orbit around Earth; while the distributions of global temperature and carbon dioxide are proportional to the time-series as Fig. 1 and Fig. 2 show. And then we assumed that Moon's gravitational wave could disturb the process of the global warming; and, then we tried to measure the order of magnitude of the assumed disturbance by Moon to Earth (global temperature), with two assumptions in the Generalized Classical Regression Models: Pure Heteroskedasticity and First-Order Autoregressive Process, and one Nonlinear Model.

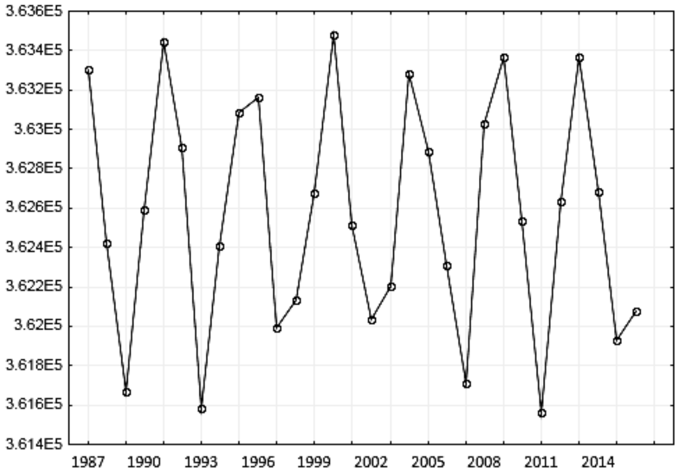

Fig. 4. Distance between Moon and Earth

The results of First-Order Autoregressive Process of Generalized Classical Regression Model suggests large disturbance of Moon to the process of global warming, which is as same as the result of Least Squares Estimation of Classical Regression Model; although, the results of the analysis with the assumptions of Pure Heteroskedasticity and Nonlinear Model suggest the opposite.

The reasons of these differences, which are observed in analysis in these four models, are supposed to be related to the nature of Moon's movement on the oval orbit, which gives larger variance and covariance, which are taken in different ways by different models.

CONCLUSION AND RECOMMENDATIONS

We assumed that the gravitational wave from Moon to Earth influenced the global temperature of Earth; and, then, the result of the Least Squares Estimation of Classical Regression Model suggested such effect to exist. However, we also found that the calculated standard error of the estimated coefficient of the gravitational wave was large.

And, then, we examined Generalized Classical Regression Model, to see if the magnitude of standard error changes, by assuming Pure Heteroskedasticity and First Order Autoregressive Process, which added more different variances and covariances in the regression models. The results indicated that the expected influence of Moon's gravitational wave was large, while the standard-error was large with the assumption of First Order Autoregressive Process; while, the expected influence was small and its standard error was also small when Pure Heteroskedasticity is assumed. However, we don't know if the assumption of Pure Heteroskedasticity is appropriate for modeling Moon's rotational movement.

Also, we tested the nonlinear Cobb-Douglas function to simulate the impacts from Moon's gravitational wave and carbon dioxide to the global warming, and the result showed more influence of carbon dioxide. However, we don't' know any reasonable theory to justify the nonlinear function, yet, rather we examined it, only to observe how the coefficients change in comparison with those of Least Squares Estimation of Classical Regression Model.

Upon above observations, we cannot deny our assumption that Moon's gravitational wave could disturb the process of global warming, yet; while, the results also suggest that uncertainty exists because of Moon's rotational movement, which is different from the processes of rising global temperature and carbon dioxide.

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