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Representation of solutions for fractional kinetic equations with deviation time variable

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ABSTRACT

We present a formula for classical solutions for time- and space-fractional kinetic equation (also known as fractional diffusion equation) and deviation time variable is given in terms of the Fox's H-function, using the step by step method. This equations describe fractal properties of real data arising in applied fields such as turbulence, hydrology, ecology, geographic, air pollution, economics and finance.

Keywords: fractal, diffusion equation, Fox's H-function, deviation variable, step by step method

1. INTRODUCTION

In recent years it has turned out that many phenomena in engineering physics, chemistry and other sciences can be described very successfully by models using mathematical tools from fractional calculus.

The concept of fractal is one of fundamental paradigms of modern theoretical and experimental physics, radio physics and radar, and fractional calculus is the mathematical basis of fractal physics, geothermal energy and cosmic electrodynamics.

Fractional differential equations have gained importance and popularity mainly due to exact description of nonlinear phenomena, especially in fluid mechanisms, e.g. nano-hydrodynamics, where continuum assumption does not well, and fractional model can be considered to be a best candidate. Hence great attention has been given to finding solutions of fractional differential equations. Although the mathematical formalism of fractional calculus is now well developed, the widespread use of fractional integrals (CI) and the fractional derivatives is constrained by lack of clear physical interpretation of them. A direct link between the CI and the Cantor's fractal set has been shown [1]. If the total number of remaining states at each stage of the partition of the set is normalized to unity, the share of the remaining states, ν, included in the CI index is exactly equal to the fractal dimension of the Cantor set ν, and $0 < \nu < 1$.

New mathematical models of various transport process of substances in porous media that have a fractal structure are investigated in book [2], such as the movement of ground water, soil moisture, and salt, evolution of small perturbations in channels with fractal walls, the dynamics of the micro-meteorology regime under irrigating large areas. The results on the analytic theory of heat and mass transfer are represented in [3], with a view to the development of computational technics to determine the fluxes of matter and heat at the interface, including the presence of chemical reactions. Last but not least, the concepts of fractal geometry have entered recently in optics, where they have been successfully used for classifications and characterization of rough surfaces and solving numerous related applied problems [4-6]. The experimental results of the study of statistical, correlations and fractal parameters, which characterize the real component of the Jones-matrix image of polycrystalline networks of flat layers of the main types of human amino acids, are presented in [7]. The use of fractional calculus in mathematical modeling of non-local process has been studied by A. M. Nakhushev [8-9], V. A. Nakhusheva [10], Y. Z. Povstenko [11-14]. It has been noted [9] that the fractional differential and integral calculus in the theory of fractals and systems with memory becomes as important as the classical analysis in physics (mechanics) continua. Thus, fundamental research on non-local problems for pseudo-differential equations is well-timed and relevant. The Cauchy problem has been extensively investigated for equations of fractal diffusion

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containing regularized fractional time derivative of the variable and the second-order derivatives in the space variables [15-17]. Details description of the studies dealing with the non-local boundary value problems are represented in the monograph [18]. As has been stated [19], in recent years, fractional reaction-diffusion models are studied due to their usefulness and importance in many areas of science and engineering. The reaction-diffusion equations arise naturally as description models of many evaluation problems in the real world, as the chemistry [20, 21], biology [22], problems in finance [23-25] and hydrology [26]. Burke at [27] obtained solutions for enzyme-suicide substrate reaction with an instantaneous point source of substrate. In 1993 Grimson and Barker [28] introduced a continuum model for the spatio-temporal growth of bacterial colonies on the surface of a solid substrate with utilizes a reaction-diffusion equation for growth. Many cellular and sub-cellular biological processes [29] can be described in terms of diffusing and chemically reacting species (e.g. enzymes). A traditional approach to the mathematical modelling of such reaction-diffusion processes is to describe each biochemical species by its (spatially depend) concentration. In recent time interest in fractional reaction-diffusion equation [30-36] has increased because the equation exhibits self-organization phenomena and introduces a new parameter, the fractional index into the equation. Additionally, the analysis of fractional reaction-diffusion equations is of great importance from the analytical and numerical point of view.

2. FRACTIONAL KINETIC EQUATION

We consider the following fractional kinetic equation with deviation time variable

$$\frac{\partial^\beta u(t,x)}{\partial t^\beta} + \mu(1-\Delta)^{\gamma/2}(-\Delta)^{\alpha/2}u(t,x) = f(t,x,u(t-h,x)) \tag{1}$$

$t > h$, $x \in \mathbf{R}^n$, $\mu > 0$

subject to the nonlocal initial condition

$$u(t,x)\,|_{0\le t\le h} = u_0(t,x),\ x\in\mathbb{R}^n, \tag{2}$$

where $u = u(t, x)$, $0 < t \le T$, $x \in \mathbf{R}^n$, is the kinetic field and $\beta \in (0, 1)$, $\gamma \ge 0$, $\alpha > 0$ are fractional parameters, Δ is the n-dimential Laplace operator. In [37] present a spectral representation of the mean-square solution of the fractional kinetic equation with random initial condition. The operator $-(I - \Delta)^{\gamma/2}$, $\gamma \ge 0$, and $(\Delta)^{\alpha/2}$, are interpreted as inverses of the Bessel and Riesz potentials respectively (see [37]).

The time derivative of order $\beta \in (0, 1]$ is defined as follows

$$\frac{\partial^\beta u(t,x)}{\partial t^\beta} = \begin{cases} \dfrac{\partial u(t,x)}{\partial t}, & if\ \beta = 1, \\ (D_t^\beta u)(t,x), & if\ \beta \in (0,1), \end{cases}$$

where

$$(D_t^\beta u)(t,x) = \frac{1}{\Gamma(1-\beta)}\left[\frac{\partial}{\partial t}\int_h^t (t-\tau)^{-\beta}u(\tau,x)d\tau - \frac{u(h,x)}{(t-h)^\beta}\right], 0 < t \le T$$

is regularized fractional derivative or fractional derivative in the Caputo-Djrbashian sense (see [37, p.3]). The fractional Laplace define as

$$(-\Delta)^{\alpha/2}u(t,x) = F^{-1}_{\xi\to x}[|\sigma|^\alpha F_{x\to\xi}[u(t,x)]],$$

and $F_{x\to\sigma}[u(t,x)] = v(t,\sigma)$, $F^{-1}_{\sigma\to x}[\mathrm{v}(t,\sigma)]$ denote the Fourier and inverse Fourier transforms of u respectively (interpretation as hypersingular integrals [15, 16]).

We prove the solvability of the Cauchy problem (1), (2) using the step by step method. We denote the space of k-times continuously differentiable functions by C^k and $C^0 := C$. The Riemann-Liouville fractional integral of order $\alpha \geq 0$ is defined for $\alpha > 0$ as

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau = (g_{\alpha} * f)(t),$$

where

$$g_{\alpha}(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$$

is the Riemann-Liouville kernel and * denotes the convolution in time.

Definition. Let $0 < \beta \leq 1$, $\gamma \geq 0$, $0 < \alpha \leq 2$. Suppose $u_0 \in C([0,\infty)\times\mathbf{R}^n)$, $f \in C([0,\infty)\times\mathbf{R}^n; C([0,\infty)\times\mathbf{R}^n))$. Then a function $u \in C([0,\infty)\times\mathbf{R}^n)$ is a classical solution of the Cauchy problem (1), (2) if

1) $F^{-1}_{\sigma\to x}[(1+|\sigma|^2)^{\gamma/2}|\sigma^2|]F_{x\to\sigma}[u(t,x)]$ defines a continuous function of $x \in \mathbf{R}^n$ for each $t > 0$;

2) for every $x \in \mathbf{R}^n$, the fractional integral $J^{1-\alpha}u$, is continuously differentiable with respect to $t > 0$, and

3) the function $u(t, x)$ satisfies the integro-partial differential equation of (1) for every $(t, x) \in (h, \infty) \times\mathbf{R}^n$ and the initial condition (2) for every $(t, x) \in (0, h) \times\mathbf{R}^n$.

In the non-deviation time variable situation the fractional-in-time diffusion equations, with formally corresponds to (1) with $\gamma = 0$, $\alpha = 2$, has been studied by many authors (see references in [37]). In [38] prove the solvability of the Cauchy problem for a quasilinear pseudodifferential equation with fractal derivative with respect to time t of order $\alpha \in (0, 1)$, second derivative with respect to spatial argument x and deviation time variable using step by step method.

For (1) where the pseudodifferential operator $Au(t, x)$ is define by a nonsmooth symbol $a(\sigma) = (1+|\sigma|^2)^{\gamma/2}|\sigma|^{\alpha}$ the posed problem with condition (2) is solved in the present paper for the first time.

In the class of rapidly decreasing functions, a pseudo-differential operator is defined by the formula

$$(Au)(t,x) = (2\pi)^{-n} \int_{\mathbb{R}^n} e^{i(x,\sigma)} a(\sigma) \hat{u}(t,\sigma) d\sigma, x \in \mathbb{R}^n, t > 0,$$

$$\tilde{u}(t,\sigma) = \int_{\mathbb{R}^n} e^{-i(\sigma,y)} u(t,y) dy, \sigma \in \mathbb{R}^n, t > 0.$$

3. STEP METHOD

By the step method, we reduce the Cauchy problem for a pseudo-differential equation with deviating argument to the Cauchy problem for an equation with nondeviating argument.

Let $h < t \leq 2h$, $x \in \mathbf{R}^n$, and let $f(t, x, u_0(x, t-h)) \equiv f_0(t, x, h)$. Then $u(x, t-h) = u_0(x, t-h)$ and problem (1), (2) takes the form

$$\frac{\partial^{\beta} u(t,x)}{\partial t^{\beta}} + (Au)(t,x) = f_o(t,x,h), \; x \in \mathbf{R}^n, \; h < t \leq 2h, \quad (3)$$

$$u(t,x)|_{t=h} = u_0(h,x), \; x \in \mathbf{R}^n. \quad (4)$$

In terms of the Fourier transform, we get the following problem:

$$\frac{\partial^{\beta}\tilde{u}(t,\sigma)}{\partial t^{\beta}}+a(\sigma)\tilde{u}(t,\sigma)=\tilde{F}(t,\sigma,h)\text{ , } \sigma\in \mathbf{R}^n,\ h<t\le 2h, \tag{5}$$

$$\tilde{u}(t,\sigma)\,|_{t=h}=\tilde{u}_0(t,\sigma)\text{ , } \sigma\in \mathbf{R}^n, \tag{6}$$

where

$$\tilde{F}(t,\sigma,h)\equiv\frac{1}{\Gamma(1-\alpha)}(t-h)^{-\beta}\tilde{u}_0(h,\sigma)+\tilde{f}(t,\sigma,h)\text{ , } h<t<2h,\ \sigma\in \mathbf{R}^n,$$

$$F(u)\equiv\tilde{u}(t,\sigma)\text{ , } h<t<2h,\ \sigma\in \mathbf{R}^n,$$

$$\tilde{f}(t,\sigma)=\int_{\mathrm{R}^n}\exp\{-i\sigma x\}f(t,x,u_0(t-h,x))dx\text{ , } h<t<2h,\ \sigma\in \mathbf{R}^n.$$

The solution of the Cauchy problem (5), (6) we will found in form

$$u(\mathrm{t},\sigma)\equiv I_t^{\beta}v(t,\sigma)=\frac{1}{\Gamma(\alpha)}\int_h^t\frac{v(\tau,\sigma)}{(t-\tau)^{1-\beta}}d\tau\text{ , } h<t<2h,\ \sigma\in \mathbf{R}^n. \tag{7}$$

When we substitute (7) in (5) we will get integral equation

$$v(t,\sigma)=\tilde{F}(t,\sigma,h)+\int_h^t\frac{a(\sigma)}{\Gamma(\beta)(t-\tau)^{1-\beta}}v(\tau,\sigma)d\tau\text{ ,} \tag{8}$$

$h<t<2h$, $\sigma\in \mathbf{R}^n$, where function

$$K_1(t,\ \sigma)\ \equiv\frac{a(\sigma)}{\Gamma(\beta)t^{1-\beta}} \tag{9}$$

is its kernel and with the help of (9) repeated kernels and resolventa

$$K_n(t,\sigma)=\frac{(-1)^n a(\sigma)^{2n}}{\Gamma(n\beta)t^{1-n\beta}}\text{ , } n\in \mathbf{N},$$

$$R(t,\sigma)=\sum_{n=1}^{\infty}\frac{a(\sigma)^n}{\Gamma(n\beta)}t^{n\beta-1}, \tag{10}$$

are constructed.

Function

$$E_{\beta,\delta}(t)=\sum_{k=0}^{\infty}\frac{t^k}{\Gamma(\beta k+\delta)}\text{ ,}$$

$t>0$, $\beta>0$, $\delta>0$ is the Mittag-Leffler function of order β and type δ. Then function $R(t,\ \sigma)$ from (10)

$$R(t,\sigma)=t^{-1}\nu E_{\beta,\beta}(\nu)\text{ ,}$$

where $\nu=a(\sigma)t^{\beta}$, $t>0$.

The solution of integral equation (8) is

$$v(t,\sigma)=\tilde{F}(t,\sigma,h)+\int_h^t R(t-\tau,\sigma,h)\tilde{F}(\tau,\sigma,h)d\tau \equiv \tilde{F}(t,\sigma,h)+(R*\tilde{f})(t,\sigma,h)\,, \qquad (11)$$

$t > h,\ \sigma \in \mathbf{R}^n$. Since the inequality

$$\frac{d}{dt}=E_{\beta,1}(-a(\sigma)t^{\beta})=\sum_{k=0}^{\infty}\frac{\beta k(-a(\sigma)t^{\beta})^{k-1}t^{\beta-1}}{\Gamma(\beta k+1)}=\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n}{\Gamma(n\alpha)}t^{n\beta-1}\equiv R(t,\sigma)$$

is true after term differentiation of series for $E(-a(\sigma)t)$ then we can present function $v(t,\sigma)$ as

$$v(t,\sigma)=\int_h^t \frac{d}{dt}E_{\beta,1}(-a(\sigma)(t-\tau)^{\beta})\tilde{F}(\tau,\sigma)d\tau+\tilde{F}(t,\sigma,h)\,.$$

When we act on (11) operator I_t^{β} we will get $\tilde{u}(t,\sigma)$. Indeed

$$I_t^{\beta}(R*\tilde{F})=\frac{1}{\Gamma(\beta)}\int_h^t\frac{d\tau}{(t-\tau)^{1-\beta}}\int_h^{\tau}\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n\tilde{F}(\lambda,\sigma,h)}{\Gamma(n\beta)(t-\lambda)^{1-n-\beta}}d\lambda=$$

$$=\frac{1}{\Gamma(\beta)}\int_h^t\left(\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n}{\Gamma(n\beta)}\int_{\lambda}^t\frac{1}{(t-\tau)^{1-\beta}}\frac{1}{(t-\lambda)^{1-n\beta}}d\tau\right)\tilde{F}(\lambda,\sigma,h)d\lambda\,.$$

In inner integral by variable τ we replace variable τ by μ according to the formula $t-\tau=\mu(t-\lambda)$. Then

$$\int_{\lambda}^t\frac{d\tau}{(t-\tau)^{1-\beta}(t-\lambda)^{-n\beta}}=\frac{1}{(t-\lambda)^{1-n\beta-\beta}}\int_0^1\frac{d\mu}{\mu^{1-\beta}(1-\mu)^{1-n\beta}}=$$

$$=(t-\lambda)^{(n+1)\beta-1}B(\beta,n\beta)=\frac{\Gamma(\beta)\Gamma(n\beta)}{\Gamma((n+1)\beta)}(t-\lambda)^{(n+1)\beta-1}$$

So,

$$I_t^{\alpha}(T*\tilde{F})=\int_h^t\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n(t-\lambda)^{n\beta}}{\Gamma(n\beta+\beta)(t-\lambda)^{1-\beta}}\tilde{F}(\lambda,\sigma,h)d\lambda\,.$$

If we add $I_t^{\beta}\tilde{F}(t,\sigma,h)$ here and take into account that $I_t^{\beta}v(t,\sigma)\equiv\tilde{u}(t,\sigma)$, we get the formula

$$\tilde{u}(t,\sigma)=\int_h^t\frac{1}{(t-\tau)^{1-\beta}}\left[\frac{1}{\Gamma(\beta)}+\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n(t-\tau)^{n\beta}}{\Gamma(n\beta+\beta)}\right]\tilde{F}(\tau,\sigma,h)d\tau\ \equiv$$

$$\equiv\int_h^t\frac{E_{\beta\beta}(-a(\sigma)(t-\tau)^{\beta})}{(t-\tau)^{1-\beta}}\tilde{F}(\tau,\sigma,h)d\tau\,, \qquad (12)$$

$t > h,\ \sigma \in \mathrm{R}^{n}$.

If consider that

$$\tilde{F}(t,\sigma,h)\equiv\frac{1}{\Gamma(1-\beta)}(t-h)^{-\beta}\tilde{u}_0(h,\sigma)+\tilde{f}(t,\sigma,h),\ t>h,\ \sigma\in\mathrm{R}^n,$$

then from (12) we get

$$\tilde{u}(t,\sigma)=\int_h^t \frac{E_{\beta,\beta}(-a(\sigma)(t-\tau)^{\beta})}{\Gamma(1-\beta)(t-\tau)^{1-\beta}(t-h)^{\beta}}d\tau\tilde{u}_0(h,\sigma)+\int_h^t \frac{E_{\beta,\beta}(-a(\sigma)(t-\tau)^{\beta})}{(t-\tau)^{1-\beta}} f(\tau,\sigma,h)d\tau \equiv$$

$$\equiv Q_1(t,\sigma,\beta,h)\tilde{u}_0(h,\sigma)+\int_h^t Q_2(t-\tau,\sigma)\,\tilde{f}(\tau,\sigma,h)d\tau. \qquad (13)$$

For function $Q_1(t, \sigma, h)$ from (13) if expressing $E_{\beta\beta}(-a(\sigma)(t-\tau)^{\beta})$ through series and change the order of summation and integration, then

$$Q_1(t,\sigma,\alpha,h)=\frac{1}{\Gamma(\beta)\Gamma(1-\beta)}\int_h^t \frac{d\tau}{(-\tau)^{1-\beta}(t-h)^{\beta}}+$$

$$+\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n}{\Gamma(1-\beta)\Gamma(n\beta+\beta)}\int_b^t \frac{(t-\tau)^{n\beta}}{(t-\tau)^{1-\beta}(\tau-h)^{\beta}}d\tau\,.$$

Since

$$\int_h^t \frac{d\tau}{(\mathrm{t}-\tau)^{1-a}(t-h)^{1-b}}=\frac{\mathrm{B}(a,b)}{(t-h)^{1-a-b}}=(t-h)^{a+b-1}\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

then when $a = \beta$, $b = 1 - \beta$ and $a = n\beta + \beta$, $b = 1 - \beta$ then we get that first and second integrals equals to $\Gamma(\beta)\Gamma(1-\beta)$ and $(t-h)^{n\beta}\dfrac{\Gamma(n\beta)\Gamma(1-\beta)}{\Gamma(n\beta+1)}$ respectively. Then

$$Q_1(t,\sigma,\beta,h)=1+\sum_{n=1}^{\infty}\frac{(-a(\sigma))^n(t-h)^{n\beta}}{\Gamma(n\beta+\beta)\Gamma(1-\beta)}\frac{\Gamma(n\beta+\beta)\Gamma(1-\beta)}{\Gamma(n\beta+1)}=$$

$$=1+\sum_{n=1}^{\infty}\frac{(-a(\sigma)(t-h)^{\beta})^n}{\Gamma(n\beta+1)}=E_{\beta,1}(-a(\sigma)(t-h)^{\beta})\,, \qquad (14)$$

$t > h$, $\sigma \in \mathrm{R}^{\boldsymbol{n}}$. Second term in (13) is expressed using the Mittag-Leffler function $E_{\beta,1}$ from (14) as formula

$$Q_2(t,\sigma,\beta,h)=D_t^{1-\beta}E_{\beta,1}(-a(\sigma)t^{\beta})=\frac{1}{\Gamma(\beta)}\frac{d}{dt}\int_h^t \frac{E_{\beta,1}(-a(\sigma)\tau^{\beta})}{(t-\tau)^{\beta}}d\tau\,, \qquad (15)$$

So we have proved

Theorem 1. The solution of Cauchy problem (5), (6) takes the form (13), where the functions Q_1, Q_2 defines by the equalities (14) and (15) respectively.

In [17] proven that function $E_{\beta}(-a(\sigma)t^{\beta})$ has a Fourier transform, so the functions

$$G_i(t,x,\beta,h)=F^{-1}_{\sigma\to x}[Q_i(t,\sigma,\beta,h)]\,,\ i = 1, 2,$$

exist. This functions was investigate in [37].

When $\beta = 1$, $\gamma = 0$, $\alpha = 1$ for the first time

$$G(t,x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\pi^{\frac{n+1}{2}}} \frac{\mu t}{((\mu t)^2 + |x|^2)^{\frac{n+1}{2}}}$$

is defines in [39, p. 93].

From equality (13) after applying the inverse Fourier transform and theorem on Fourier transform of product we get the formula for solution $u(t, x)$ of Cauchy problem (4), (5) as a sum of convolution

$$u(t,x) = \int_{\mathrm{R}^n} G_1(t, x-\xi, \beta, h) u_0(h, \xi) d\xi + \int_h^t d\tau \int_{\mathrm{R}^n} G_2(t-\tau, x-\xi, \beta, h) f(\tau, \xi, h) d\xi \qquad (16)$$

$h \le t \le 2h$, $x \in \mathbf{R}^n$.

The vector-function (G_1, G_2) is called Green function of Cauchy problem (3), (4), moreover $G_2(t, x, a, h) = \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} G_1(t, x, a, h)$.

We prove (similarly as [40, pp. 189-191]) that function $u(t, x)$ defined (16) satisfies the equation (3) and initial condition (4).

4. MAIN THEOREM

Theorem 2. The solution of problem (3), (4) is exists and determined by the formula (19).

The next step is to continue the solution to the interval $kh \le t \le (k + 1)h$, $x \in \mathbf{R}^n$, e.g. construction functions $u_0(kh, \xi)$, $f(t, \xi, kh)$, $G_1(t, x-\xi, \beta, kh)$, $G_2(t-\tau, x-\xi, \beta, kh)$ such that on this interval the solution of corresponding Cauchy problem is described in the form (16) with constructed components. So, we have

Theorem 3. The solution of problem (1), (2) is exists and determined as sum of convolution

$$u(t,x) = \int_{\mathrm{R}^n} G_1(t, x-\xi, \beta, kh) u_0(kh, \xi) d\xi + \int_h^t d\tau \int_{\mathrm{R}^n} G_2(t-\tau, x-\xi, \beta, kh) f(\tau, \xi, kh) d\xi ,$$

$kh \le t \le (k + 1)h$, $x \in \mathbf{R}^n$.

Remarks. When $\beta \in (0, 1)$, $\gamma = 0$, $\alpha = 2$ two-point problem is studied in [40]. The regularized fractional derivatives is defined in [41]. The boundary problem for equation of fractal diffusion with argument deviation is studied in [42].

CONCLUSION

The concept of fractal is one of fundamental paradigms of modern theoretical and experimental physics, radio physics and radar, and fractional calculus is the mathematical basis of fractal physics, geothermal energy and cosmic electrodynamics. Fractional differential equations have gained importance and popularity mainly due to exact description of nonlinear phenomena, especially in fluid mechanisms, e.g. nano-hydrodynamics, where continuum assumption does not well, and fractional model can be considered to be a best candidate. Hence great attention has been given to finding solutions of fractional differential equations.

By the step method we reduce the Cauchy problem for a pseudo-differential equation with deviating argument to the Cauchy problem for an equation with nondeviating argument on interval with length h on t. We constructed the Green's function for the above described problem. The properties of the Green's function allow us to describe the solution of this problem as the sum of the convolutions of the fundamental solution with the data of the problem. Using the method of steps the obtained solution is continued to an arbitrary numerical interval and we present a formula for classical solution for time and space-fractional kinetic equation (also known as fractional diffusion equation) and deviation time variable.

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