Inquiry-based learning for enhancing students' interest in mathematical research: a case study on approximation theory and Fourier series

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Abstract
This paper investigates how to develop students' interest in mathematical research by using inquiry-based learning (IBL) as a pedagogical approach. We conducted a case study on the application of IBL to the teaching of approximation theory and Fourier series, which are important topics in mathematics and computer science. We surveyed the students who participated in the IBL workshops and measured their emotional state using the Differential Emotion Scale (DES) by Izard. The results showed that the IBL environment reduced the students' negative emotions and increased their positive emotions, which in turn enhanced their engagement and motivation in the mathematical research activities. We conclude that IBL is an effective method for fostering students' interest in mathematical research and suggest some implications for future practice and research.

Keywords
inquiry-based learning, mathematical research, approximation theory, Fourier series, emotional state, differential emotion scale

1. Introduction

Higher education aims to develop scientific competencies in future professionals and academics, which are essential for their successful career advancement. Research activities are one of the mechanisms to foster such competencies, as they enable students to create new methods, ideas, and approaches that meet the changing demands of the modern world [1, 2, 3]. This is especially relevant for mathematical education, where research activities can enhance students'

3L-Person 2022: VII International Workshop on Professional Retraining and Life-Long Learning using ICT: Person-oriented Approach, October 25, 2022, Kryvyi Rih (Virtual), Ukraine
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performance and creativity in their professional or academic endeavors [4, 5, 6, 7, 8]. Therefore, the issue of organizing research activities in Mathematics remains a topical challenge in pedagogical studies.

However, traditional teacher-centered methods do not facilitate active student involvement in research activities [9]. According to the reports of the European Association for Quality Assurance in Higher Education, the European University Association, and the Higher School Teachers European Society [10], the effectiveness of developing students' research skills depends on the choice of learning strategy that prioritizes student-centered methods. This is related to the diversity and increasing expectations of higher education, which require fundamental changes in its delivery and focus on flexible ways of engaging students in research activities. One of the methods that implements such an approach is inquiry-based learning, which has been widely adopted in many countries [11, 12, 13]. Inquiry-based learning is based on the student-centered paradigm, where students construct their learning and knowledge acquisition in a similar way as scientists do [14]. In Mathematics, this is emphasized by Sandoval and Reiser [15], Jahnke et al. [4], Artigue and Blomhøj [16], Dorier and Maass [17], who argue that inquiry is one of the most important contexts for learning mathematics. Thus, the issue of organizing research activities in Mathematics through inquiry-based learning aligns with the current trends and demands of higher education.

Many researchers have highlighted the need to support active student research activities. Lithner [18] noted that an international trend in Mathematics education is to acquire mathematical knowledge not only in terms of context but also in terms of developing skills related to conducting mathematical research. Bonwell and Eison [19], cited in [20], stated that students should do more than just listen. They should read, discuss, and investigate problems. Jones et al. [21] affirmed that it is necessary to foster creative thinking and investigative skills in students at every level of their university education. Scholars stress that engaging students in research activities during their studies promotes the development of research competence, which is vital for solving practical problems and for adapting quickly to the changing conditions of the modern era and enhancing their skills continuously. We also considered the views of Dreyfus et al. [22], who regarded research activities during Mathematics learning as a natural part of the educational process, which aims to develop research competence among students.

According to Yore [9], the formation of interest in research activities is the first stage during the development of research competencies while learning Mathematics. This idea is consistent with the conclusions by Hernandez-Martinez and Vos [23], who have described the critical state of the matter to form students' interest in research activities. Scientists emphasized the importance of organizing students' activities, the formation of their positive attitude to research projects, and the use of inquiry-based learning as a pedagogical approach.

Inquiry-based learning is a student-centered method that involves posing questions, problems, or scenarios and encouraging students to explore them using their own prior knowledge and scientific facts. Inquiry-based learning also requires students to restructure their previous ideas about the scientific concept by adding new studied information, take into account each other, monitor and evaluate their own learning, and transfer new knowledge into a real context [24].

Approximation theory is a branch of mathematics that deals with finding simple functions that approximate complex or unknown functions. It has many applications in various fields such as computer science, engineering, physics, biology, and economics. For example, approximation theory can be used to compress data, model natural phenomena, optimize systems, and solve differential equations [25, 26, 27].

In order to organize practice-focused research activities using approximation theory, scientists offer to use special courses dedicated to special scientific researches in the priority areas of modern Mathematics. This fact is evidenced by the opinion of Yarullin et al. [28], Biza et al. [29], Telegina et al. [30] about the significant potential in the researches on forming a positive attitude to students' research activities using the materials of different mathematical branches. In scientists' opinion, the use of interesting mathematical theories encourages students to get a more meaningful education of theoretical materials, facts, and methods of solving mathematical problems and it allows getting particular experience. We can also meet the confirmation of this opinion in the works by Matejko and Ansari [31], Sevinc and Lesh [32], who investigated the organization of research activities related to particular branches of Mathematics.

The idea caught on, that is why guided by the conclusions made by the above-mentioned scientific researches we decided to research the formation of students' interest in research activities on Mathematics through the implementation of practice on approximation theory following inquiry-based learning. The choice of this branch results from its extensive use in practice. This is explained by the fact that the modern stage of science and technology development is characterized by the use of a considerable amount of information. As experience shows this tendency will only enhance in the future – the development of computer science, telecommunication, and registration equipment lead to steady growth of the data amount. Therefore, the tools and methods of their processing and analysis are growing. The creation of a single methodical approach based on general mathematical principles is actual for several tasks such as to get, model, register, and process data. The series finds a mass use as a tool to represent a considerable class of functions, carrying out analytical transformations, approximate calculations in many applied tasks. Algorithmic and computer software that is created on their basis is characterized by high universality and is included in computer and hardware-computer complexes of different purposes.

The research is aimed at investigating the effects of inquiry-based learning and approximation theory on students' interest in research activities on mathematics, as measured by a self-report questionnaire and a performance-based assessment.

2. Method

At the first stage of the research, we used a survey method to assess students' interest in Mathematics research activities. We used the Differential Emotions Scale by Izard [33] to survey students. The relevance of involving this methodology to assess students' interest in research activities is proven by the researches where the direct dependency between the subject's interest in cognitive activities and their emotional state during its implementation is emphasized. Since the feeling is a dynamic component of the emotion (Panksepp [34]) and two psychobiological processes are connected with it – fascination and individuation (Langer [35]), motivating, managing, and informative functions of feelings allow them to capture or simplify and organize the thing that can become (especially in difficult situations) a great number of impulses in concentrated cognitive processes. During 2015–2019 we surveyed master's degree students of Physics-Mathematics departments of Kryvyi Rih State Pedagogical University and Berdyansk State Pedagogical University. 49 master's students took part in the survey (17 male students and 32 female students aged from 20 to 28). The use of the online survey, first through Google form, posted on the Internet, and then, moved to the forum of the platform "Higher School Mathematics Teacher" [36] had an advantage in comparison to the survey on paper as it encouraged the respondents' frankness and prevented missed questions.

According to the chosen methodology, we selected the Likert scale to assess each of the basic emotions where 1 – "feeling is completely absent"; 2 – "feeling is slightly expressed"; 3 – "feeling is moderately expressed"; 4 – "feeling is strongly expressed"; 5 – "feeling is fully expressed". At the beginning of the research, the most significant (> 9 points) positive emotion related to the experience of Mathematics research activities was "interest", negative – "shame" and "fear". Students usually face the last two emotions while learning Mathematics.

Students believe that the key problem of learning mathematical theory is the absence of the connection between theory and practice and the abstract character of the subject.

At the second stage of the research, we determined the structure of practice regarding Approximation theory and the main aspects of the content that ensure its correspondence to inquiry-based learning. While selecting resources for the analysis of possibilities to use inquiry-based learning we were focused on those that represent the efficiency of its use during the education. Among them, we can name TeachThought [37], Lesley University [38], The National Academies Board on Science Education [39], Alberta Education [40] (table 1).

Table 1
The analysis of the resources that represent the efficiency of using inquiry-based learning.

Resources	Used while learning a subject	Features	What are the efficiency grounds
Teach Though	Biochemistry and Molecular Biology Education, Mathematics	Joint activities	The solid knowledge foundation through an active part
Lesley University	Mathematics, Life sciences	Constructing knowledge based on experience	Possibility for the full cycle of education
The National Academies Board on Science Education	Biological sciences	Structure and sequence of education are directed at creating a challenging situation	Integration of learning activity with laboratory experience
Alberta Education	Librarianship, work with information	Student's involvement in metacognition; encouragement of critical and creative thinking	Focus on achieving defined learning outcomes in different subjects

We also found out what the purpose of using inquiry-based learning by other scientists was. Cheng et al. [41] noted the efficiency of using the approach to increase the motivation of students' learning. Duran and Duran [42] describe the use of inquiry-based learning in programs of professional development in education. Supasorn and Promarak [43] see the use of inquiry-based learning as an efficient method of improving students' understanding of natural processes.

In conclusions of scientific researches done by Bybee et al. [44], Abdi [45], Ong et al. [46] we also find the confirmation of the efficiency to use the above-mentioned approach to improve students' achievements in science. Considering it, we believe that inquiry-based learning will encourage the alignment of teaching processes with the formation of better students' understanding of scientific knowledge and skills during practice.

The practice program consists of six classes.

1. The history of the development of approximation theory and Fourier series.
2. The ways of periodic function classification.
3. Approximation methods that are based on matrix series summing.
4. Main tasks of approximation theory: approximation of individual function, class approximation, precise, and asymptomatically precise ratio.
5. Examples of researches by subject.
6. Examples of using approximate aggregates in computer complexes of broad purpose.

The practice was aimed at the formation of students' interest in research activities through their implementation in the real process of using series in applied tasks.

The practice was held for a group of 7–8 students twice a month for three months. Every class included two hours of classwork and three hours of extracurricular work. The classes were held by the prominent teachers of Mathematics departments who took part in the development of the practice and looked for the method, the implementation of which would encourage the formation of students' interest in research activities during the practice.

During the organization of practice classes, we developed recommendations for every practice stage that have to encourage the increase in students' interest in mathematics research activities.

At the first stage, the teacher has to determine what students already know regarding the concept that is considered and what kind of knowledge they still need. In order to master new educational material, it is necessary to help students to revise Mathematics sections such as Algebra, Mathematical Analysis, Functional Analysis, and Function Theory. Moreover, at this stage, the teacher is only a consultant who helps students to prepare short reports encouraging students' interest and motivation. For this purpose, the teacher presents the actuality of the researches dedicated to learning approximate features of approximation methods that are generated by certain transformations of partial sums of Fourier series and allow building the sequence of trigonometric polynomials that would equally coincide for any function (table 2).

Table 2
Recommendation for the teacher on the organization of the first stage.

Appropriate	Inappropriate
encourage students to raise their questions offer to compare their ideas with others	read the lecture give definitions to terms explain or give tasks

The second stage is aimed at strengthening students' activities regarding knowledge and skills. At this stage, students can revise the tasks that use the methods of Approximation theory on special subjects that they learn. As a rule, students cite examples of tasks on periodic signal approximation in the theory of control engineering, pattern recognition, nondestructive testing, etc. Students can discuss and write down approximation methods in every particular case. The teacher is only a consultant who offers students such research methods as observation, hypothesis generation, forecasting. Students' communication and work in groups without the direct teacher's involvement are encouraged to equally coincide for any function (table 3).

Table 3
Recommendation for the teacher on the organization of the second stage.

Appropriate	Inappropriate
encouragement of search for several ways to solve the problems comparison of ideas self and mutual survey	use of traditional explanation implementation and involvement of a great amount of terminology

At the next stage, students can describe their point of view regarding the search for solving extreme problems of approximation theory. After this, the teacher has to introduce common terminology and acquaint the students with the general scheme of researching integral images of trigonometric polynomial variations that are generated by linear methods of summing Fourier series, from periodic functions. Generating students' new ideas on methods of approximation improvement, their comparison with the ideas of the previous stage is possible. At this stage, the teacher also has to prevent possible mistakes while explaining misconceptions that could arise at the stage of engagement and exploration. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes (table 4).

After getting an explanation about the research main scheme regarding integrated images of trigonometric polynomial variations during the classes of periodic functions it is important to involve students in further research activities. Further work includes significant analytical calculations connected with exact and approximate methods. Starting from the integral image students can learn asymptotic behavior of exact upper bounds of deviations of trigonometric polynomials from periodic functions to infinity. The stage is aimed at helping students to develop a deeper understanding of general methods of mathematical analysis and the use of approximation processes in practical tasks. Students can carry out additional researches, develop new approximation methods, exchange ideas, and use acquired research experience to integrate Approximation theory in practice (table 5).

The practice of working in small groups is important at this stage. The lessons include planning and preparation of students' proper development on using the considered approximation methods from every group of students. It is possible to create an algorithmic and program-algorithmic product based on the created methods. As the simplest and at the same time the most natural example of a linear process of approximation of continuous periodic functions of the real variable can be the approximation of these functions using the sequence

Table 4
Recommendation for the teacher on the organization of the third stage.

Appropriate	Inappropriate
teacher's explanation expression of the ideas using generally accepted terms idea review and formation of new ones	forming a great amount of terminology focus on independent work

Table 5
Recommendation for the teacher on the organization of the fourth stage.

Appropriate	Inappropriate
enhancement of understanding through strengthening the ideas acquired by experience use of algorithms that are close to new situations grounds for conclusions support of forming student's proper ideas	development of the ideas that are not connected with previous experience generating a great number of ideas without deepening in the essence of the theory

elements of partial sums of Fourier series, the greater majority of students have a basic idea about the techniques of using these methods while creating an algorithmic product. But, as it is well known, the sequences of partial sums of Fourier series $S_n(f;x)$ are not equally similar for the entire class of continuous periodic functions. Thus, a considerable number of students' developments in this area are directly dedicated to the learning of approximate features of other approximation methods that are generated by particular transformations of partial sums of its Fourier series for this function and allow building the sequence of trigonometric polynomials that would be completely similar for every function [47]. Fejer sums $\sigma_n(f;x)$ are arithmetic averages for the first n of partial Fourier sums for this function and, as it is known, the sequence of polynomials $\sigma_n(f;x)$ equally coincides with its function. Sums of de la Vallee Poussin $V_{n,p}(f;x)$ are a synthesis of sums $\sigma_n(f;x)$ and have approximate features that depend a lot on the parameter p. Trigonometric polynomials $V_{n,p_1,p_2}(f;x)$ that are generated by the repeated use of de la Vallee Poussin summation method are the further synthesis of classical Fourier methods, de la Vallee Poussin and Fejer [48]. Choosing particular parameters p_1 and p_2 these polynomials coincide with the sums $S_n(f;x)$, $V_{n,p}(f;x)$, $\sigma_n(f;x)$. The works of practice participants should be dedicated to the learning of approximate features of such approximation methods showing graphically the advantages of its use (figure 1, 2). For the visualization, students can be recommended a system of computer mathematics Maple that includes developed graphic means.

The demonstration of the efficiency of the selected approximation methods can be done by comparing the results of numerical experiments held simultaneously for the operators $S_n(f;x)$, $V_{n,p}(f;x)$ and $V_{n,p_1,p_2}(f;x)$. Meanwhile, it is necessary to pay students' attention to the fact that the aggregate of all the harmonicas that are used to build the operators $S_n(f;x)$;

$$s[f] = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n n^3 \sin nx$$

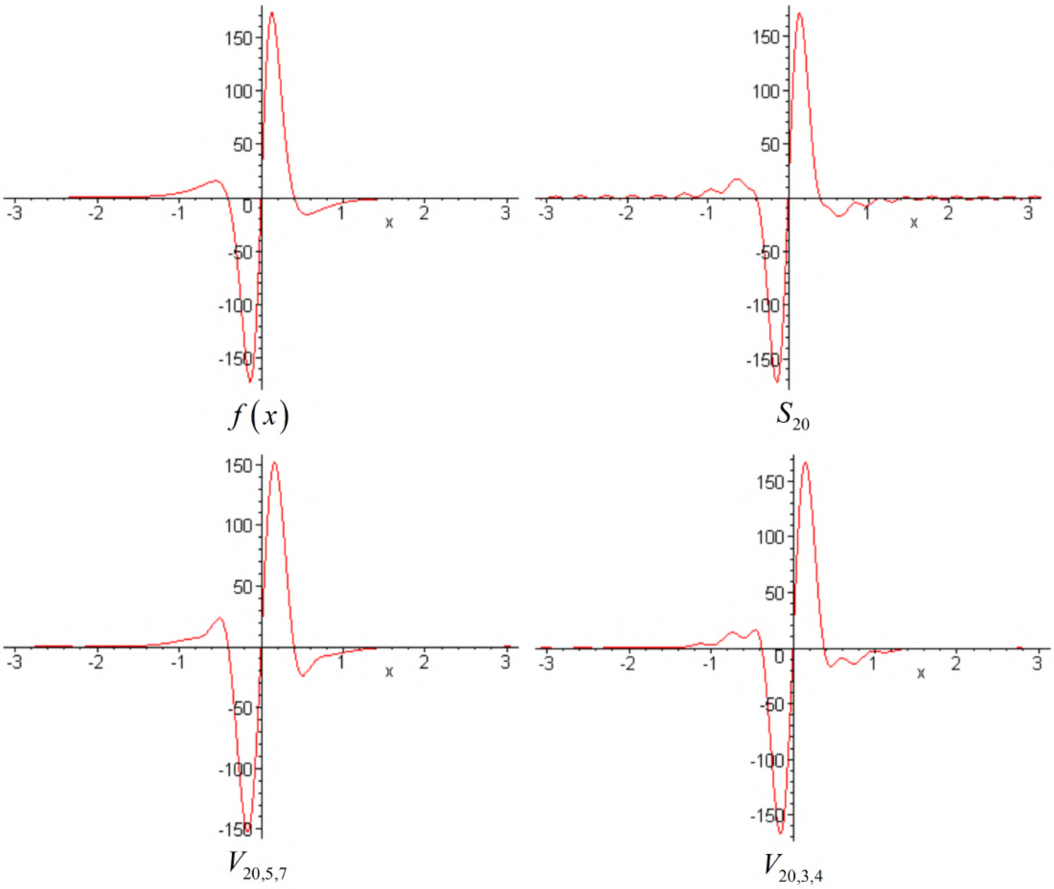
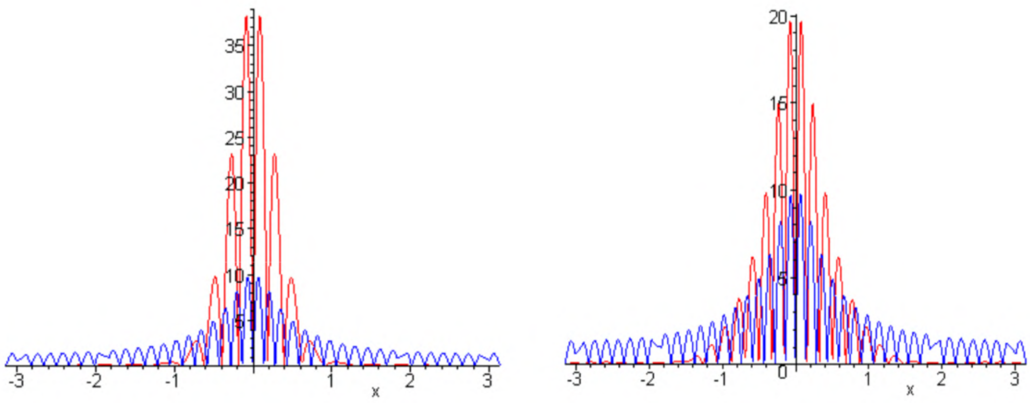


Figure 1: Visualization of functions and trigonometric sums that are generated in different methods of summarizing Fourier series in the system of computer mathematics Maple.

$V_{n,p}(f;x)$ coincides with a similar aggregate for the operator $V_{n,p_1,p_2}(f;x)$. At the same time, the program for the numeric implementation of the values $S_n(f;x)$, $V_{n,p}(f;x)$ and $V_{n,p_1,p_2}(f;x)$ can be developed using Python. This tool is easy to use for students–non-programmers and is suitable for easy calculations.

The final stage of practice is dedicated to evaluation. Evaluation is considered to be a permanent process during which the teacher only observes the students and supports them during report presentations, idea introduction, and question tasks. The use of peer assessment is relevant. Such a form of evaluation can be complemented by students' self-assessment of their level. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes (table 6).

The use of inquiry-based learning does not oblige the teacher to strictly follow the indicated stages. If necessary, it is possible to repeat them several times (Bybee and Landes [49]). This



$|f - S_{20}|, \quad |f - V_{20,5,7}|$ $\qquad$ $|f - S_{20}|, \quad |f - V_{20,3,4}|$

Figure 2: Visualization of deviation of Fourier series and repeated de la Vallee Poussin repeated series from the function $f(x)$ in the system of computer mathematics Maple.

Table 6
Recommendation for the organization of the final stage.

Appropriate	Inappropriate
evaluate the progress in general in comparison to the initial level evaluate the ability to use approximate methods to solve complex problems give students feedback regarding the feasibility of their ideas encourage questions that enhance a deeper understanding of the influence of individual function features on the approximation order	evaluate single facts and separate elements of approximation theory offer a survey in a test form

fact proves the flexibility of using this approach for the implementation of scientific practice.

3. Results

During the preparation stage, we selected the target type as a selection strategy, because the selection had to include the students who have a high achievement level in mathematical branches. By high level, we understand the absence of the final mark “satisfactory” and lower following the national 4–level scale “unsatisfactory”, “satisfactory”, “good”, “excellent” for each of the subjects “Algebra”, “Mathematical analysis”, “Functional analysis” and “Function theory”. The target selected analysis provided us with a sample size n=49 of students that represents 23% of the general number of master’s degree students of the first year during 2015–2019. At the stage of organizing data collection, we used the tool of express-evaluation of positive and negative emotional states called the Differential Emotion Scale (Izard [33]), which ensures diagnostics of a wide range of emotional states. Each of the ten basic emotions (x_i, $i = 1, 2, ..., 10$) is represented by three independent changeable 5–character scales for factors that describe emotional states. The points on every scale correspond to the level of emotional feedback and can be in total from 3 to 15 points. The stage of data analysis of every profile implies the selection of significant (>9 points) emotions, creation of "emotion profile", determination of the dominant emotional state.

At the beginning of the research, the most significant positive emotions regarding the experience of research activities are "interest", negative – "shame" and "fear" (table 7).

Table 7
Distribution of significant emotions at the beginning of the research.

Emotion	Number of students who have this emotion as dominant (>9 points)	Comparison with the general number of students
Interest	32	65.3%
Fear	45	91.8%
Shame	27	55.1%

While processing every profile we defined the indexes of emotional states that characterize the level of subjective students' emotional attitude to the present experience of research activities. The Index of positive emotions and Index of critically negative emotions could range from 9 to 45 points, the Index of anxious–depressive emotions ranged from 12 to 60 points. We defined that the positive emotional state turned out to be dominant among 69.4% of students; a strong level (> 36 points) of expressing a positive emotional state was marked only among 6.1% of respondents. Also, a distinct (from 29 to 36 points) level of positive emotional state was fixed among 10.2% of students. Other students (53.1%) showed moderate (from 20 to 28 points) and weak (< 20 points) level. So, most students' attitude to the research process can be mainly characterized as positive. However, this positive attitude is weakly expressed, unstable, and cannot ensure the proper motivation in overcoming difficulties that inevitably arise during research activities. This fact plays an important (if not the most important) role in the failure of attempts to involve an unprepared student in research activities in any area, including Mathematics.

The dominant critically negative emotional state regarding the present experience of research activities was fixed among 12.2% of respondents, half of whom had a strong (>32 points) or distinct (from 25 to 32 points) level. It is important that among all the students who had the critically negative state as dominant, the factor "Dull" took no less than 4 points, and, accordingly, made the greatest contribution to the calculation. It testifies a stereotype regarding the complexity and absence of interest in research activities among young people. We considered this aspect while searching for methods of practice implementation.

As mentioned above, the emotions "fear" and "shame" were detected as significant among 91.8% and 55.1% of respondents. These emotions are included in the third group of emotions that determine the anxious–negative emotional state of the subject regarding the experience of research activities. Despite this fact, the given state is dominant only among 18.4% of students. It demonstrates that these two emotions influence the formation. 4.1% of respondents have strong (> 30 points) level of emotional state, distinct (from 21 to 30 points) – 10.2%, moderate (from 12 to 20 points) and 4.1% of respondents – weak (< 12 points). Such a noticeable selection of two emotions in the general image of the emotional state confirms the idea that fear and shame prevent students from implementing their interest in the research process and take an active position while conducting research.

The repetitive survey was carried out after finishing the practice. The distribution of significant emotions after taking practice is represented (table 8).

Table 8
Distribution of significant emotions after taking practice.

Emotion	Number of students who have this emotion as dominant (>9 points)	Comparison with the general number of students
Interest	44	89.7%
Surprise	18	36.7%
Fear	39	79.5%
Shame	5	10.2%

Interest turned out to be a significant positive emotion among 44 students. We can note that the number decrease in students who had shame as a significant negative emotion is well seen – 17 respondents. At the same time, the number decrease of students who had fear as a significant emotion is minor – 6 students (figure 3).

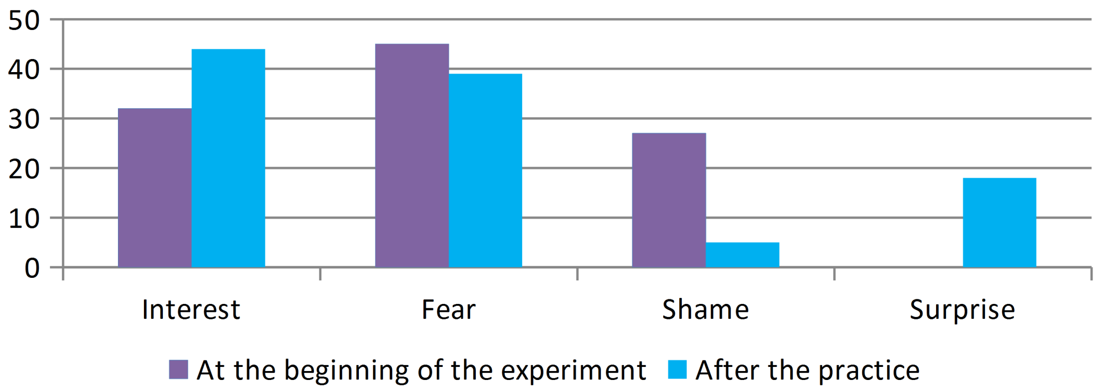


Figure 3: Distribution of a significant emotion.

Despite this fact it is impossible to claim that this emotion in the context of the given research is badly adapted. The profile analysis of respondents' emotions shows the decrease of fear expression to varying degrees among 77.5% of students. The presence of surprise among the significant emotions, as well as interest, which is included in the positive group, is predictable.

More detailed analysis of the feasibility of implementing practice that was carried out using the index calculations of students' emotional states. We detected the increase of students with the dominant positive emotional state up to 81.7%, where 63.2% of respondents had a strong and distinct level. At the beginning of the practice, the same indicator was 16.3%. Thus, we managed to form a stable positive attitude to research activities among more than half of the practice participants.

The number of students who have a critically negative emotional state as dominant remained at the level of 12.2%, though the qualitative structure of this subgroup changed. In our opinion, it is connected with a greater amount of working practice in small groups during classes in comparison to individual work. As teachers pointed out certain students perceived such a format negatively.

The dominant anxious–negative subject's attitude to experience of research activities after taking a practice was fixed among 6.1% of students. Among them 4% of respondents have moderate and 2.1% – weakly expressed level of emotional state. The comparative analysis of the students' number regarding dominant emotional states is displayed (figure 4).

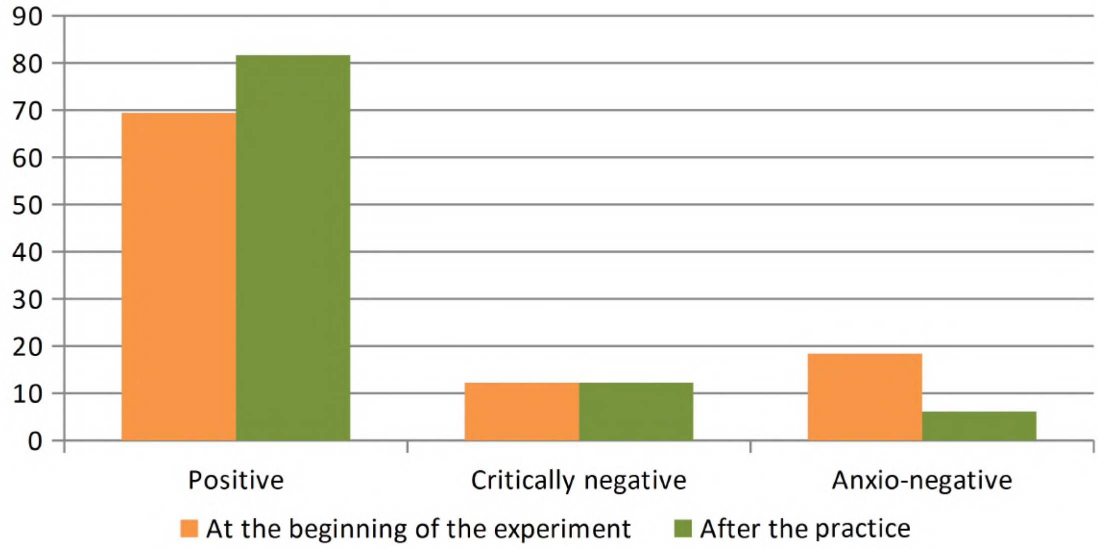


Figure 4: Distribution of dominant states.

The analysis of the results proved that creating the environment based on inquiry-based learning during the scientific practice where students did not feel negative emotions to research activities encouraged the increase of their interest in research activities.

4. Discussion

Searching for ways of forming students' interest in research activities on mathematics we faced the researches done by Sandoval and Reiser [15], Rocard et al. [12]. The scientists point out that in order to form students' impression of the real world it is necessary to show them how to organize their activities as real scientists do during the process of learning and knowledge grounding. Fallon et al. [20] offered to seek the possibilities to organize students' research activities through the method selection and forms of a learning organization that influences active students' involvement.

Traditional educational methods, which are focused on the teacher, don't provide an active students' involvement in research activities [9, 50, 51]. The scientists emphasize the importance of searching for educational models that encourage the strengthening of students' learning activities. The Deductive Content Analysis Method helped us to choose inquiry-based learning as the foundation of developing a scientific environment for students' education.

The efficiency of inquiry-based learning to encourage students' research activities is proved by Duran and Duran [42], Bybee and Landes [49], Supasorn and Promarak [43], Cheng et al. [41]. Also, we support the opinion by Vlasenko et al. [51, 52, 53, 54], who believe that learning has to be built so that students can research, explain, extend and estimate their progress, and the introduction of ideas assumes students' awareness of the reason or necessity of their use. The indicated aims are fully agreed with the content of inquiry-based learning.

Alshehri [55] believes that while organizing research activities it is necessary to direct students to the main models of subject matters. One of the key subject matters of Mathematics is Approximation theory, its broad influence on the modern state of innovation and technology development is widely known. The research is aimed at searching for ways of implementing a practice on Approximation theory to form students' interest in Mathematics research activities. The main research result testifies that the use of the approach inquiry-based learning influenced efficiently the formation of students' positive attitude towards research activities. Within this approach, the involvement of the practice on Approximation theory encouraged the increase of the level of expressing students' positive emotional state (particularly interest, surprise increase) and decrease of anxiety level. These results are agreed with the conclusions by Chin and Lin [56], Abdi [45], Jung et al. [57], Ong et al. [46], who studied the connection between interest growth and a person's emotional state. This justifies the use of methodology Differential Emotions Scale by Izard [33] during the experiment.

5. Conclusion

In this paper, we have explored the use of inquiry-based learning and approximation theory as a way to foster students' interest in research activities on mathematics. We have developed and implemented a practice on approximation theory that follows the principles of inquiry-based learning. We have also measured the effects of this practice on students' emotional states and attitudes towards research activities.

Our results show that inquiry-based learning and approximation theory can create a positive and engaging learning environment that enhances students' interest in research activities. This can help students develop their research competence, which is a key component of professional competence in mathematics and related fields.

However, our research has some limitations that need to be addressed in future work. For instance, we only focused on one branch of mathematics and one type of inquiry-based learning. We also did not compare our practice with other methods or assess its long-term effects on students' performance and motivation. Therefore, future research could extend our work by exploring other branches of mathematics, other forms of inquiry-based learning, and other outcomes of interest.

References

[1] P. P. Nechypurenko, V. N. Soloviev, Using ICT as the Tools of Forming the Senior Pupils' Research Competencies in the Profile Chemistry Learning of Elective Course "Basics of Quantitative Chemical Analysis", in: A. E. Kiv, V. N. Soloviev (Eds.), Proceedings of the 1st International Workshop on Augmented Reality in Education, Kryvyi Rih, Ukraine, October 2, 2018, volume 2257 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2018, pp. 1–14. URL: https://ceur-ws.org/Vol-2257/paper01.pdf.

[2] P. Nechypurenko, S. Semerikov, T. Selivanova, T. Shenayeva, Selection of ICT tools for the development of high school students' research competencies in specialized chemistry training, Educational Technology Quarterly 2021 (2021) 617–661. doi:10.55056/etq.22.

[3] P. P. Nechypurenko, M. P. Chernova, O. O. Evangelist, T. V. Selivanova, Enhancing student research activities through virtual chemical laboratories: a case study on the topic of Solutions, Educational Technology Quarterly 2023 (2023) 188–209. doi:10.55056/etq.603.

[4] H. N. Jahnke, R. Chuaqui, G. Lachaud, D. Pimm, G. A. Goldin, A. H. Schoenfeld, E. M. Bologna, S. Fujimori, D. E. Scott, R. J. Shumway, G. Booker, J. Easley, F. Pluvinage, R. W. Scholz, L. P. Steffe, J. Yates, A. Bessot, L. G. Callahan, R. Hollands, F. K. Reisman, G. Schubring, M. Abdeljaouad, P. S. Jones, J. Rogalski, G. Schubring, D. Woodrow, W. Zawadowski, J. Kilpatrick, H. J. A. Rimoldi, R. Sumner, R. Rees, K. C. Fuson, S. Sato, C. Comiti, T. E. Kieran, G. Steiner, C. Taylor, A. P. French, R. Karplus, G. Vergnaud, E. Esty, G. Glaeser, H. Halbertstam, Y. Hashimoto, T. A. Romberg, C. Keitel, B. Winklemann, R. Lesh, R. R. Skemp, L. Buxton, N. Herscovics, S. J. Bezuszka, K. Hart, Research in mathematics education, in: M. J. Zweng, T. Green, J. Kilpatrick, H. O. Pollak, M. Suydam (Eds.), Proceedings of the Fourth International Congress on Mathematical Education, Birkhäuser Boston, Boston, MA, 1983, pp. 444–545. doi:10.1007/978-1-4684-8223-2_13.

[5] R. Turner, Exploring mathematical competencies, Research Developments 24 (2010). URL: https://research.acer.edu.au/resdev/vol24/iss24/5.

[6] A. Vintere, A. Zeidma, Engineers' mathematics education in the context of sustainable development, in: L. Malinovska (Ed.), Proceedings of 15-th International Scientific Conference Engineering for Rural Development, volume 15, Jelgava, 2016, pp. 1121–1127. URL: http://tf.llu.lv/conference/proceedings2016/Papers/N218.pdf.

[7] J. Proulx, Mathematics education research as study, For the Learning of Mathematics 35 (2015) 25–27. URL: http://www.jstor.org/stable/44382684.

[8] B. Koichu, A. Pinto, Developing education research competencies in mathematics teachers through TRAIL: Teacher-Researcher Alliance for Investigating Learning, Canadian Journal of Science, Mathematics and Technology Education 18 (2018) 68–85. doi:10.1007/s42330-018-0006-3.

[9] L. D. Yore, What is meant by constructivist science teaching and will the science education community stay the course for meaningful reform?, The Electronic Journal for Research in Science & Mathematics Education 5 (2001). URL: https://ejrsme.icrsme.com/article/view/7662.

[10] Standards and Guidelines for Quality Assurance in the European Higher Education Area (ESG), EURASHE, Belgium, 2015. URL: https://www.enqa.eu/wp-content/uploads/2015/11/ESG_2015.pdf.

[11] National Research Council, Inquiry and the national science education standards: A guide for teaching and learning, 2000. doi:10.17226/9596.
[12] M. Rocard, P. Csermely, D. Jorde, D. Lenzen, H. Walberg-Henriksson, V. Hemmo, Science Education NOW: A renewed Pedagogy for the Future of Europe, Office for Official Publications of the European Communities, Luxembourg, 2007. URL: https://www.eesc.europa.eu/en/documents/rocard-report-science-education-now-new-pedagogy-future-europe.
[13] National Research Council, America's lab report: Investigations in high school science, 2006. URL: https://www.nap.edu/catalog/11311/americas-lab-report-investigations-in-high-school-science.
[14] L. Hrynevych, N. Morze, V. Vember, M. Boiko, Use of digital tools as a component of STEM education ecosystem, Educational Technology Quarterly 2021 (2021) 118–139. doi:10.55056/etq.24.
[15] W. Sandoval, B. Reiser, Explanation-driven inquiry: Integrating conceptual and epistemic scaffolds for scientific inquiry., Science Education 88 (2004) 342–375. doi:10.1002/sce.10130.
[16] M. Artigue, M. Blomhøj, Conceptualizing inquiry-based education in mathematics, ZDM 45 (2013) 797–810. doi:10.1007/s11858-013-0506-6.
[17] J.-L. Dorier, K. Maass, Inquiry-based mathematics education, in: S. Lerman (Ed.), Encyclopedia of Mathematics Education, Springer International Publishing, Cham, 2020, pp. 384–388. doi:10.1007/978-3-030-15789-0_176.
[18] J. Lithner, Mathematical reasoning and familiar procedures, International Journal of Mathematical Education in Science and Technology 31 (2000) 83–95. doi:10.1080/002073900287417.
[19] C. C. Bonwell, J. A. Eison, Active Learning: Creating Excitement in the Classroom, Technical Report 1, The George Washington University, School of Education and Human Development, Washington, D.C., 1991. URL: https://files.eric.ed.gov/fulltext/ED336049.pdf.
[20] E. Fallon, S. Walsh, T. Prendergast, An activity-based approach to the learning and teaching of research methods: Measuring student engagement and learning, Irish Journal of Academic Practice 2 (2013). URL: https://arrow.tudublin.ie/ijap/vol2/iss1/2. doi:10.21427/D7Q72W.
[21] K. Jones, L. Black, A. Coles, Marking 21 years of Research in Mathematics Education, Research in Mathematics Education 21 (2019) 1–5. doi:10.1080/14794802.2019.1592336.
[22] T. Dreyfus, M. Artigue, D. Potari, S. Prediger, K. Ruthven (Eds.), Developing Research in Mathematics Education, 1st. ed., Routledge, London, 2018. doi:10.4324/9781315113562.
[23] P. Hernandez-Martinez, P. Vos, "why do i have to learn this?" a case study on students' experiences of the relevance of mathematical modelling activities, ZDM 50 (2018) 245–257. doi:10.1007/s11858-017-0904-2.
[24] L. Mathiassen, Collaborative practice research, in: R. Baskerville, J. Stage, J. I. DeGross (Eds.), Organizational and Social Perspectives on Information Technology: IFIP TC8 WG8.2 International Working Conference on the Social and Organizational Perspective on Research and Practice in Information Technology June 9–11, 2000, Aalborg, Denmark, Springer US, Boston, MA, 2000, pp. 127–148. doi:10.1007/978-0-387-35505-4_9.
[25] H. S. Malvar, Signal processing with lapped transform, Artech House, Norwood, 1992.
[26] A. N. Pankratov, M. A. Gorchakov, F. F. Dedus, N. S. Dolotova, L. I. Kulikova, S. A.

Makhortykh, N. N. Nazipova, D. A. Novikova, M. M. Olshevets, M. I. Pyatkov, V. R. Rudnev, R. K. Tetuev, V. V. Filippov, Spectral analysis for identification and visualization of repeats in genetic sequences, Pattern Recognition and Image Analysis 19 (2009) 687. doi:10.1134/S105466180904018X.

[27] O. Rovenska, O. Novikov, On approximation of classes of analytic periodic functions by Fejer means, Chebyshevskii Sbornik 21 (2020) 218–226. doi:10.22405/2226-8383-2020-21-4-218-226.

[28] I. F. Yarullin, N. A. Bushmeleva, I. I. Tsyrkun, The research competence development of students trained in mathematical direction, International Electronic Journal of Mathematics Education 10 (2015) 137–146. URL: https://www.iejme.com/article/the-research-competence-development-of-students-trained-in-mathematical-direction.

[29] I. Biza, V. Giraldo, R. Hochmuth, A. S. Khakbaz, C. Rasmussen, Research on Teaching and Learning Mathematics at the Tertiary Level, 2366-5947, 1st. ed., Springer International Publishing, 2016. doi:10.1007/978-3-319-41814-8.

[30] N. V. Telegina, S. E. Drovosekov, D. G. Vasbieva, V. L. Zakharova, The use of project activity in teaching mathematics, Eurasia Journal of Mathematics, Science and Technology Education 15 (2019) em1738. URL: https://www.ejmste.com/article/the-use-of-project-activity-in-teaching-mathematics-7695. doi:10.29333/ejmste/108439.

[31] A. A. Matejko, D. Ansari, Contributions of functional Magnetic Resonance Imaging (fMRI) to the Study of Numerical Cognition, Journal of Numerical Cognition 4 (2018) 505–525. URL: https://jnc.psychopen.eu/index.php/jnc/article/view/5825. doi:10.5964/jnc.v4i3.136.

[32] S. Sevinc, R. Lesh, Training mathematics teachers for realistic math problems: a case of modeling-based teacher education courses, ZDM 50 (2018) 301–314. doi:10.1007/s11858-017-0898-9.

[33] C. Izard, Differential Emotions Theory, Springer, Boston, 1977.

[34] J. Panksepp, Damasio's error?, Consciousness & Emotion 4 (2003) 111–134. URL: https://www.jbe-platform.com/content/journals/10.1075/ce.4.1.10pan. doi:10.1075/ce.4.1.10pan.

[35] S. Langer, Mind: An Essay on Human Feeling, Jonhs Hopkins University Press, Baltimore, 1967.

[36] K. Vlasenko, I. Sitak, Higher school mathematics teacher, 2019. URL: http://formathematics.com.

[37] Lesley University Online, What is the 5E model? a definition for teacher, 2017. URL: https://www.teachthought.com/learning/what-is-the-5e-model-a-definition-for-teachers.

[38] Lesley University Online, Empowering students: the 5E model explained, 2019. URL: https://lesley.edu/article/empowering-students-the-5e-model-explained.

[39] R. W. Bybee, The BSCS 5E Instructional Model and 21st century skills, 2009. URL: https://sites.nationalacademies.org/cs/groups/dbassesite/documents/webpage/dbasse_073327.pdf.

[40] Alberta Learning, Focus on inquiry: A teacher's guide to implementing inquiry-based learning, 2004. URL: https://www.academia.edu/9913211/Focus_on_Inquiry_A_Teachers_Guide_to_Implementing_Inquiry-based_Learning.

[41] P.-H. Cheng, Y.-T. C. Yang, S.-H. G. Chang, F.-R. R. Kuo, 5E mobile inquiry learning approach for enhancing learning motivation and scientific inquiry ability of university students,

IEEE Transactions on Education 59 (2016) 147–153. doi:10.1109/TE.2015.2467352.
[42] L. B. Duran, E. Duran, The 5E instructional model: A learning cycle approach for inquiry-based science teaching, The Science Education Review 3 (2004) 49–58. URL: https://files.eric.ed.gov/fulltext/EJ1058007.pdf.
[43] S. Supasorn, V. Promarak, Implementation of 5e inquiry incorporated with analogy learning approach to enhance conceptual understanding of chemical reaction rate for grade 11 students, Chem. Educ. Res. Pract. 16 (2015) 121–132. doi:10.1039/C4RP00190G.
[44] R. W. Bybee, J. A. Taylor, A. Gardner, P. V. Scotter, J. Powell, A. Westbrook, N. Landes, The BSCS 5E instructional model: Origins, effectives, and applications, 2006. URL: https://media.bscs.org/bscsmw/5es/bscs_5e_full_report.pdf.
[45] A. Abdi, The effect of inquiry-based learning method on students' academic achievement in science course, Universal Journal of Educational Research 2 (2014) 37–41. URL: http://www.hrpub.org/journals/article_info.php?aid=944. doi:10.13189/ujer.2014.020104.
[46] E. T. Ong, A. Govindasay, S. M. Salleh, N. M. Tajuddin, N. A. Rahman, M. T. Borhan, 5E Inquiry Learning Model: Its Effect on Science Achievement among Malaysian Year 5 Indian Students, International Journal of Academic Research in Business and Social Sciences 8 (2018) 348–360. doi:10.6007/IJARBSS/v8-i12/5017.
[47] O. Rovenska, Approximation of analytic functions by repeated de la Vallee Poussin sums, Computer Research and Modeling 11 (2019) 367–377. doi:10.20537/2076-7633-2019-11-3-367-377.
[48] O. Novikov, O. Rovenska, Approximation of classes of Poisson integrals by repeated Fejer sums, Lobachevskii J. Math. 38 (2017) 502–509. doi:10.1134/S1995080217030209.
[49] R. Bybee, N. Landes, Science for life and living: An elementary school science program from biological sciences curriculum study, The American Biology Teacher 52 (1990) 92–98.
[50] J.-L. Lin, M.-F. Cheng, Y.-C. Chang, H.-W. Li, J.-Y. Chang, D.-M. Lin, Learning activities that combine science magic activities with the 5E instructional model to influence secondary-school students' attitudes to science, Eurasia Journal of Mathematics, Science and Technology Education 10 (2014) 415–426. doi:10.12973/eurasia.2014.1103a.
[51] K. Vlasenko, O. Chumak, I. Sitak, I. Lovianova, O. Kondratyeva, Training of mathematical disciplines teachers for higher educational institutions as a contemporary problem, Universal Journal of Educational Research 7 (2019) 1892–1900. doi:10.13189/ujer.2019.070907.
[52] K. V. Vlasenko, S. V. Volkov, D. A. Kovalenko, I. V. Sitak, O. O. Chumak, A. A. Kostikov, Web-based online course training higher school mathematics teachers, CEUR Workshop Proceedings 2643 (2020) 648–661.
[53] K. Vlasenko, O. Chumak, V. Achkan, I. Lovianova, O. Kondratyeva, Personal e-learning environment of a mathematics teacher, Universal Journal of Educational Research 8 (2020) 3527–3535. doi:10.13189/ujer.2020.080828.
[54] K. Vlasenko, D. Kovalenko, O. Chumak, I. Lovianova, S. Volkov, Minimalism in designing user interface of the online platform "Higher school mathematics teacher", CEUR Workshop Proceedings 2732 (2020) 1028–1043.
[55] M. A. Alshehri, The impact of using (5e's) instructional model on achievement of mathematics and retention of learning among fifth grade students, IOSR Journal of Research and Method in Education 6 (2016) 43–48. URL: https://www.iosrjournals.org/iosr-jrme/papers/Vol-6%20Issue-2/Version-1/G06214348.pdf.

[56] E.-T. Chin, F.-L. Lin, A survey of the practice of a large-scale implementation of inquiry-based mathematics teaching: from Taiwan's perspective, ZDM 45 (2013) 919–923. doi:10.1007/s11858-013-0546-y.

[57] N. Jung, C. Wranke, K. Hamburger, M. Knauff, How emotions affect logical reasoning: evidence from experiments with mood-manipulated participants, spider phobics, and people with exam anxiety, Frontiers in Psychology 5 (2014) 570. URL: https://www.frontiersin.org/article/10.3389/fpsyg.2014.00570. doi:10.3389/fpsyg.2014.00570.