The Wheel of Mathematics Learning Methods

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Abstract. The article is devoted to the creation of a wheel of mathematics learning methods. The study analyzed how Bloom's taxonomy is used by the educational community. Special attention is paid to the study of the issue of bringing to a single system a set of disparate goals and tasks of learning in new models of education, which provide for the openness of learning to new technologies. Analysis of scientific works and resources helped to determine the structuring of the Wheel of Mathematics Learning Methods (WMLM). The authors of the article identified the areas of didactics that should fill the wheel: Competencies, Motivation, Bloom's Taxonomy, Activities, and Technologies. A structured set of hints for the teacher is presented in the form of 5 sectors of the wheel: Apply, Analyze, Evaluate, Create, Remember Understand. Each sector is a set of constituent components Methods, Forms, Tools, Activities and Active Verbs. The analysis of the results of the survey of 58 respondents proved that Bloom's taxonomy is important both in the selection of teaching methods and means and in the determination of effective forms of organization of educational activities. It was concluded that the idea of developing a methodology wheel should be based on the structuring of a set of tips for the teacher at all stages of activity from planning to implementation of mathematics education.

1. Introduction

When you come across an exciting new method for learning Mathematics, you think: 'This is cool, can I use this method in my classroom?', but then you realize that you need to think about how the use of this method can contribute to the achievement of the educational goals of the program you are teaching. It was this issue and the desire to help Mathematics teachers make the right decisions when choosing teaching methods that led to the emergence of the Wheel of Mathematics Learning Methods (WMLM). The methodology Wheel combined several different areas of didactic thinking in one diagram. In this integrated structure were placed the methods, forms and means of learning related to the educational purpose that they are most likely to serve. This then enabled mathematics teachers to identify the pedagogical place and purpose of their various technology-based learning and teaching activities in the context of their overall course objectives and regarding the wider developmental needs of their students.

What goals and specific tasks does the teacher set for himself before entering the classroom? How are they related? And how to check whether the set goals are being achieved? Bloom asked these questions in the 1950s. Trying to bring a set of disparate goals and tasks to a single system, Bloom created a theory that has been the subject of heated debate and discussion for the sixth time. In his fundamental work 'Taxonomy of Educational Objectives: The Classification of Educational Goals', Bloom [1] tried to construct a hierarchy of educational goals covering the cognitive domain, which would describe step by step the levels of human thinking and the learning tasks that follow from this. Bloom [1] divided the goals of education into three areas: cognitive (requirements for mastering the content of the subject), psychomotor (development of motor, neuromuscular activity) and affective (emotional value area, attitude to the subject). The first taxonomy, covering the cognitive domain, includes six categories of goals with their internal more fractional division.

From Bloom's point of view, learning goals directly depend on the hierarchy of mental processes, such as remembering, understanding, applying, analyzing, evaluating, and creating. At the same time, each level of the cognitive pyramid, according to Bloom, is based on the previous one. The basis of everything is memorization (knowledge), and the highest point of both cognitive abilities and learning goals is the ability to independently evaluate. The idea is clear: without memorization and knowledge, understanding is impossible, without understanding, use is impossible, without mastering the initial levels, analysis and synthesis are impossible, and without all this, one cannot imagine a creative assessment of phenomena and events.

Bloom's taxonomy has become a very important element in the educational community and has been adopted by many educational institutions in the United States and abroad [2], but in the future, due to increasing criticism, it began to be used less. According to scientists, concepts of different orders are mixed in this hierarchy, namely, specific learning results (memorization, understanding, application) and mental operations necessary to achieve these results (analysis, synthesis, evaluation). However, this did not prevent teachers from different countries from using the proposed hierarchy and creating meaningful and systematized tasks aimed at the intellectual development of students. The team of scientists Anderson et al. [3] revised the taxonomy, changing its content and swapping its levels.

After the modernization of the taxonomy, a model called 'Pedagogical Wheel' ('Pedagogical Wheel', from 'iPad) appeared in Carrington's blog [4], in which they found intersection points of the goals of Bloom's taxonomy and options for using useful iPad applications for the appropriate group.

The taxonomy also attracted the attention of scientists Ellerton et al. [5], who showed the use of the Wheel of didactics, focusing on non-trivial mathematical tasks in secondary school. The scientists described the selection of methods, forms and means of learning by young teachers according to pre-selected types of activity with the help of the developed wheel. Scientists presented fragments of classes demonstrating the use of the Wheel in high school practice.

The idea of involving the Wheel during training captured Ping and Hua [6]. The researchers developed a didactic Wheel and used it during elementary school student's mastery of division. To develop the components of the Wheel, scientists used a survey, which helped in the selection of means that can accompany the division process. The involvement of the Wheel during the educational process contributed to positive dynamics in students' learning of division actions.

The place of taxonomy in their developments was found by Bobis et al. [7], who developed a Wheel of mathematical instructions and used it to improve the motivation of high school students and their involvement in the educational process. The development of the Wheel was based on the main types of activity proposed by Bloom's taxonomy. The Wheel also offers technologies that can provide the specified activity.

Kim [8] devoted his study to the involvement of students in certain types of activity. The scientist described his experience in developing the methodology Wheel as a multidisciplinary system. This study embodies the advances from the development and application of an educational model that considers the learning of both the humanities (History, Geography, and Bibliography) and the five domains of STEAM (Science, Technology, Engineering, Art, and Mathematics) [9].

To expand the limits of creative activity of teachers and students, the Intel "Teach to the Future" program is being implemented in different countries [10]. This program has already reached more than 1.75 million teachers from 35 countries around the world. Among the tasks of the program is the promotion of the development of student and teacher skills that involve the use of high-level thinking. This list [11] is also based on Bloom's taxonomy.

So, one can endlessly argue about the values of Bloom's classification of pedagogical goals. But there is an obvious fact – taxonomy does not lose its relevance. Moreover, it is used not only in the framework of traditional education but also in completely new models that provide for the interactivity of learning and its openness to new technologies.

The purpose of our research is to develop a Wheel of Mathematics Learning Methods.

2. Method

To develop the WMLM, we used the analysis of the answers of mathematics teachers and master's students of pedagogical and classical Institutions of Higher Education. 58 respondents took part in the survey. The questionnaire contained 18 questions and was developed using an open online service and posted on the Internet at the link `https://docs.google.com/forms/d/e/1FAIpQLScpCJ6Vq4aiVqvOC-5Ln5LTDgiTOSwlxJohmnsHFBaK75rDfQ/viewform`.

All teachers agreed with the idea that several different areas of didactics should be combined in one diagram, proposed in the form of a wheel. What kind of areas is this? The driving force of any method is motivation. Motivation makes students active and is supported by the use of certain learning technologies. The chosen technologies should provide the main types of activity proposed by Bloom's taxonomy. The given types of activities should be adjusted according to the mathematical competencies that should be formed in students during the mastery of mathematical subjects (figure 1). We consider each area as a sieve through which the process of developing Mathematics learning methods is filtered. Therefore, the specified areas of didactics work like wheels that turn and move each other. There are five such wheels. It is considered in more detail.

2.1. Competencies

Competencies are at the heart of learning design. Mathematical competencies meet the long-term goals of educational programs and ensure the student's employability. The teacher must constantly review the educational program and check how much it contributes to the development of certain abilities. This means that each teacher, taking into account the opinions of employers, must formulate his expectations from students. This can be helped by asking, 'How does what I do meet certain expectations? Can the content I create help students change to meet the expectations of it?'.

2.2. Motivation

Motivation is the most important component of any method. It moves both the student and the teacher, each time providing an answer to the question 'Why am I doing this again?'. By choosing learning outcomes, and selecting types of activities, the teacher designs content and selects learning technologies to ensure student motivation, which was so well presented by Pink in his TEDtalk 'The Puzzle of Motivation' [12].

2.3. Bloom's taxonomy

Bloom's taxonomy is a tool that can help a teacher develop educational goals that ensure the formation of thinking in students, the order of which is constantly growing. It all starts with

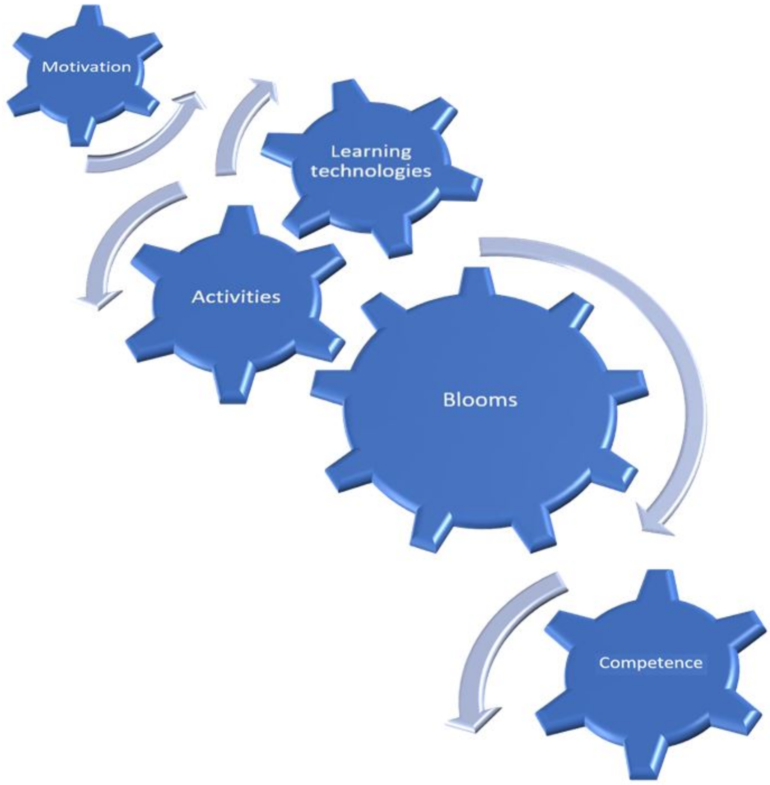

Figure 1. Wheels of didactics.

simple 'memorization and understanding', growing into 'analysis and creation' ensuring the transformation of students. This is made possible by obtaining at least one learning goal from each of the categories that work on the formation of higher-order thinking.

2.4. Activities

The transformation of students' thinking takes place in the process of organizing their activities. The correspondence of students' activities to each type of thinking is ensured through the involvement of special tasks and exercises aimed at students' mastery of knowledge and skills, the development of intellectual and cognitive processes and sustainable professional motivation to achieve success, as well as the creation of a friendly atmosphere that contributes to the disclosure and manifestation of the student's personality.

Having decided on the types of activities, we have been ready to choose learning technologies.

2.5. Technologies

The organization of certain student activities is ensured through the developed technology of teaching and learning (methods, forms, and means). Classical methods of teaching [13] and forms of organization of the learning process are accompanied by the use of learning tools that help create the necessary conditions for achieving the goal of learning. The Wheel offers learning technologies that help sustain activities by stimulating student motivation. The use of the mentioned teaching technologies and Active Verbs has a twofold nature: firstly, the teacher, implementing his educational project, promotes the development of high-level thinking skills in students, and secondly, the teacher can make sure that the implementation of such a project contributes to the development of students.

2.6. How it works

The wheel should be seen as a structured set of prompts that provide opportunities to reflect on teaching, from planning to implementation. These prompts are interconnected, like mechanical mechanisms that ensure the development of methods for learning Mathematics and the selection of its components. During their selection, one should be prepared for the fact that a decision made in one of the areas often affects other decisions (figure 2).

Figure 2. The Wheel of Mathematics Learning Methods.

3. Results

The results of an online survey of Mathematics teachers and master's students of pedagogical and classical higher education institutions, which was carried out during the study, showed the following.

Among the teaching methods that contribute to better memorization of the material by students, teachers could choose not only classical methods according to Lerner and Skatkin [14] but also active (gaming), project methods, etc. Respondents preferred the explanatory and illustrative methods (73.6%), the research method (66%), and the problem-search method (62.3%). However, active methods (business games) received support from almost 50% of respondents.

Regarding the formation of students' understanding of the material, 67.9% of teachers recognized the problem-search method as the most appropriate for this, 62.3% – chose the explanatory and illustrative method, 52.8% – the research method, and the method of building mathematical models was supported by almost 50% of teachers.

In addition, the research method was chosen by the majority of teachers (73.1%) as the most appropriate for forming students' ability to use the material, and the method of building mathematical models was chosen by 53.8%. It is interesting that, along with classical methods, teachers also actively chose project methods (52%) and game methods (52%).

The results of the questionnaire regarding the most popular methods according to the objectives of Bloom's taxonomy of methods are shown in figure 3.

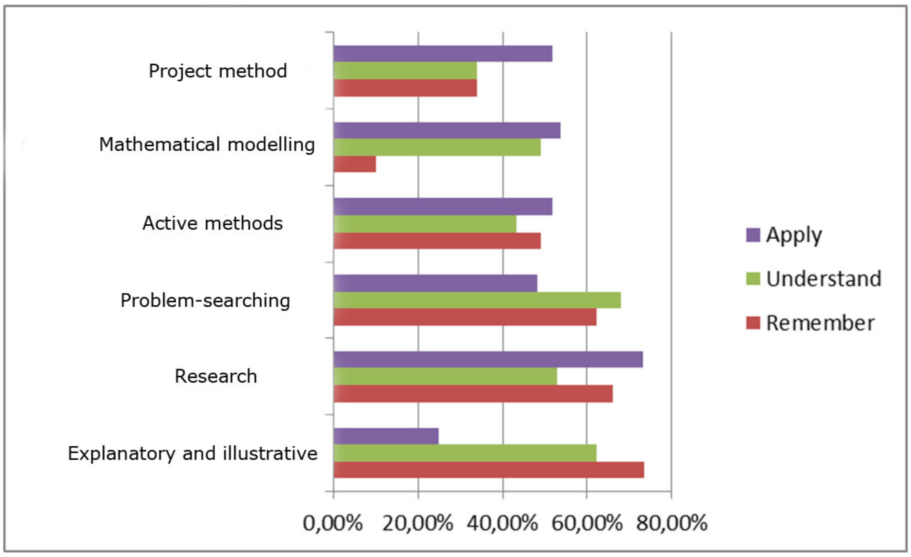

Figure 3. Distribution of teaching methods for the formation of memorization, understanding and application of the material by students.

The problem-search method (76.9%) and logical methods of cognition (75%) were recognized as the most appropriate for the formation of student's ability to analyze and synthesize. It should be noted that the research method was popular among teachers. Thus, it was chosen by 71.2% of teachers as the most appropriate for forming students' ability to evaluate phenomena and events. And logical methods of cognition were chosen by 55.8% of teachers to achieve this goal (figure 4).

As for the teaching means that are most appropriate for students to remember and understand the material, the majority of respondents chose materialized ones (images, diagrams, tables, reference notes), and noted the use of presentations, videos, slide lectures, online calculators, etc. However, most teachers (63.5%) gave preference to intellectual means (educational texts and their media: textbooks, manuals; systems of problems, questions) for the formation of students' ability to use the material. These same means were chosen by the majority of teachers (69.2%)

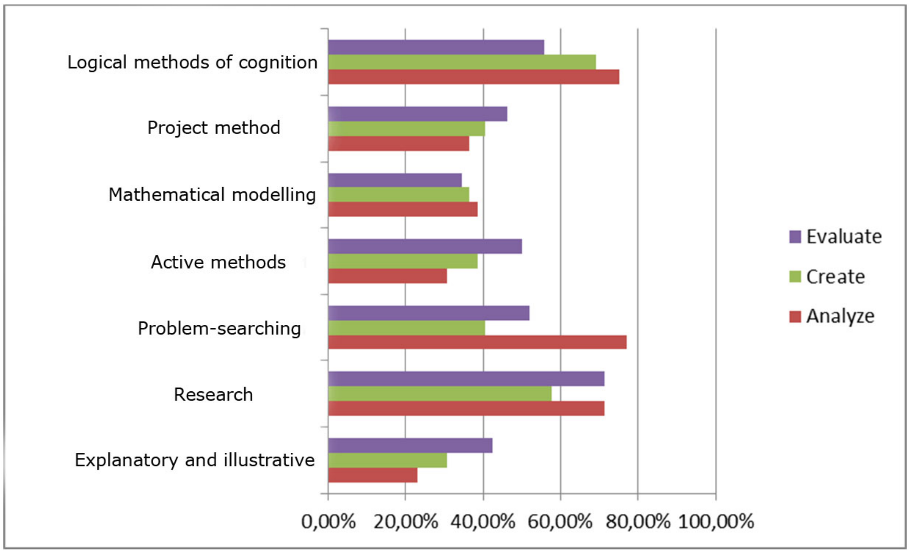

Figure 4. Distribution of teaching methods for the formation of the ability to analyze, synthesize and evaluate phenomena.

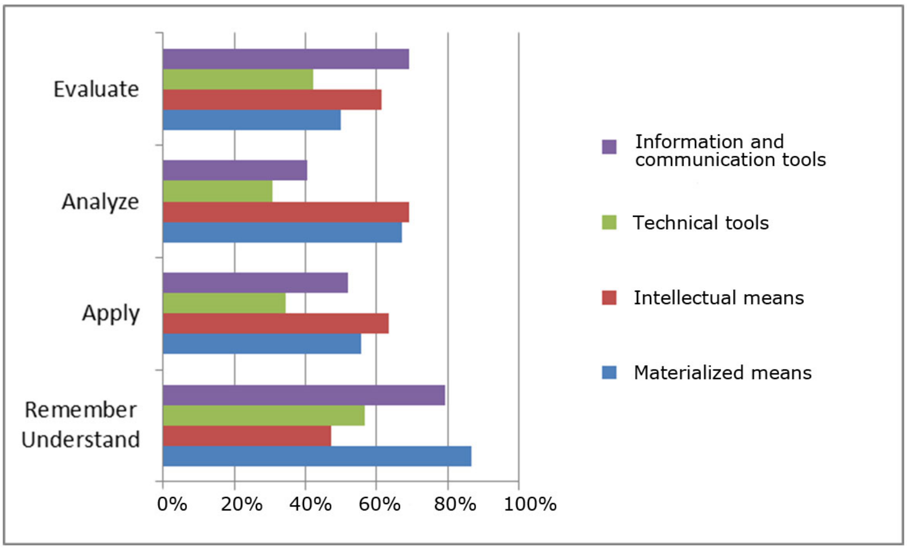

Figure 5. Distribution of learning means to achieve goals according to Bloom's taxonomy.

as the most appropriate for the formation of analytical skills.

According to the majority of respondents, information and communication tools (69.2%) and intellectual means (61.5%) are the most appropriate for forming students' ability to evaluate phenomena and events. The distribution of funds is shown in figure 5.

Organization's forms of educational activities to achieve goals concerning Bloom's taxonomy, according to respondents, were distributed as follows:

- for memorizing the material, individual forms of work were chosen as the most appropriate (66%);
- it was chosen group and pair work (64.2%) and practical and laboratory work (64.2%) to develop students' understanding of the material;

- 92.3% of respondents chose practical and laboratory work to form students' ability to use the material;
- 65.4% chose individual forms of work to develop student's analytical skills;
- practical and laboratory work (72.5%) was chosen as a leader among the forms of organization of educational activities for the formation of synthesis skills in students
- practical and laboratory work (76.9%) received the maximum number of votes for students' ability to evaluate phenomena and events.

Thus, the analysis of the received respondents' answers was laid as the basis of the WMLM.

4. Discussion

Analysis of the research papers of Carrington [4] (on the creation of the 'Pedagogical Wheel' model), Ellerton et al. [5] (on the use of the didactic Wheel), Ping and Hua [6] (about the involvement of the Wheel during learning) confirmed the relevance of the construction of the methodology Wheel, as a way of bringing to a single system a set of disparate goals and tasks of learning Mathematics. During the construction of the methodology Wheel, the authors of these studies decided on a system of principles: humanity and professional orientation, systematicity and flexibility, dynamism and variability. The author's choice of such a system of principles is consistent with Perogonchuk's research (2018) [15], which highlighted the idea of using several scientific approaches during pedagogical modelling [15].

The approach to the development of the WMLM was as follows.

It was chosen the method of analysis of theoretical and practical investigations in the field of development and involvement of the methodology Wheel, as well as the analysis of the answers of Mathematics teachers and students of higher education institutions. The need to develop a questionnaire to identify the needs of teachers and students in the learning process was confirmed by the research of Knowles and Kalata [16], Kebritchi et al. [17], Porter [18]. The analysis of respondents' answers made it possible to single out those areas of didactics that should fill the WMLM, namely: Competencies, Motivation, Bloom's Taxonomy, Activities, Technologies.

The authors of this article presented the WMLM as a structured set of tips in the form of a ring divided into 5 sectors: Apply, Analyze, Evaluate, Create, Remember Understand. The concentric layers of the ring, in turn, define a set of components, such as Active Verbs, Activities, Methods, Forms, and Tools. Thus, the authors propose a two-dimensional lattice in the form of a wheel, with each cell having a double characteristic. Bloom's taxonomy fills the content of each cell according to its characteristics. For example: in a cell characterized by the following Analyze-Methods parameters, we find the following tips corresponding to Bloom's taxonomy: reproductive, problem teaching, partial search, and research.

In this way, the WMLM contains structured tips for teachers, from which it becomes clear: 1) which teaching methods should be selected following the formation of high-level thinking skills according to Bloom's taxonomy; 2) how to decide on the types of means most appropriate for mental activity; such as memorization, understanding, analysis, evaluation of phenomena, etc.; 3) how to choose effective forms of organization of educational activities to achieve goals according to Bloom's taxonomy.

Developing structured tips for Mathematics teachers, the authors of this paper take into account the opinion of Shaikh and Khoja [19] and Vlasenko et al. [20–24], according to whom the teacher's professional activity at all its stages from planning to implementation should be aimed at achieving goals of student development.

5. Conclusions

The analysis of resources and scientific research confirmed the conclusion about the need to bring to a single system a set of different goals and tasks of learning Mathematics in the conditions of applying new education models. The educational community's use of Bloom's classification of educational goals is relevant today. This prompted the authors of this study to create the Wheel of Mathematics learning Methods, a system of structured prompts for the teacher at all stages of learning.

The development of the WMLM consisted of several stages. In the first stage, the authors of the study decided on the areas of didactics that would be presented in the form of a wheel. The results of the survey of Mathematics teachers made it possible to determine the following areas: Competencies, Motivation, Bloom's Taxonomy, Activities, and Technologies. At the next stage, it was determined the content of the 5 sectors of the Wheels. The authors filled each sector (Apply, Analyze, Evaluate, Create, Remember Understand) with tips and hints that allow the teacher to implement Activities, Active Verbs of Bloom's taxonomy and components of the classical methodological system such as methods, forms, and means.

The authors of the article established that Bloom's taxonomy has a direct advantage both at the stage of selecting teaching methods and means or tools and at the time of determining effective forms of organization of educational activities. This allowed the authors of this study to establish types of teaching methods and means that are most effective for the formation of certain high-level thinking skills concerning Bloom, as well as to rank the forms according to their effectiveness in achieving the goals based on Bloom's taxonomy.

The vector of further research is the study of the effectiveness of the WMLM on the results of Mathematics learning of students of higher education institutions.

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