Oksana Bezushchak[1], Bogdana Oliynyk[2]

Steinitz numbers and primary decompositions of unital locally matrix algebras

[1] Taras Shevchenko National University of Kyiv, Kyiv, Ukraine.

[2] National University of Kyiv-Mohyla Academy,
Kyiv, Ukraine.

Let F be a ground field. A unital F-algebra A is said to be a locally matrix if an arbitrary finite collection of elements $a_1, \ldots, a_s$ from A lies in a subalgebra B with 1 of the algebra A, and B is isomorphic to a matrix algebra $M_n(F)$, $n \geq 1$. A unital locally matrix algebra A over a field F is called primary if the subalgebra B from above is isomorphic to a matrix algebra $M_p^n(F)$, $n \geq 1$, where p is a fixed prime number.

We say that the decomposition

$$A = \bigotimes_{p \in \mathbb{P}} A_p$$

of a unital locally matrix algebra A over F is a primary decomposition if each algebra A_p is primary for all $p \in \mathbb{P}$.

We outline the construction of a unital locally matrix algebra of uncountable dimension that does not admit a primary decomposition. It gives negative answers to the question posed in [1]. We also assign a Steinitz number $\mathbf{st}(A)$ to an arbitrary unital locally matrix algebra A and show that for an arbitrary infinite Steinitz number s there exists a unital locally matrix algebra A having the Steinitz number s and being not isomorphic to a tensor product of finite dimensional matrix algebras.

[1] *V. M. Kurochkin. On the theory of locally simple and locally normal algebras.* Mat. Sb., Nov. Ser.*,* **22(64)** *(1948), no. 3, 443–454.*

E-mail: ✉[1] *bezusch@univ.kiev.ua,* ✉[2] *oliynyk@ukma.edu.ua.*