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DETERMINATION OF GROUPS ALL OF WHOSE PROPER SUBGROUPS HAVE A COMMUTATOR SUBGROUP OF ORDER EQUAL OR LESS THAN *p* (*p* > 3): CORRIGENDUM

It is shown that the nonabelian p-groups all of whose proper subgroups have a derived subgroup of order equal or less then p for $p > 2$ is either groups with commutator subgroup of order p or groups, which are either 2-generated group from isoclinism families Φ_3, Φ_6, Φ_8, Φ_{14} or 3-generated groups from isoclinism families Φ_4, Φ_{11}. The misprints and mistakes of a previous paper [1] are corrected[1].

Let $\mathcal{P}$ be a class of finite p-groups, all of whose proper subgroups have a commutator subgroup of order equal or less then p. The obvious examples of such groups are the abelian p-groups and p-groups with commutator subgroup of order p.

In a previous paper [1] the $\mathcal{P}$-groups with a commutator subgroup of order bigger than p for $p > 2$ have been investigated. Next properties have been established.

Let G be a group from $\mathcal{P}$ where $p > 2$ and commutator subgroup G_2 has an order bigger than p. Thus

1. G_2 is an abelian group. It is either elementary abelian of order equal or less then p^3 or a cyclic of order p^2.
2. If G_2 is a cyclic of order p^2 or G_2 is not a subset of the center $Z(G)$ then the group generators minimal number $d(G)$ of the group G is equal 2.
3. The isoclinism families containing these groups are pointed.
4. The description for all such group by the generators and relations is given.

The aim of this note is to correct some mistakes and misprints of [1] and give a more compact description of these groups, which offer a better insight into their structure.

The statement (a) of the Theorem 6 of [1] is incorrect: the 4-generated groups from the family Φ_{12} are not $\mathcal{P}$-groups.

Thus the groups from $\mathcal{P}$ with commutator subgroup of order bigger than p, ($p > 2$) are exhausted by the following:

1) the 2-generated groups of the family Φ_{14}, i.e. all groups with relations
$$G = \langle g_1, g_2 \mid [g_2, g_1] = c,\ [c, g_1] = 1,\ [c, g_2] = 1,\ c^{p^2} = 1,\ g_1^{p^m} = c^{\alpha},\ g_2^{p^n} = c^{\beta} \rangle,$$
where $m \geq n \geq 2$, and α, β are equal to 1, 0 or p;
2) the 2-generated groups of the family Φ_8, i. e. all groups with relations
$$G = \langle g_1, g_2 \mid [g_2, g_1] = c,\ c^{p^2} = 1,\ [c, g_1] = 1,\ [g_2, c] = c^p,\ g_1^{p^m} = c^{\alpha},\ g_2^{p^n} = c^{\beta} \rangle,$$
where $m \geq 1$, $n \geq 2$, and α, β satisfy the following conditions:

[1] Робота частково підтримана Державним фондом фундаментальних досліджень Ф25./157-2008 № ДР 0107U010499 та Міжнародним благодійним фондом відродження Києво-Могилянської академії.

- if $m=1$ then $\alpha=1$ and if $n=2$ then $\beta=0$, else β is equal 0 or p;
- if $m>1$ then $\alpha\equiv 0 \bmod p$, β is equal 0 or p;

3) the 3-generated groups of the family Φ_4, i.e. all groups with the relations

$$G=\langle g_1,g_2,g_3 \mid [g_2,g_1]=z_1, [g_3,g_1]=z_2, [g_2,g_3]=1, [g_i,z_j]=1, [z_1,z_2]=1, z_j{}^p=1, g_i^{p^{n_i}}=z_1^{\alpha_i}z_2^{\beta_i}, (\alpha_i, \beta_i=0..p-1; i=1,2,3; j=1,2)\rangle;$$

4) the 3-generated groups of the family Φ_{11}, i.e. all groups with the relations

$$G=\langle g_1,g_2,g_3 \mid [g_1,g_2]=z_3, [g_1,g_3]=z_2, [g_2,g_3]=z_1, [g_i,z_j]=1, [z_i,z_j]=1, z_j^p=1, g_i^{p^{n_i}}=z_1^{\alpha_i}z_2^{\beta_i}z_3^{\gamma_i}, (\alpha_i, \beta_i, \gamma_i=0..p-1; i,j=1,2,3)\rangle;$$

5) the 2-generated groups of the family Φ_3, i.e. all groups with relations

$$G=\langle g_1,g_2 \mid [g_2,g_1]=c, [c,g_1]=z, [c,g_2]=1, [g_i,z]=[c,z]=1, z^p=c^p=1, g_1^{p^m}=z^\alpha, g_2^{p^n}=z^\beta, (\alpha, \beta=0..p-1; i=1,2)\rangle;$$

6) the 2-generated groups of the family Φ_6, i. e. all groups with relations

$$G=\langle g_1,g_2 \mid [g_1,g_2]=c, [g_1,c]=z_1, [g_2,c]=z_2, [g_i,z_j]=[c,z_j]=[z_1,z_2]=1, z_j{}^p=c^p=1, g_i^{p^{n_i}}=z_1^{\alpha_i}z_2^{\beta_i}, (i,j=1,2,)\rangle,$$

where $\alpha_i, \beta_i=0..p-1$.

Unfortunately, the paper mentioned above contains many misprints appeared through carelessness of editor program and the other technical reasons. We will point main of them.

So, the statement given the structure of the p-group with commutator subgroup on the 3-th line from bottom the page 28 of [1] is misquoted (put $\varepsilon_i=0$ if $n_i=0$ instead $\varepsilon_i=1$ if $n_i\geq 0$).

There are some misprints in the statements of the lemma 2 (put $a, b\in G$ instead $a, d\in G$), Theorem 4(b) (put $a^{p^m}=g^\alpha$ instead $a^{p^m}=g^{p^{\alpha_1}}$), Theorem 6(b) (put Φ_{11} instead Φ_1). Fortunately one can reconstruct the correct meaning here from the text of proof.

Misprints in the text of proof are numerous but can be easy corrected.

[1]. *Čepulić V., Pyliavska O.* Determination of groups all of whose proper subgroups have a commutator subgroup of order equal or less than p, $(p\geq 3)$ // Наукові записки НаУКМА.— Т. 39. Фізико-математичні науки.— 2005.— С. 28–34.

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ГРУПИ, В ЯКИХ КОЖНА ВЛАСНА ПІДГРУПА МАЄ КОМУТАНТПОРЯДКУ НЕ БІЛЬШОГО НІЖ p $(p>3)$: CORRIGENDUM

Показано, що неабелеві p-групи, у яких кожна власна підгрупа має комутант порядку не більшого ніж p (p > 2), вичерпуються p-групами з комутантом порядку p і групами, що мають комутант порядку p^2 і є або 2-породженими групами сімейств ізоклінності Φ_3, Φ_6, Φ_8, Φ_{14}, або 3-породженими групами сімейств ізоклінності Φ_4, Φ_{11}. Виправлено помилки та опечатки статті [1].