ПРОГРЕСИВНІ ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ, ВИСОКОПРОДУКТИВНІ КОМП'ЮТЕРНІ СИСТЕМИ

UDC 519.004.942
DOI: 10.20535/SRIT.2308-8893.2019.3.01

CALCULATING ENERGY DENSITY AND SPIN MOMENTUM DENSITY OF MOON'S GRAVITATIONAL WAVES IN RECTILINEAR COORDINATES

Y. MATSUKI, P.I. BIDYUK

Abstract. In this research the energy density was calculated and the spin momentum density of Moon's gravitational waves in the rectilinear coordinates' system of Moon's gravity and Earth's global temperature. At first, we assumed an action principle that combines the gravitational field and gravitational waves, which formulate a closed system, together with Earth's global temperature. And, then, we calculated the energy densities of those energy field and waves, which are calculated as their variances in the rectilinear coordinates, also to calculate their coefficients and standard errors of the calculated coefficients. The calculated results are consistent with the findings of our previous research [1], which shows the negative contribution of gravitational waves to Earth's global temperature, while the gravitational field positively contributes to the global temperature. We also calculated spin momentum of Moon's gravitational waves in the system of rectilinear coordinates.

Key words: Moon, Earth, global temperature, gravitational field, gravitational waves, rectilinear coordinates, energy density, spin momentum density.

INTRODUCTION

Our previous research [1] showed a relation between Moon's gravitational field and gravitational waves. After that report we continued investigating the characters of gravitational waves, using the same data set, but this time with the theories of relativity and quantum mechanics.

Our mathematical method starts from an action principle, which assumes that there is an action integral that describes the motion of the waves, which must be stationary to be consistent with the law of conservation of energy within the boundary of a closed space. We also assume the rectilinear coordinates of the flat-space for time and space by tensors that represent the energy field as well as the pseudo-tensors that represent the flow of energy.

Dirac [2, 3] predicted that the pseudo-tensor can be built in the coordinates of the tensors' field only when the motion of the gravitational waves, which is expressed by the pseudo-tensors, occurs in one direction. In addition, he also created a basic equation that describes the gravitational field and the other energy flows, such as motion of the gravitational waves, and they can be added together linearly, which agrees with the theory of the special relativity [2].

And then, upon the theory [2, 3], we set up an equation to describe the gravitational waves in the gravitational field. Then we solved the equation to calculate the energy densities and coefficients of the gravitational field, and the gravitational waves toward Earth's global temperature in the space between Moon and Earth. After that we also investigate the character of the gravitational waves by calculating the momentum density of the spin of the gravitational waves.

DATA

Table 1 shows the descriptive statistics of the data, from 1987 till 2009, of the global temperature (increased degree Celsius since 1978) [4], the distance between Moon and Earth (r : kilometers, km) [5], and calculated $\frac{1}{r^2}$ ((kilometers)$^{-2}$, km^{-2}), which we use for our calculations.

Table 1. Descriptive statistics

Variable	**Global temperature (°C) ***	**Distance between Moon and Earth (r : km)**	$\frac{1}{r^2}$ **((km)$^{-2}$)**
Mean	0,29130	$3{,}62618 \cdot 10^5$	$7{,}60509 \cdot 10^{-12}$
Standard deviation	0,12125	$5{,}98411 \cdot 10^2$	$2{,}51097 \cdot 10^{-14}$
Minimum	0,10000	$3{,}61583 \cdot 10^5$	$7{,}56999 \cdot 10^{-12}$
Maximum	0,43000	$3{,}63483 \cdot 10^5$	$7{,}64865 \cdot 10^{-12}$
Skewness	-0,21063	-0,15249	0,15787
Kurtosis	1,29401	1,67498	1,67879
Valid number of observations	23	23	23

* Increased degree Celsius since 1978

CALCULATIONS

Gravitational waves in gravitational field

Dirac [2] created the basic equation for a quantum theory of the Born-Infield electro-dynamics in the rectilinear coordinates, which agrees with the special relativity theory. It defines the action integral of the motions of particles in the electro-magnetic field: $I = \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})d^4x}$, where, $\delta I = \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}$ is the energy density of the electro-magnetic field that provides the action principle of a particle in this energy field, $g_{\mu\nu}$ are fundamental tensors,

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

$F_{\mu\nu}$ are tensors that describe the field quantities of the electro-magnetic field, which are made by two vectors, x_μ and x_ν, while $\mu = 0,1,2,3$ and $\nu = 0,1,2,3$, where each suffix represents each of 4 coordinates of the flat-space. We use this mathematical formula in order to calculate the energy density of the gravitational waves in the gravitational field, and have made $\delta I = \sqrt{Y - \left\{ ax^0 + bx^3 + c\frac{1}{r^2} \right\}}$, where a, b, and c are coefficients that are to be constants; Y is Earth's global temperature as we assume that it is influenced by the motion of gravitational waves; x^0 is the metric that refers the time-coordinate of the flat-space, which is made of a time vector ($=1$), where it is constant. (Because in the theory of the special relativity, nothing can exceed the speed of light; therefore, nothing moves and it is $=1$.) And x^3 is the metric in the flat-space, which we consider as only one direction with a distance r between Moon and Earth.

For calculating the energy density of the gravitational waves, we assume that it is $c\frac{1}{r^2}$; because, we assume that the solutions of $g^{\mu\nu} g_{\rho\sigma,\mu\nu} = 0$ are the gravitational waves traveling with the velocity of light, which satisfies d'Alambert equation [3]. Here, $g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ is made of contravariant vectors, and $g_{\mu\nu}$ are made of their covariant vectors, while $g_{\rho\sigma,\mu\nu} = \frac{\partial^2 g_{\rho\sigma}}{\partial x^\mu \partial x^\nu}$, where x^μ, x^ν are the contravariant vectors that are described in the rectilinear coordinates of the flat-space. Here, μ, ν, ρ, σ are the suffixes that indicate the coordinates of time and space. Because of $g_{\rho\sigma,\mu\nu} = \frac{\partial^2 g_{\rho\sigma}}{\partial x^\mu \partial x^\nu}$, the gravitational waves must have dimension of $\frac{1}{r^2}$. Now we set $H^2 = \left\{ Y - \left(a - br + c\frac{1}{r^2} \right) \right\}^2$.

Now calculate the coefficients a, b and c, with the constraints: $\frac{\partial E(H^2)}{\partial a} = 0$, $\frac{\partial E(H^2)}{\partial b} = 0$, and $\frac{\partial E(H^2)}{\partial c} = 0$, And then the equations are transformed to: $\frac{\partial}{\partial a} E(H^2) = -2E(H) = 0$, with $E(H) = 0$; $\frac{\partial}{\partial b} E(H^2) = -2E(Hr) = 0$, with $E(Hr) = 0$; $\frac{\partial}{\partial c} E\left(H\frac{1}{r^2} \right) = 0$, then $E\left(H\frac{1}{r^2} \right) = 0$, where $E(H^2)$ is the expected value of H^2 and where $L = \left(a - br + c\frac{1}{r^2} \right)$ is Lagrangean. (Note: here in this Lagrangean, the minus-sign of the space coordinate appears as the minus-sign of the coefficient b, because in the special theory of relativity, the geodesic of the time and space is described by the expression: $(dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$, where dx^0 is a displacement of the time-vector, and dx^1, dx^2 and dx^3 are displacements of the space-vectors). And then we can calculate these coefficients: a, b and c, algebraically as follows: at first, we make a matrix $X = \{x_1, x_2, x_3\}$, where $x_1 = 1$ (time-coordinate), $x_2 = r$ (space-coordinate, which is a distance between Moon and Earth), and $x_3 = \frac{1}{r^2}$. Here, the above mentioned constraints, $E(H) = 0$, $E(Hr) = 0$ and $E\left(H\frac{1}{r^2}\right) = 0$ are generally described by the matrices $X'H = 0$, then:

$$Q = X'X = \begin{bmatrix} 23{,}00000 & -8{,}34022 \cdot 10^{6} & 1{,}74917 \cdot 10^{-10} \\ -8{,}34022\text{e} \cdot 10^{6} & 3{,}02432 \cdot 10^{12} & -6{,}34278 \cdot 10^{-5} \\ 1{,}74917 \cdot 10^{-10} & -6{,}34278 \cdot 10^{-5} & 1{,}33027 \cdot 10^{-21} \end{bmatrix},$$

where X' is the transposed matrix of X; $A = Q^{-1}X'$, where Q^{-1} is the inverse matrix (reciprocal matrix) of Q; $\beta = AY$, and where β is the vector of three coefficients, a, b and c; $N = XA$; $M = I - N$, where I is a unity matrix, in which all the diagonal elements are 1, and non-diagonal elements are 0; $e = MY$; $\Sigma = e'eQ^{-1}/(n-k)$, where Σ is the matrix that contains variances and covariances of the variables; e' is the transposed vector of e; n is the number of data (in this analysis 23); and k is the degree of freedom (number of variables, in this analysis $k = 3$).

The results of the calculations are as follows:

$$\beta = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5{,}72334 \cdot 10^{3} \\ 1{,}05217 \cdot 10^{-2} \\ -2{,}50844 \cdot 10^{14} \end{bmatrix};$$

$$\Sigma = \begin{bmatrix} 1{,}26538 \cdot 10^{8} & 2{,}32669 \cdot 10^{2} & -5{,}54473 \cdot 10^{18} \\ 2{,}32669 \cdot 10^{2} & 4{,}27814 \cdot 10^{-4} & -1{,}01952 \cdot 10^{13} \\ -5{,}54473 \cdot 10^{18} & -1{,}01952 \cdot 10^{13} & 2{,}42964 \cdot 10^{29} \end{bmatrix}.$$

Standard errors of the coefficient vector

$$\sigma_b = \begin{bmatrix} \sqrt{1{,}26538 \cdot 10^{8}} \\ \sqrt{4{,}27814 \cdot 10^{-4}} \\ \sqrt{2{,}42964 \cdot 10^{29}} \end{bmatrix} = \begin{bmatrix} 1{,}12489 \cdot 10^{4} \\ 2{,}06837 \cdot 10^{-2} \\ 4{,}92914 \cdot 10^{14} \end{bmatrix}.$$

Spin momentum density of gravitational waves

From the above calculations, we found that the energy of gravitational waves has negative coefficient, $c = -2{,}50844 \cdot 10^{14}$ to Earth's global temperature, while the coefficient of the gravitational field has positive coefficient, $b = 1{,}05217 \cdot 10^{-2}$.

It means that the flow of the gravitational waves doesn't increase the potential energy of this system between Moon and Earth. Then we investigated the character of the gravitational waves, assuming that the vectors of the coordinates formulate the motion of the waves, with the theory made by Dirac [3] upon the equation of the motion of the gravitational waves, $g^{\mu\nu}g_{\rho\sigma,\mu\nu}=0$.

At first, $g_{\mu\nu,\sigma}=u_{\mu\nu}l_{\sigma}$, while $g_{\mu\nu,\sigma}=\dfrac{\partial g_{\mu\nu}}{\partial x^{\sigma}}$, where x^{σ} are the contravariant vectors that describe the coordinates of time and space, and μ, ν, σ are the suffixes that indicate those coordinates; while we analyze only one direction of the space, μ, ν, σ= 0, or 3, where 0 is for time, and 3 is for one direction of the space. Also, we put $u_{\mu\nu}u^{\nu\mu}=u_{\mu}{}^{\mu}=u$, where $u^{\mu\nu}$ are contravariant two-vector tensors and $u_{\mu\nu}$ are covariant two-vector tensors, and $u_{\mu\nu}=u_{\nu\mu}$; here l_{σ} are constants, which satisfy $g^{\rho\sigma}l_{\rho}l_{\sigma}=0$. Then it was assumed that the gravitational waves are traveling in the empty space where only the gravitational field exists, and then this condition leads to $g^{\mu\nu}u_{\mu\rho}l_{\nu}=\frac{1}{2}g^{\mu\nu}u_{\mu\nu}l_{\rho}=\frac{1}{2}ul_{\rho}$, and then we get $u_{\rho}{}^{\nu}l_{\nu}=\frac{1}{2}ul_{\rho}$.

Now, when $\rho=0$, we have:

$$u_{\rho}{}^{\nu}l_{\nu}=\sum_{\nu=0}^{3}u_{0}{}^{\nu}=u_{0}{}^{0}l_{0}+u_{0}{}^{1}l_{1}+u_{0}{}^{2}l_{2}+u_{0}{}^{3}l_{3}=u_{0}{}^{0}+0+0-u_{0}{}^{3}=$$
$$=g^{00}u_{00}-g^{33}u_{03}=u_{00}-(-1)u_{03}=u_{00}+u_{03}=\frac{1}{2}ul_{\rho}=\frac{1}{2}ul_{0}=\frac{1}{2}u\,,$$

where $l_0=1$, $l_1=0$, $l_2=0$ and $l_3=-1$.

When $\rho=1$, then

$$u_{\rho}{}^{\nu}l_{\nu}=\sum_{\nu=0}^{3}u_{1}{}^{\nu}=u_{1}{}^{0}l_{0}+u_{1}{}^{1}l_{1}+u_{1}{}^{2}l_{2}+u_{1}{}^{3}l_{3}=u_{1}{}^{0}+0+0-u_{1}{}^{3}$$
$$=g^{00}u_{10}-g^{33}u_{13}=u_{10}-(-1)u_{13}=u_{10}+u_{13}=\frac{1}{2}ul_{1}=0\,.$$

When $\rho=2$, we have:

$$u_{\rho}{}^{\nu}l_{\nu}=\sum_{\nu=0}^{3}u_{2}{}^{\nu}=u_{2}{}^{0}l_{0}+u_{2}{}^{1}l_{1}+u_{2}{}^{2}l_{2}+u_{2}{}^{3}l_{3}=u_{2}{}^{0}+0+0-u_{2}{}^{3}=$$
$$=g^{00}u_{20}-g^{33}u_{23}=u_{20}-(-1)u_{23}=u_{20}+u_{23}=\frac{1}{2}ul_{2}=0\,.$$

When $\rho=3$, then we have

$$u_{\rho}{}^{\nu}l_{\nu}=\sum_{\nu=0}^{3}u_{3}{}^{\nu}=u_{3}{}^{0}l_{0}+u_{3}{}^{1}l_{1}+u_{3}{}^{2}l_{2}+u_{3}{}^{3}l_{3}=u_{3}{}^{0}+0+0-u_{3}{}^{3}=$$
$$=g^{00}u_{30}-g^{33}u_{33}=u_{30}-(-1)u_{33}=u_{30}+u_{33}=\frac{1}{2}ul_{3}=-\frac{1}{2}u\,.$$

Thus,

$$(u_{00}+u_{03})-(u_{30}+u_{33})=u_{00}+u_{03}-u_{30}-u_{33}=u_{00}-u_{33}=\frac{1}{2}u-\left(-\frac{1}{2}u\right)=u\ ,$$

where $u_{03}=u_{30}$. Also, $u_{11}=g_{11}u_1{}^1l_1=0$, and $u_{22}=g_{22}u_2{}^2l_2=0$, therefore

$$u_{00}-u_{33}=u=u_{00}-u_{11}-u_{22}-u_{33}\text{; and } u_{11}+u_{22}=0\ .$$

Also, $2u_{03}=u-2u_{00}=(u_{00}-u_{33})-2u_{00}=-u_{00}-u_{33}=-(u_{00}+u_{33})$, because $u_{00}+u_{03}=\frac{1}{2}u$.

Here, $g^{00}=1$, $g^{11}=g^{22}=g^{33}=-1$, and

$$g^{01}=g^{02}=g^{03}=g^{10}=g^{12}=g^{13}=g^{20}=g^{21}=g^{23}=g^{30}=g^{31}=g^{32}=0\ .$$

On the other hand the general formula of the action integral for the waves moving in one direction is:

$$\int Ld^4x=-\int\frac{1}{4}g^{\mu\nu}(u^{\rho}_{\mu}l_{\sigma}+u^{\rho}_{\sigma}l_{\mu}-u_{\mu\sigma}l^{\rho})(u^{\sigma}_{\nu}l_{\rho}+u^{\sigma}_{\rho}l_{\nu}-u_{\nu\rho}l^{\sigma})dx^0dx^1dx^2dx^3\ ,$$

where L is Lagrangean that describes the motion of the waves [3]. With the constraint, $\delta L=0$, the general solution of the pseudo-tensor $t_{\mu}{}^{\nu}$ that represents the spin momentum densities of the gravitational waves are:

$$16\pi\ t_{\mu}{}^{\nu}=\frac{1}{2}(u_{\alpha\beta}u^{\alpha\beta}-\frac{1}{2}u^2)l_{\mu}l^{\nu}\ ,$$

where l_{α} is one direction, in which the waves are moving in. Here,

$$u_{\alpha\beta}u^{\alpha\beta}-\frac{1}{2}u^2=u_{00}u^{00}+u_{11}u^{11}+u_{22}u^{22}+u_{33}u^{33}+2u_{01}u^{01}+2u_{02}u^{02}+$$

$$+2u_{03}u^{03}+2u_{12}u^{12}+2u_{23}u^{23}+2u_{31}u^{31}-\frac{1}{2}u^2=$$

$$=u_{00}g^{00}g^{00}u_{00}+u_{11}g^{11}g^{11}u_{11}+u_{22}g^{22}g^{22}u_{22}+u_{33}g^{33}g^{33}u_{33}+2u_{01}g^{00}g^{11}u_{01}+$$

$$+2u_{02}g^{00}g^{22}u_{02}+2u_{03}g^{00}g^{33}u_{03}+2u_{12}g^{11}g^{22}u_{12}+2u_{23}g^{22}g^{33}u_{23}+$$

$$+2u_{31}g^{33}g^{11}u_{31}-\frac{1}{2}u^2=u_{00}{}^2+u_{11}{}^2+u_{22}{}^2+u_{33}{}^2+(-1)2u_{01}{}^2+(-1)2u_{02}{}^2+$$

$$+(-1)2u_{03}{}^2+2u_{12}{}^2+2u_{23}{}^2+2u_{31}{}^2-\frac{1}{2}(u_{00}-u_{33})^2=u_{11}{}^2+u_{22}{}^2+2u_{12}{}^2=$$

$$=\frac{1}{2}(u_{11}-u_{22})^2+2u_{12}{}^2\ ;$$

and here $u^{\mu\nu}=g^{\mu\mu}g^{\nu\nu}u_{\mu\nu}$, $g^{00}=1$, $g^{11}=-1$, $g^{22}=-1$, $g^{33}=-1$, $-2u_{03}{}^2=-\frac{1}{2}(u_{00}{}^2+2u_{00}u_{33}+u_{33}{}^2)$, $\frac{1}{2}(u_{00}-u_{23})^2=\frac{1}{2}(u_{00}{}^2-2u_{00}u_{33}+u_{33}{}^2)$,

${u_{10}}^2 = {u_{01}}^2 = {u_{13}}^2 = {u_{31}}^2$ because $u_{10} + u_{13} = 0$, and, ${u_{20}}^2 = {u_{02}}^2 = {u_{23}}^2$ because $u_{20} + u_{23} = 0$, and from $u_{11} + u_{22} = 0$, $u_{22} = -u_{11}$, and then ${u_{11}}^2 + {u_{22}}^2 = {u_{11}}^2 + (-u_{11})^2 = 2{u_{11}}^2$, and finally: $(u_{11} - u_{22})^2 = (u_{11} - (-u_{11}))^2 =$, $= (2u_{11})^2$ so ${u_{11}}^2 + {u_{22}}^2 = 2{u_{11}}^2 = \frac{1}{2}(2u_{11})^2 = \frac{1}{2}(u_{11} - u_{22})^2$.

And then the spin momentum density of the gravitational waves becomes of the form:

$$16\pi\, {t_0}^0 = \frac{1}{4}\{(u_{11} - u_{22})^2 + {u_{12}}^2\}, \text{ with } {t_0}^3 = {t_0}^0 .$$

Now assume an infinitesimal rotation operator, R, in the plane of contravariant vectors $x^1 x^2$. If it is applied to any vector, A_1, A_2, it has the effect: $RA_1 = A_2$, $RA_2 = -A_1$, and $R^2 A_1 = -A_1$, so iR must have the eigenvalues ± 1 when applied to the vector [3]; here, $iR = \pm 1$. So, the operator R makes antisymmetric change of the vectors. When we apply this infinitesimal rotation operator, R, to $u_{\mu\nu} = A_\mu A_\nu$, the rotations will occur as follows:

$$Ru_{11} = R(A_1 A_1) = (RA_1)A_1 + A_1(R(A_1) = A_2 A_1 + A_1 A_2 = u_{21} + u_{12} = 2u_{12},$$

where $u_{21} = u_{12}$;

$$Ru_{12} = R(A_1 A_2) = (RA_1)A_2 + A_1(RA_2) = A_2 A_2 + A_1(-A_1) = u_{22} - u_{11};$$

$$Ru_{22} = R(A_2 A_2) = (RA_2)A_2 + A_2(RA_2) = -A_1 A_2 + A_2(-A_1) = -u_{12} - u_{21} = -2u_{12};$$

$$R(u_{11} + u_{22}) = R(A_1 A_1 + A_2 A_2) = (RA_1)A_1 + A_1(RA_1) + (RA_2)A_2 + A_2(RA_2) =$$

$$= A_2 A_1 + A_1 A_{21} - A_1 A_2 - A_2 A_1 = 0;$$

$$R(u_{11} - u_{22}) = R(A_1 A_1 - A_2 A_2) = A_2 A_1 + A_1 A_2 + A_1 A_2 + A_2 A_1 = 4 A_1 A_2 = 4u_{12};$$

$$R^2(u_{11} - u_{22}) = R(R(u_{11} - u_{22})) =$$

$$= R(Ru_{11} - Ru_{22}) = R(2u_{12} - (-2u_{12}) = 2Ru_{12} + 2R_{12} =$$

$$= 2(u_{22} - u_{11}) + 2(u_{22} - u_{11}) = 4(u_{22} - u_{11}) = -4(u_{11} - u_{22}).$$

Thus, $u_{11} + u_{22}$ is invariant (constant), while iR has the eigenvalues ± 2 when applied to $u_{11} - u_{22}$ or u_{12}. Therefore, the components of $u_{\alpha\beta}$ that contribute to the momentum density of gravitational waves correspond to spin 2 [3].

Upon the above theory, it was calculated the spin momentum of the gravitational waves by assuming $l_\sigma x^\sigma$ as coordinates of 4 dimensional flat-space (rectilinear coordinates), x_0, x_1, x_3, x_4, and then, we examined the parity of the $u_{\mu\nu}$, where each element of vector $x_0 = 1$ (time-coordinate), $x_1 = 0$, $x_2 = 0$ and $x_3 = -r$, and each of those are 23×1 vector.

Now $X=\{x_0,x_1,x_2,x_3\}$, and then we calculated the $u_{\mu\nu}$ as shown below:

$$u_{\mu\nu}=X'X=\begin{bmatrix} 23.0 & 0 & 0 & -8{,}34022\cdot 10^6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8{,}34022\cdot 10^6 & 0 & 0 & 3{,}02432\cdot 10^{12} \end{bmatrix}.$$

We see that: $u_{00}=23$, $u_{01}=u_{02}=u_{10}=u_{20}=u_{11}=u_{12}=u_{13}=u_{21}=u_{22}=$ $=u_{23}=u_{31}=u_{32}=0$, $u_{30}=u_{03}=-8{,}34022\cdot 10^6$, and $u_{33}=3{,}02432\cdot 10^{12}$.

And then the rotation operator was applied, R, to $u_{\mu\nu}$, to see the following:

$$Ru_{11}=u_{21}+u_{12}=0+0=2u_{12},\; Ru_{12}=u_{22}-u_{11}=0,\; Ru_{22}=-u_{12}-u_{21}=-2u_{12}.$$

So, $R(u_{11}+u_{22})=0$, $R(u_{11}-u_{22})=4u_{12}$, and $R^2(u_{11}-u_{22})=-4(u_{11}-u_{22})$. This result shows that the calculated $u_{\mu\nu}$, shown above, are consistent with the report by Dirac [3] about the infinitesimal rotational operator and the spin momentum of the gravitational waves.

THEORETICAL JUSTIFICATION OF OUR CALCULATIONS

Our equation for calculating the energy densities of gravitational energy field and gravitational waves is: $H=\left\{Y-\left(a-br+c\frac{1}{r^2}\right)\right\}$, where $L=\left(a-br+c\frac{1}{r^2}\right)$, Y is the global temperature. And then we calculated the coefficients a, b and c of $L=\left(a-br+c\frac{1}{r^2}\right)$, after giving the constraints: $\frac{\partial E(H^2)}{\partial a}=0$, $\frac{\partial E(H^2)}{\partial b}=0$, and $\frac{\partial E(H^2)}{\partial c}=0$, where $E(H^2)$ is the expected value of H^2.

Below we show a theoretical justification of this our calculation. In general, if $f=f(q_n,p_n)$ and $g=g(q_n,p_n)$ are arbitrary functions, and if $[f,g]=\frac{\partial f}{\partial q_n}\frac{\partial g}{\partial p_n}-\frac{\partial f}{\partial p_n}\frac{\partial g}{\partial q_n}$, and then, for example, $\dot g=[g,H]+U_m[g,\varphi_m]$, where $m=1,....,M$, which distinguishes independent functions $\varphi_m(q,p)$. And if $[\varphi_m,H]=0$, it gives the constraint to find the solutions of the problem. Here now we assume that, ϕ_m's are the functions that describe the gravitational waves, where H is named as Hamiltonian, where q_n are coordinates, and $n=1,......N$, while N is the number of degrees of freedom. Also, $H=p_n\dot q_n-L$, where $L=L(q,\dot q)$ is Lagrangean, $\dot q_n=\frac{\partial q_n}{\partial t}$, t is time-coordinate, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_n}\right)=\frac{\partial L}{\partial q_n}$, and $p_n=\frac{\partial L}{\partial \dot q_n}$ are momenta. In the theory of special relativity, N is finite; but, in the theory of general relativity of 4 dimensional curved space, N is infinite; and then, $\partial L = \int p_x \delta \dot{q}_x$, where the coefficient of $\delta \dot{q}_x$ in the integrand in ∂L is defined to be momenta p_x [2].

The action integral of Born-Infield electro-dynamics is $I = \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}) d^4 x}$, where $F_{\mu\nu}$ give the electromagnetic field. Here, the coordinates of electrodynamics is A_r, where $r = 0,1,2,3$; and, the related momenta D^r are the components of electric induction [2]. A_r and D^r satisfy $[A_r, D'^s] = g_r^s \delta(x - x')$, where $\delta(x - x')$ are the changes of the coordinates from r to s, where g_r^s is the Kronecker delta function and $\delta(x - x')$ is the delta function of $x - x'$. Now, only A remains as Hamiltonian, H. And then, $B^r = \frac{1}{2} \varepsilon^{rst} F_{st} = \varepsilon^{rst} A_{t,s}$, where $A_{t,s}$ are differentials of A_t, differentiated by the coordinate vector s, and $\varepsilon^{rst} = 1$ when $(rst) = (1,2,3)$. Here there are only 3 coordinates, because in electromagnetic dynamics the time-coordinate $r = 0$ doesn't have meaning. And now, $[B^r, D'^s] = \varepsilon^{rst} \delta,_t (x - x')$. Then the momentum density is $K_r = F_{rs} D^s$. Also the energy density is:

$$K_\perp = \{\Gamma^2 - \gamma_{rs}(D^r D^s + B^r B^s) - \gamma^{rs} F_{rt} F_{su} D^t D^u\}^{\frac{1}{2}},$$

where γ_{rs} is the metric in three-dimensional surface and $-\Gamma^2 = \det \gamma_{rs}$.

In these calculations Earth's global temperature Y is assumed to be, $p_n \dot{q}_n$; and the coefficients, a, b, and c are translated as p_n of the Lagrangean, L. Also here, H is the only Hamiltonian; and now, the energy densities of the gravitational waves and the gravitational field are calculated with

$$H = \left\{ Y - \left(a - br + c \frac{1}{r^2} \right) \right\}.$$

Here, $a - b \times r$ is generalized as $\Gamma^2 = -\det \gamma_{rs}$; and, $c \frac{1}{r^2}$ is generalized as $-\gamma_{rs}(D^r D^s + B^r B^s) - \gamma^{rs} F_{rt} F_{su} D^t D^u$. Also, $\frac{\partial E(H^2)}{\partial a} = 0$, $\frac{\partial E(H^2)}{\partial b} = 0$, and $\frac{\partial E(H^2)}{\partial c} = 0$ are the constraints that we used in our calculations. Here the generalized expression of our constraint $X'H = 0$ is $[X, H] = 0$, and X represents the energy densities that includes the gravitational waves $\varphi(q, p)$. Here we have to note that the number of order of freedom of coordinates is finite, $N = 4$, as we assumed only 4 vectors, x^0, x^1, x^2, and x^3, of the rectilinear coordinates in our calculation.

Similarly, in our calculation about the spin momentum of the Gravitational waves, the generalized form of the pseudo tensor is:

$$t_{\mu}{}^{\nu}\sqrt{-\det(g_{\mu\nu})} = \frac{\partial L}{\partial q_{n},_{\nu}} q_{n},_{\mu} - g_{\mu}{}^{\nu} L \text{ is } [L, q_n],$$

where, $L = -\frac{1}{4} g^{\mu\nu} ({}_{\mu}{}^{\rho} l_{\sigma} + u_{\sigma}{}^{\rho} l_{\mu} - u_{\nu\sigma} l^{\rho})(u_{\nu}{}^{\sigma} l_{\rho} + u_{\rho}{}^{\sigma} l_{\nu} - u_{\nu\rho} l^{\sigma})$, is the Lagrangean that describes the motion of the gravitational waves. And then, $[L, q_n] = 0$, gives the constraint to calculate the spin momentum of the gravitational waves, which is followed by calculations we showed above. In addition, $[B^r, D'^s] = \varepsilon^{rst} \delta,_t (x - x')$, in the theory of the electromagnetic field is the original idea of the infinitesimal rotation operator, R, in our analysis shown above. It changes the variables, but it doesn't change the physical system, which is the consistent argument that agrees with the theory of relativity [2].

Here we have to report one more aspect of the gravitational waves: the general solution of the momentum density of the gravitational waves, $16\pi\, t_{\mu}{}^{\nu} = \frac{1}{2}(u_{\alpha\beta} u^{\alpha\beta} - \frac{1}{2} u^2) l_{\mu} l^{\nu}$, involves both contravariant vectors, x^{μ}'s, and covariant vectors, x_{μ}'s. Dirac [3] predicted that the gravitational waves appear only in one direction. Then, the momentum density of the gravitational wave becomes $16\pi\, t_0{}^0 = \frac{1}{4}\{(u_{11} - u_{22})^2 + u_{12}{}^2\}$, where $t_0{}^3 = t_0{}^0$. These contravariant vectors, x^{μ}'s, and covariant vectors, x_{μ}'s, are exchanged each other through fundamental tensors, $g^{\mu\nu}$'s, as we showed as $u^{\mu\nu} = g^{\mu\mu} g^{\nu\nu} u_{\mu\nu}$, and this operation changes the sign ($\pm$) of the vectors. And the momentum density of gravitational waves is calculated as the scalar-products of those two different coordinates' systems. However, contravariant vectors and covariant vectors are in different coordinates' systems, and the momentum density can be calculated when two different coordinates' systems meet, although the contravariant vectors are not yet observable in the real physical system. This issue may be further investigated for explaining the negative contribution of the gravitational waves to the gravitational energy field.

CONCLUSIONS AND RECOMMENDATION

In our previous research, [1], we compared the influences of $\frac{1}{r}$ (as a surrogate for Newton's gravitational field) and $\frac{1}{r^2}$ (as the surrogate for the gravitational waves' movement) to Earth's global temperature, assuming as if they are independent variables for the Least Squares Estimation of Classical Regression Model. Instead in this report, we have calculated the energy density of gravitational waves in the rectilinear coordinates of time and space (the empty space where only gravitational field exists). For these calculations we set an action integral in a rectilinear coordinate system, which linearly combines the gravitational field, the gravitational waves and Earth's global temperature, where each of them describes the field of energy. Then we calculated the coefficients of those energy fields from the energy densities algebraically with a constraint, in which the derivatives of the energy density are zero, and as a result we found that the gravitational field has more effect on Earth's global temperature, while the energy of Moon's gravitational waves has a negative contribution to it.

In order to investigate the nature of the negative contribution of gravitational waves to the Earth's global temperature, we also examined the spin momentum of the assumed gravitational waves, in the rectilinear coordinate system. Although the spin momentum is very small and it doesn't raise the potential energy in the theory of quantum mechanics, it must exist on theory [6]. The result or our calculation indicated that the gravitational waves in our coordinate system had the spin 2. On the other hand in this analysis, we calculated the scalar products of contravariant vectors and covariant vectors, while contravariant vectors are not observable in the real physical field, which leaves the issue for the further investigation.

REFERENCES

1. *Matsuki Y.,* Empirical Investigation on Influence of Moon's Gravitational-Field to Earth's global temperature / Y. Matsuki, P.I. Bidyuk // System Research & Information Technology, 1/2019, 2019. — N 2. — P. 107–118.
2. *Dirac P.A.M.* Lectures on Quantum Mechanics. — New York: Belfer Graduate School of Science, Yeshiva University, 1964. — 87 p.
3. *Dirac P.A.M.*, General Theory of Relativity. Florida State University. — New York: John Wiley & Sons, 1975. — 69 p.
4. *UK* Department of Energy and Climate Change (DECC). — Available at: http://en.openei.org/datasets/dataset/b52057cc-5d38-4630-8395-b5948509f764/resource/f42998a9-071e-4f96-be52-7d2a3e5ecef3/download/england.surface.temp1772.2009.xls
5. *Moon* Distance Calculator – How Close is Moon to Earth? Available at: https://www.timeanddate.com/astronomy/moon/distance.html?year=1987&n=367
6. *Dirac P.A.M.* The Principle of Quantum Mechanics / PAM DITAC. — Oxford: Clarendon Press, 1958. — 312 p.

Receive 06.02.2019

From the Editorial Board: the article corresponds completely to submitted manuscript.