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ABOUT ONE PROBLEM FOR EQUATION OF FRACTAL DIFFUSION WITH ARGUMENT DEVIATION

For a quasilinear pseudodifferential equation with fractional derivative by time variable t with order $\alpha \in (0, 1)$, *second derivative by space variable x and argument deviation with help of step method we prove the solvability of boundary problem with two unknown function by variable t.*

1. Formulation of the problem

We should determine the solution of the boundary value problem

$$D_t^{\alpha} u(t,x) = a^2 \frac{\partial^2 u(t,x)}{\partial x^2} - B(t)p(t,x) + f(t,x,u(t-h,x)), \quad t > h, x \in \mathrm{P}, \tag{1}$$

$$u(t,x)|_{0 \le t \le h} = u_0(t,x), \quad x \in \mathrm{P}, \tag{2}$$

$$u((k+1)h,x) = \varphi(x), \quad x \in \mathrm{P}, \tag{3}$$

where

$$D_t^{\alpha} u(t,x) = \frac{1}{\Gamma(1-\alpha)}\left[\frac{\partial}{\partial t}\int_h^t \frac{u(\tau,x)d\tau}{(t-\tau)^{\alpha}} - (t-h)^{\alpha} u_0(h,x)\right]$$

is a regularized fractional Riemann-Liouville derivative of $\alpha \in (0,1)$ order, $t > h$, $x \in \mathrm{P}$, h is a number, $k \in \mathrm{N}$, f, u_0, φ are known functions, u, p are unknown functions.

The problem (1) – (3) contains fractal integro-differential equations, which are used in physical, mechanical and other disciplines. We note that the Cauchy problem for an equation with fractional derivatives is sufficiently complete and thoroughly analyzed in the papers of A.N. Kochubey and S.D. Eidelman [1, 2, 3] and many of their later papers.

As a solution of the problem (1) – (3) we mean a pair of functions $u(t,x)$, $p(t,x)$ with properties [4].

2. Definition and properties if Green function

We denote

$$G_i(t,x,\alpha,h) = F^{-1}_{\sigma\to x}[Q_i(t,\sigma,\alpha,h)], i=1,2 \qquad (4)$$

where $F^{-1}_{\sigma\to x}$ is a inverse Fourier transform,

$$Q_1(t,\sigma;\alpha,h) = E_{\alpha,1}(-a^2|\sigma|^2(t-h)^\alpha), t>h, \sigma\in \mathrm{P},$$

$$Q_2(t,\sigma;\alpha,h) = D_t^{1-\alpha}E_{\alpha,1}(-a^2|\sigma|^2(t-h)^\alpha), t>h, \sigma\in \mathrm{P},$$

are expressed in the Mittag-Leffler function

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty}\frac{t^k}{\Gamma(\alpha k+\beta)}, t>0, \alpha>0, \beta>0$$

[4, p. 25]. For functions G_i, $i=1,2$, from (4) such estimates are true [2, lemmas 1, 2]:

$$|D_x^m G_1(t,x,\alpha,h)| \le Ct^{-\frac{\alpha(1+m)}{2}}\exp\{-c\rho(t,x,h)\}, m\le 3, \qquad (5)$$

$$|D_t^\alpha G_1(t,x,\alpha,h)| \le C(t-h)^{-\frac{3}{2}\alpha}\exp\{-c\rho(t,x)\},$$

$$|D_x^m G_2(t,x,\alpha,h)| \le C(t-h)^{-\frac{\alpha(1+m)}{2}-1+\alpha}\exp\{-c\rho(t,x,h)\}, m\le 3,$$

$$|D_t^\alpha G_2(t,x,\alpha,h)| \le C(t-h)^{-\frac{\alpha}{2}-1}\exp\{-c\rho(t,x,h)\},$$

where $\rho(t,x,h) = (|x|(t-h))^{\frac{2}{2-\alpha}}$, $t>0$, $x\in \mathrm{P}$.

3. The classical solution of (1) – (3) problem

Let's construct a classical solution of the original problem (1) – (3) when $kh\le t\le (k+1)h$, $x\in \mathrm{P}$ in the band $\Pi = (h,(k+1)h)\times \mathrm{P}$ [4]

$$u(t,x) = \int_{-\infty}^{\infty} G_1(t,x-\xi;\alpha,kh)u_0(kh,\xi)d\xi +$$

$$+\int_{kh}^{t} d\tau \int_{-\infty}^{\infty} G_2(t-\tau,x-\xi;\alpha,kh)[f(\tau,\xi;kh)-B(\tau)p(\tau,\xi)]d\xi, \qquad (6)$$

where the term $B(t)p(t,x)$ moved to the right-hand side of equation. Let's satisfied the condition (3) from (6), then we obtain the equation

$$\int_{kh}^{(k+1)h} d\tau \int_{-\infty}^{\infty} G_2((k+1)h-\tau,x-\xi;\alpha,kh)B(\tau)p(\tau,\xi)d\xi =$$

$$= [G_1(t,x;\alpha,kh)*u_0(kh,x)+G_2(t,x;\alpha,kh)**f(t,x;kh)]|_{t=(k+1)h} -$$

$$-\varphi(x) \equiv \Psi(x;\alpha,kh,(k+1)h), \qquad (7)$$

which is a integral Fredholm equation of first kind for definition of function $p(t,x)$. The solution of equation (7) we find as

$$p(t,x) = G_1(t,x;\alpha,kh)C, \qquad (8)$$

where C is constant.

If we substitute (8) in (7) we obtain linear equation for definition of C, whose solution is

$$C = \Psi(x;\alpha,kh,(k+1)h)\{\int_{kh}^{(k+1)h} B(\tau)d\tau \int_{-\infty}^{\infty} G_2((k+1)h-\tau, x-\xi;\alpha,kh)\times$$

$$\times G_1(\tau,\xi;\alpha,kh)d\xi\}^{-1}.$$

If we take into account the convolution formula, we obtain

$$C = \Psi(x;\alpha,kh,(k+1)h)\{(G_2(t,x;\alpha,h))\int_{kh}^{(k+1)h} B(\tau)d\tau\}^{-1}.$$

So from (8) we obtain

$$p(t,x) = G_1(t,x;\alpha,kh)\Psi(x;\alpha,kh,(k+1)h)\{G_2(t,x;\alpha,h)\int_{kh}^{(k+1)h} B(\tau)d\tau\}^{-1}. \quad (9)$$

If we substitute (9) in (6) we obtain the formula for $u(t,x)$. So the pair of functions $u(t,x)$ from (6) and $p(t,x)$ from (9) defines the classical solution of the problem (1) - (3). Based on the inequalities for Green functions, we estimate pair $(u(t,x), p(t,x))$ of searched functions

$$|p(t,x)| \le Ct^{-\frac{\alpha}{2}} |B| (|u_0|_C + |\varphi|_C + |f|_C), \quad (10)$$

$$|u(t,x)| \le C(|u_0|_C + |f|_C + |B|_C + |\varphi|_C), \quad (11)$$

where $|\cdot|_C$ denotes norm in space continuously functions.

Note that function $p(t,x)$ is not uniquely defined, since in (8) we can add a multiplier $m(t)$.

The two-point boundary value problem for the diffusion equation with the operator of fractional differentiation with respect to the t is considered in the [5]. So we have the following theorem

Theorem. *The classical solution of the problem (1)–(3) defines by pair of functions (6), (9) for which the estimates (10), (11) are correct.*

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