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Application of the Cobb-Douglas function for the formation of would-be economists' analytical skills

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Abstract. Training high-quality specialists in the economic field is crucial. This task can be achieved by training students in economic specialties in mathematical modeling. Solving problems using production functions, particularly Cobb-Douglas functions, can become the ground for mastering mathematical modeling. The article looks into the issue of the combination of table and clustering methods to systematize such kinds of problems. The involvement of students in real production situations while mathematical training contributes to the development of analytical skills needed for analyzing economic processes, modeling decisions, working with data, and coming to conclusions. The article presents a system of problems involving the Cobb-Douglas production function. The study shows the implementation of these problems while mastering 'Multivariate Functions' by economic majors. Also, the work presents the practicality of such a system when developing their analytical skills. The run experiment confirmed that the developed system of problems results in the progression of would-be economists' analytical skills.

1. Introduction

There is always an urgent issue of training economists whose skills will be able to ensure stable development in any country. When preparing such specialists, they should be taught mathematical modeling, which means that they should understand in practice how to build a mathematical model of a real problem and solve it with the help of a mathematical apparatus. Creating economic models that characterize the dependence of the volume of production on various factors or their ratios in the conditions of the national economy requires applying production functions. Such models usually contain the production volume in natural or value terms and spent volumes of resources. They are called factors of production. There are two types of production functions:

(i) single-factor, establishing the dependence of the volume of production on one factor; this variety includes linear, parabolic, static, and indicative functions;

(ii) two-factor establish the dependence of the production volume on the ratio of two factors; this variety includes Cobb-Douglas, Leontiev, Solow, and Allen functions.

The Cobb-Douglas function belongs to the class of two-factor production functions. For this function, capital and labor are chosen as the two main factors of production. A certain proportionality of their combination creates conditions for obtaining a product. The purpose of the Cobb-Douglas production function is to reflect the technological ratio of the labor input and capital required to produce a particular product in the required quantity.

The concept of a production function, which relates output to the factors that underlie it, has a long history. In the early 1900s, economists used that idea to explain phenomena ranging from diminishing returns to exhaustion of the product under conditions of maximum productivity. Malthus' iron law of wages, Ricardo's theory of rent, the trend of the relative share of income in a developing economy, the first-order conditions for optimal factor employment, and Euler's addition theorem – all based on the Cobb-Douglas production function [1]. For the first time, the Cobb-Douglas function was used by the Swedish economist Knut Veksell. Its statistical verification was performed from 1927 to 1947 by two scientists – Charles Cobb and Paul Douglas [2]. The production function developed by Cobb and Douglas is the first aggregate production function. Its application made it possible to model both small-scale processes and entire sectors of the economy. Statistical confirmation of this function was the beginning of a new stage of macroeconomic development, which allows the assessment of the efficiency of production at the standard of the national economy.

The literature analysis results in the conclusion that the Cobb-Douglas production function is used to estimate the production capacity of the economic sector in different countries of the world. Skrynkovskyy et al. [3] used the Cobb-Douglas production function to forecast trends in the state's industrial development using the example of Ukraine. Rana et al. [4] highlighted the methods of raising domestic animals and the socio-economic conditions for this in the example of Cuchia farmers in the Bogura district of Bangladesh. The Cobb-Douglas production function was used to determine the contribution of the most important variables in the production process of Cuchia farming. Tirfi and Oyekale [5] analyzed the influence of some climatic variables on the yield of maize in Ethiopia. Augmented Cobb-Douglas production function was used for data analysis.

Jandhana et al. [6] devoted their research to assessing and analyzing the sustainability of the metallurgical sector in Indonesia using the Cobb-Douglas production function. The analysis shows how labor costs, capital investment, and total factor productivity contribute to industry growth. This study and its results contributed to the enlargement of sustainability measurement research in the metallurgical sector and other industries. The German economist Koch [7] believes that the Cobb-Douglas production function is central to economic growth theory and microeconomics. Convenient mathematical properties explain its use. Some of these properties describe a set of (partial) differential equations.

Despite the Cobb-Douglas function applicability, there is debate as to whether its analytical form makes sense from a dimensional point of view. Economist Labini [8] joined the discussion and noted that despite several harsh criticisms, the function should be used 'in the first approximation'. Mathematician Vîlcu [9] obtained an interesting connection between some fundamental concepts in the theory of production functions and the differential geometry of hypersurfaces in Euclidean spaces. He established that the generalized Cobb-Douglas production function has decreasing/increasing returns to scale if and only if the corresponding hypersurface has positive/negative Gaussian curvature. Furthermore, this production function has constant returns to scale if the corresponding hypersurface can be developed. By defining the Cobb-Douglas function with variable output elasticity, it can be shown that a large class of production functions can be written as the Cobb-Douglas function with non-constant output elasticity. This framework has several advantages in contrast with standard flexible functions such as the Translog function. Its inclusion does not require using the second-order approximation, facilitating the deduction of the linear function of input demands without invoking the duality theorem. This makes it possible to generalize the demand elasticity function understandably to the case where the elasticity of substitution between each pair of inputs is not necessarily the same. Also, it provides a more general and flexible structure compared to the traditional approach of nested elasticity functions while facilitating the analysis of substitution properties of nested elasticity functions [10].

Experimental studies rarely address the shape and nature of the production function, which is useful for obtaining optimal levels of resource substitution in increasingly resource-constrained environments. For example, the function of educational production can be investigated [11] due to the rapid spread of online education as a replacement for traditional, against the background of full-scale temporary replacement due to the coronavirus pandemic. The results show that the estimates are consistent with the general form of the Cobb-Douglas production function and imply that the blended learning approach is optimal. Conclusions [11] result in the rapid expansion of educational technologies worldwide and their continued replacement of traditional education.

Therefore, the presented conclusions show that the Cobb-Douglas production function keeps a crucial place in economic research, so its study in mathematics classes while training students in economics major does not raise doubts. Methodists Smeureanu and Isaila [12] believe that thanks to the visualization of fundamental concepts from the field of economic and mathematical modeling, it is possible to organize knowledge in coherent scenarios presented in the form of an educational game to attract attention and influence the spirit of competition of students and thus stimulate their imagination, which is used to build mental models, and intuition, which is capitalized in the process of discovering knowledge while solving a certain system of problems.

Such scientists as Landgärds [13], Hoag and Benedict [14], Semenenko and Tsibulya [15] devoted their works to the fundamental training of students of economic specialties through the involvement of a system of problems. All of them emphasize that the high-quality training of such specialists is provided by developing their mathematical literacy and knowledge of functions. After all, economists have to deal with large data sets, visually interpret data, with numbers, and with mathematical models and functions. Therefore, learning mathematical modeling is based on the mastery of functions. The elementary function consideration begins with elementary ones and moves on to studying bivariate functions. Function mastery is ensured by solving a certain system of problems. Analysis of articles [13, 14, 16]) testifies to a wide range of applications of problem systems when learning functions and mathematical modeling with their help. These are the most common application areas of such

1. Optimization of production processes: solving the problems of production optimization, resource allocation, and planning of production processes.
2. Project management: using problem systems to plan, control, and evaluate various aspects of projects in diverse industries, including construction, information technology, and research and development.
3. Logistics and supply chains: solving the problems of optimization of supply chains, distribution of goods and resources, routing, and inventory.
4. Transport and transport systems: using problem systems to develop efficient and optimal transport routes, planning traffic schedules, and managing transport networks.
5. Finance and banking: solving problems of managing portfolios, risks, and financial planning with the help of problem systems.

6. Medicine and health care: using problem systems to optimize treatment processes, allocate resources in medical facilities, and plan health services.
7. Energy and resource conservation: solving energy management problems, optimizing using resources, and developing energy conservation strategies.

These areas are just examples of the problem systems application while learning using mathematical modeling. Nunokawa [17] determined the relationship between problem-solving and learning mathematics. He emphasized that choosing the right problems is crucial to providing students with an opportunity for 'authentic' problem-solving. Caerols and Vogt-Geisse [18] gave examples of problems to motivate students' critical mathematical thinking while learning Mathematics. Lubienski [19] focused on improving students' intellectual abilities in mathematics education through problem-solving. Al-Khateeb [20] devoted his work to checking the impact of solving mathematical problems using mobile learning on students' ability to develop analytical and mathematical skills. Klang et al. [21] confirmed that solving mathematical problems is a paramount area of mathematics learning and emphasized the need for research into such approaches to mathematics learning. Jacinto [22] substantiated that solving mathematical problems while training would-be economists causes the student's interest to rise and improve their mathematical skills and specific abilities. So, the research analysis proved that a correctly selected system of problems for the production functions application, particularly the Cobb-Douglas functions, while training students of economic specialties, will allow us to increase their motivation and develop their analytical, mathematical, intellectual, economic, and financial abilities.

Mathematical training for students majoring in economics is aimed at developing various skills and abilities that are important for analyzing economic processes, modeling decisions, and working with financial data. One of the crucial is analytical skills: students learn to understand, analyze, and solve complex mathematical problems related to economic theories and models. Analytical skills include a person's ability to understand, analyze, and draw conclusions based on available information. This includes identifying relationships between facts, distinguishing the important from the minor, formulating and testing hypotheses, and making informed decisions.

Dendir et al. [23] emphasize the urgency of developing the analytical skills of students of economic specialties. Scientists believe that analytical skills form the basis for students' problem-solving ability. Suciu and Lacatus [24] believe that developing students' analytical skills during their economic education can be crucial for evolving an economist's career in the new context of a knowledge-based society.

The main aspects of analytical skills include: the ability to understand context (the ability to analyze a situation or problem in a broad context, taking into account all the factors that can affect it); logical thinking (the ability to build logical arguments, develop consistent evidence and exclude unfounded assumptions); data analysis (ability to effectively process and interpret large amounts of information using various methods of data interpretation); problem solving (the ability to identify problems, set specific goals for solving them, and find optimal ways to achieve these goals); critical thinking (the ability to evaluate ideas, evidence, or decisions using objective criteria and avoiding biases or reinforcing stereotypes); decision-making (the ability to consider alternatives, gather enough information for an informed choice and draw conclusions); communication of results (the ability to clearly and effectively express one's thoughts and analytical findings in written or oral form). There is an opinion that analytical skills can be improved by solving mathematical problems, during which new methods of analysis are taught, mathematical modeling skills are improved, and sound economic decisions are made. The validity of this opinion can be verified by developing a system of problems based on the use of the Cobb-Douglas production function.

Therefore, the purpose of our article is to systematize problems that present the use of the Cobb-Douglas production function, to implement this system of problems when

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the mathematical training of economic majors, as is also to check the practicality of its implementation while developing students' analytical skills.

2. Methods

There are many methods of systematizing problems, and the choice of a specific one depends on the nature of the problem, its complexity, and specific goals. Here are some of them:

- *Classification by features or characteristics.* This method refers to grouping problems by common specifics. For example, problems can be classified by topic, difficulty, solution time, etc.
- *Method of analogies.* In this method, problems are systematized by finding analogies between them. If two problems have similar aspects or solutions, they can be combined into one category.
- *Hierarchy method.* This method uses a hierarchical structure to systematize problems. Problems are divided into groups according to their level of generality or specificity.
- *Method of matrices or tables.* Problems can be systematized by creating matrices or tables, where one axis is responsible for one or more criteria, and the other is for different tasks. This method helps compare problems according to various aspects.
- *Method of schemes or diagrams.* Using diagrams or charts can help organize problems by visualizing the relationships between them or their characteristics.
- *Ranking method.* Problems can be organized by ranking them according to certain criteria, such as importance, priority, or impact on organizational goals.
- *Method of grouping or clustering.* Problems can be organized by dividing them into groups or clusters based on their similarities.

These methods can be used separately or in combination depending on the specific situation and the needs of problem systematization. This study uses a combination of methods. It is about tabular and clustering methods. Problems presented in the table should ensure the development of students' analytical skills while mastering the section 'Multivariate functions'.

It will describe what students need to know about the Cobb-Douglas function to work with it. The Cobb-Douglas production function formula reflects the dependence of the volume of production of a certain product on the combination of two factors of production – labor (L) and capital (K). In general, the formula looks like this:

$$Q = c \cdot K^{\alpha} \cdot L^{\beta}, \quad Q : \mathbb{R}^{+} \times \mathbb{R}^{+} \to \mathbb{R} : (K, L) \to Q(K, L), \tag{1}$$

where Q is an indicator of the production quantity, which characterizes the real cost of goods and services produced in a certain period; c – general index of technological factor productivity; this indicator is the most difficult to determine and predicts, with a certain level of error, the possibility of imperfect assessment of labor and capital input, and even the influence of other factors; L – labor costs, which are included in the production of a certain production quantity, expressed in the number of man-hours worked by all employees for the specified period; K – costs of invested capital in the production of a certain production quantity, expressed in the real cost of equipment and machines used in production; α – technological capital elasticity; β is the technological labor elasticity.

Sometimes, the Cobb-Douglas function is simplified to a form called a partial case of the function:

$$Y = c \cdot K^{\alpha} \cdot L^{1-\alpha}. \tag{2}$$

This formula is used when the technological capital elasticity and labor are related: $\beta = 1 - \alpha$.

Working with production functions, students should master certain topics of 'Multivariate functions', namely: domain of a bivariate function, partial derivatives and first-order differentials of a bivariate function, complete increment and complete differential of the first order, derivative in direction, gradient of a function, differentiation of complex functions, partial derivatives and differentials of higher orders, differentiation of implicit functions, extremum of a bivariate function, method of least squares, etc.

It is assumed that while studying production functions, students will master not only mathematical concepts but also economic ones: average labor productivity (the ratio of the production quantity (in terms of value) to the amount of labor expended), average return on funds (quantity of products (in terms of value) per unit of funds); capital equipment, which expresses the volume of fixed assets per employee; marginal labor productivity (equal to the additional value of products created by a supplemental unit of labor); marginal return on funds (equal to the supplemental value of products produced as a result of an increase in funds per unit); isoquant or production indifference curve (a line at each point of which different combinations of production factors (capital and labor) give the same amount of output). The formulation of these concepts should be presented when considering the problems in which they are involved.

Table 1 presents a system of problems based on the Cobb-Douglas function. It contains topics of Calculus, necessary skills and related problems. The ideological complexity of the problems increases, which contributes to the gradual development of students' skills in studying production functions and economic concepts and meets the didactic requirements for the system of problems.

Table 1: A system of problems based on the Cobb-Douglas function involvement.

№	**Topics of Calculus**	**Skills**	**Problems**
1	The domain of a bivariate function, the full increment of a bivariate function	The ability to identify problems, set specific goals for solving them, and find optimal	The production function is the Cobb-Douglas function. To increase output by 3%, it is necessary to either increase funds by 6% or the number of employees by 9%. In 2020, one employee produced goods worth 500 thousand conventional units (c. u.) per month. 8,000 employees work at the enterprise in c. u. The fixed capital is estimated at 1 bln. c. u. Find the production function
2	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions	The ability to analyze the situation in a broad context, concerning all the factors that can affect it	There is a production function $Y = 2K^{0.2}L^{0.8}$. Find the marginal rate of displacement of capital by labor

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№	Topics of Calculus	Skills	Problems
3	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives, and higher orders differentials, differentiation of implicit functions	The ability to evaluate alternatives, the ability to gather enough information to make a justified choice and draw conclusions, the ability to clearly and effectively express one's thoughts and analytical findings in written or oral form	The production function of an enterprise is the Cobb-Douglas function. To increase output by 1%, raising funds by 3% or the number of employees by 3% is necessary. The enterprise employs 1,000 workers, and one employee manufactures products in 10^7 c. u. The fixed assets of the enterprise are estimated at 10^9 c. u. Find the production function, the average and marginal values of labor productivity and return on capital, and the elasticity of output by labor and capital. Explain the economic meaning of these indicators
4	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives and higher orders differentials, differentiation of implicit functions	The ability to construct logical arguments, the ability to develop a consistent proof, exclude unfounded assumptions, the ability to evaluate ideas or decisions using objective criteria, and avoid stereotypes	The quantity of a firm's production of computer monitors is expressed in terms of the input's capital K and labor L by the Cobb-Douglas production function P: $\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} : (K, L) \mapsto rK^{\frac{1}{2}}L^{\frac{1}{2}}$. The firm controls the parameter $r \in \mathbb{R}_0^+$ through the technology used in the production process. The price of a unit of capital is p euro, and that of a unit of labor is q euro with p, $q > 0$. Suppose that the firm keeps its total production and its total cost constant. To achieve this, the firm currently invests K^* units of capital and L^* units of labor while the production process parameter is r^*. Moreover, $qL - pK > 0$ in a neighborhood of (K^*, L^*, r^*). Determine the impact of an increase of the parameter r with one unit on the inputs K and L. Do this by applying the implicit function theorem and calculating the necessary derivatives with respect to r

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№	Topics of Calculus	Skills	Problems
5	Partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, derivative in direction, differentiation of complex functions, partial derivatives	The ability to evaluate alternatives, the ability to gather enough information for a reasoned choice and draw conclusions, the ability to identify problems, set specific goals for their solution, and find optimal ways to achieve these goals	The quantity of a firm's production of computer monitors (CM) is expressed in terms of the input's capital K and labor L by the Cobb-Douglas production function P_{PM}: $\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} : (K, L) \mapsto rK^{\frac{1}{2}}L^{\frac{1}{2}}$. The same firm also produces TV screens in a quantity that is analogously expressed in terms of the input's capital K and labor L by the Cobb-Douglas production function P_{PM}: $\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} : (K, L) \mapsto sK^{\frac{1}{2}}L^{\frac{1}{2}}$. The firm controls the parameters r, $s \in \mathbb{R}_0^+$ independently of each other through the technology that is used in the production process. The price of a unit capital is p euro, and that of a unit labor is q euro with p, $q > 0$. Suppose that the firm keeps its total production and its total cost constant. To achieve this, the firm currently invests K^* units of capital and L^* units of labour while the production process parameters are r^* and s^*. Moreover, $qL^* - pK^* > 0$. Determine the impact of an increase of the parameter r with one unit on the input capital if the parameter s is kept constant
6	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives, and higher orders differentials	The ability to analyze the situation in a broad context, concerning all the factors that can affect it	The production output q of a firm is determined by capital K and labour L. The exact relation is unknown. However, experience has shown that • a decrease of production by 3% is to be expected when capital and labour are both reduced by 1%; • an increase of production by 2% is expected when capital is decreased by 1% and labour is increased by 3%. Management wants to cut wages and plans to reduce labour by 3%. If the current production level has to be maintained, what action should management take with respect to capital?

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№	Topics of Calculus	Skills	Problems
7	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions	The ability to create logical arguments, develop a consistent proof, and exclude unfounded assumptions	Work out the demand price elasticity for the Cobb-Douglas production function is given as a function of time: $Y(t) = A \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha}$
8	Partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, differentiation of complex functions, partial derivatives and differentials of higher orders, differentiation of implicit functions	The ability to evaluate alternatives, the ability to gather enough information for a reasoned choice and draw conclusions	Applying the Cobb-Douglas function for Beiersdorf AG, find the marginal productivity of labor and capital using their financial reports `https://www.beiersdorf.com/`
9	Partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, differentiation of complex functions	The ability to create logical arguments, develop a consistent proof, and exclude unfounded assumptions	Find the demand price elasticity for the Cobb-Douglas production function for BASF using their financial reports `https://www.basf.com/ua/`
10	The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions	The ability to create logical arguments, develop consistent proof, and exclude unfounded assumptions	Work out the Cobb-Douglas elasticity of substitution for Beiersdorf AG using their financial reports `https://www.beiersdorf.com/`

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№	Topics of Calculus	Skills	Problems
11	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives, and higher orders differentials, differentiation of implicit functions	The ability to create logical arguments, develop a consistent proof, and exclude unfounded assumptions	Consider a firm whose production can be modeled using such a production function, i.e., $q(K,L) = CK^{\alpha}L^{\beta}$ with capital denoted by K, labor by L and C, $\alpha, \beta \in \mathbb{R}_0^+$. Additionally, denote the unit price of capital and labor by w_K and w_L, respectively, and the unit price of production by p. All these prices are assumed to be positive. Then we can figure out the optimal input combination that maximizes profit. The profit function $\pi = \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R} : \pi(K,L)$ is determined by $\pi(K,L) =$ $= pq(K,L) - w_K K - w_L L =$ $= pCK^{\alpha}L^{\beta} - w_K K - w_L L$
12	The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, derivative in direction, differentiation of complex functions, partial derivatives, and higher orders differentials, extremum of a bivariate function	The ability to create logical arguments, the ability to effectively process and interpret large amounts of information using various methods of data interpretation	A sample of data that characterizes the company's work over the past 10 months is given. In this sample, each value of $Y = \{157;\ 208;\ 254\}$ – the cost of manufactured products, thousand c.u. correspond to indicators $X_1 = \{11.45;\ 12.31;\ 16.20\}$ – the price of the main production assets, thousand c. u., and $X_2 = \{20.1;\ 25.3;\ 31.1\}$ – labor costs, man-hours. It is necessary to build a multiple correlation economic model in the form of the Cobb-Douglas function, evaluate the accuracy and reliability of the model, determine the closeness of the relationship between factors, construct isoquants of interchangeability of model factors, and make an economic analysis of the obtained results according to all known production functions characteristics
13	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives, and higher orders differentials	The ability to create logical arguments, develop a consistent proof, and exclude unfounded assumptions	Calculate the Cobb-Douglas production function elasticity $Y = AK^{\alpha}L^{1-\alpha}$ at an arbitrary point $M_0(K_0, L_0)$ in terms of the variables K and L

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№	Topics of Calculus	Skills	Problems
14	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, extremum of a bivariate function	The ability to create logical arguments, the ability to analyze a situation or problem in a broad context, concerning all factors that may affect it	Consider the Cobb-Douglas production function. To increase production by 3%, it is necessary to increase production assets by 6% or the number of workers by 9%. Last year, one worker produced products worth 1 million c. o. per month, and there were 1,000 employees in the enterprise. Fixed assets were estimated at 10 billion c. u. Write down the expression for the production function and find the average return on assets
15	Partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives, and higher orders differentials, extremum of a bivariate function	The ability to analyze a situation or problem in a broad context, concerning all factors that may affect it, the ability to create logical arguments	For the Cobb-Douglas production function $y = 1000K^{\frac{1}{2}}L^{\frac{1}{3}}$, find the average and marginal labor productivity, the average and marginal return on capital, and the output elasticities for labor and capital
16	The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, differentiation of complex functions, partial derivatives, and higher orders differentials	The ability to effectively process and interpret large amounts of information using various methods of data interpretation	Construct a Cobb-Douglas production function given a date in capital changes (X) and a date in labor changes (Y): $X = \begin{pmatrix} 8.35 & 9.58 & 10.45 & 11.57 \\ 4.93 & 6.15 & 8.47 & 8.89 \end{pmatrix}$, $Y = (65.94\ \ 55.17\ \ 79.12\ \ 82.96)$. Construct a mathematical model of the problem by applying the Cobb-Douglas production function

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<table>
<tr><th>№</th><th>Topics of Calculus</th><th>Skills</th><th>Problems</th></tr>
<tr><td>17</td><td>The domain of a bivariate function, complete increment and complete differential of the first order, derivative in direction, the gradient of a function, partial derivatives and higher orders differentials, differentiation of implicit functions, least squares method</td><td>The ability to create logical arguments, the ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form</td><td>A company needs to produce 100 units of output. This amount can be produced using the following production factor combinations:
<table>
<tr><td>L</td><td>10</td><td>6</td><td>3</td><td>1</td></tr>
<tr><td>K</td><td>2</td><td>7</td><td>13</td><td>16</td></tr>
</table>
The cost of a labor unit is 100 c. u., and the capital unit cost is 60 c. u. Which of the combinations of these two factors is the most appropriate? Suppose that the labor unit cost has increased to 110 c.u. What should be the entrepreneur's decision?</td></tr>
<tr><td>18</td><td>The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, derivative in direction, the gradient of a function, differentiation of complex functions, partial derivatives, and higher orders differentials, extremum of a bivariate function, least squares method</td><td>The ability to effectively process and interpret large amounts of information using various methods of data interpretation, the ability to create logical arguments, and the ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form</td><td>A firm that produces televisions chooses one of three production technologies, each different in the combination of resources.
<table>
<tr><td></td><td colspan="6">Technologies</td></tr>
<tr><td></td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">C</td></tr>
<tr><td></td><td>L</td><td>K</td><td>L</td><td>K</td><td>L</td><td>K</td></tr>
<tr><td>1</td><td>12</td><td>4</td><td>9</td><td>6</td><td>7</td><td>8</td></tr>
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<tr><td>5</td><td>62</td><td>8</td><td>27</td><td>24</td><td>23</td><td>37</td></tr>
</table>
1. Determine which technology the firm will choose for each production.
2. Calculate the total costs for each level of production.
3. Suppose the labor cost per unit increases to 300 c.u., and the capital cost does not change. Will this change affect the choice of technology?</td></tr>
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Table 1 – continued from previous page

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19	The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, derivative in direction, the gradient of a function, differentiation of complex functions, partial derivatives, and higher orders differentials, extremum of a bivariate function, differentiation of implicit functions, extremum of the bivariate functions, least squares method	The ability to effectively process and interpret large amounts of information using various methods of data interpretation, the ability to create logical arguments, and the ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form	A firm produces T-shirts using L manual labor and K equipment. 1. The production function is as follows: $y = K \cdot L$. Determine all combinations of factor inputs producing 100 T-shirts. 2. The firm pays employees a wage of $w = 4$, and the use of equipment costs $r = 1$. How many factors of production can the firm consume at total costs $C = 50$? 3. What combination of factors minimizes the cost of producing 100 T-shirts? 4. What will be the maximum level of output at maximum costs $C = 400$? 5. How will this combination change if the wage rate decreases to $w = 1$? Fill in the table. Labor cost – 8 c. u. per unit. Capital cost – 12 c. u. per unit. Choose the best combination of resources. Option | L | K A | 11 | 2 B | 4 | 6 C | 8 | 6 D | 8 | 4 E | 3 | 9

Continued on next page

Table 1 – continued from previous page

№	Topics of Calculus	Skills	Problems
20	The domain of a bivariate function, partial derivatives of a bivariate function and its differentials of the first order, complete increment and complete differential of the first order, derivative in direction, the gradient of a function, differentiation of complex functions, partial derivatives and higher orders differentials, differentiation of implicit functions, extremum of the bivariate functions	The ability to analyze a situation or problem in a broad context, concerning all factors that may affect it, the ability to create logical arguments, and the ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form	Investigate the Cobb-Douglas production function $y = 0.2715L^{0.1113}C^{0.9356}$, calculating: 1) average efficiency of fixed assets; 2) marginal efficiency of fixed assets; 3) marginal rate of equivalent substitution of fixed assets by the number of employed population; 4) elasticity coefficient by fixed assets; 5) elasticity coefficient of resource substitution; 6) average efficiency of the number of employed; 7) marginal efficiency of the number of employed; 8) marginal rate of equivalent substitution of the number of employed by fixed assets; 9) elasticity coefficient by the number of employed; 10) elasticity coefficient of resource substitution. Conclude the economic analysis of the function you constructed
21	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, differentiation of implicit functions, extremum of the bivariate functions	The ability to create logical arguments, develop a consistent proof, and exclude unfounded assumptions	The production function is determined as the Cobb-Douglas one. To increase output by 5%, it is necessary to grow funds by 10% or the number of employees by 15%. In 2006, one worker produced products worth 2000 c.u. per month, and there were 1000 employees in total. Fixed assets were estimated at 4 million c.u. Write down the production function, the value of the average assets return and average labor productivity, and the output elasticity by labor and funds
22	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, differentiation of implicit functions, extremum of the bivariate functions	The ability to create logical arguments, the ability to gather enough information for a reasoned choice and draw conclusions	The production function is determined as the Cobb-Douglas one. To increase output by 23%, it is necessary to grow funds by 46% or the number of employees by 69%. In 2016, one employee produced goods for 3000 c. u. per the month, and there were 125 employees in total. Fixed assets are estimated at 5 million c. u. Determine the production function y, the value of the average assets return and average labor productivity, and the elasticity of output for labor $E_L(y)$ and for funds $E_K(y)$.

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Table 1 – continued from previous page

№	Topics of Calculus	Skills	Problems
23	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, differentiation of implicit functions	The ability to analyze a situation or issue in a broad context, concerning all factors that may affect it, the ability to create logical arguments	A certain production can be described by applying the Cobb-Douglas function. It is known that each employee produces goods worth 100,000 c. u. per month. The total number of employees is 1,000 people. Fixed assets are estimated at 64 million c. u. It is known that to increase production by 3%, it is necessary to increase either the cost of assets by 6% or the number of employees by 9%. Determine the Cobb-Douglas function for this enterprise using its elasticity coefficients
24	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, extremum of a bivariate function, differentiation of implicit functions	The ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form	The output Q produced by a firm is determined by the Cobb-Douglas production function $Q = 25 \cdot K^{\frac{1}{3}} \cdot L^{\frac{2}{3}}$, where K is the volume of invested capital, and L is the volume of labor. Schematically construct isoquants (level lines) corresponding to the output of $Q_0 = 50$ output units. Give an economic interpretation.
25	Partial derivatives of a bivariate function and its differentials of the first order, partial derivatives and higher orders differentials, extremum of a bivariate function	The ability to create logical arguments, the ability to clearly and effectively express one's thoughts and analytical conclusions in written or oral form	To expand production, the entrepreneur allocates 150,000 c. u. It is known that if x thousand c. u. are spent on the purchase of equipment, and y thousand c. u. on the salary of new employees, then the increase in production volume will be $Q = 0.001x^{0.6}y^{0.4}$. How should the allocated funds be distributed so that the increase in production volume is maximum?

3. Results

An experiment was conducted to check the practicality of problems collected based on the production function. The basis for the experiment were students of the following specialties: 051 Economics; 072 Finance, Banking and Insurance; 073 Management; 075 Marketing; 076 Entrepreneurship, Trade and Stock Exchange Activities from the National University of 'Kyiv Mohyla Academy' and Sumy State University.

The main objectives of the experiment were: research into the learning bivariate Functions by students of economic specialties in the course of Higher Mathematics; developing and implementing the problems system that would demonstrate the application of Cobb-Douglas functions; the impact of the problems system on the economic major's students' analytical skills, analysis of the experiment results. The control group (CG) and the experimental group (EG) were formed at the beginning of the experiment to determine the standard of students' mathematical preparation. It was testing on the topics of the section Differentiation of univariate functions. After the experiment, the students were tested again. At the beginning and end of the experiment, students in the control and experimental groups solved the same tests developed by the authors of this study.

The experiment lasted three weeks while learning bivariate functions with help the problems systematized based on the production function. 237 students participated in the experiment, of which 120 students were included in the control group and 117 students in the experimental group:

- the control group (CG) included students from the following specialties: 051 Economics; 076 Entrepreneurship, Trade and Stock Exchange Activities from Sumy State University and National University of Kyiv Mohyla Academy. Learning the section Differentiation of Bivariate Functions in these groups was carried out according to the standard methodology;
- the experimental group (EG) included students from the following specialties: 072 Finance, Banking, and Insurance; 073 Management; 075 Marketing from Sumy State University and National University of Kyiv Mohyla Academy. Learning the section's topic was with help using a developed problems system in which production functions were applied.

The control test, offered to students at the final stage of the experiment, contained three types of problems.

The first type was dedicated to the partial derivatives of a bivariate function, namely, finding the derivative in the direction, the gradient of the function, and the first-order complete differential given in the explicit form. Such types of problems test the ability to calculate partial derivatives of bivariate functions, the ability to create logical arguments, the ability to identify problems, set specific goals for their solution and find optimal ways to achieve these goals, the ability to evaluate ideas and solutions using objective criteria and avoiding biases or stereotypes (*Skill 1*).

The problems of the second type were related to the differentiation of complex functions, higher orders partial derivatives and differential of implicit functions, and extremum of bivariate functions. These types of problems were aimed at testing students' ability to work with bivariate functions, the ability to develop consistent proof and eliminate unfounded assumptions, the ability to effectively process and interpret large amounts of information, and the ability to clearly and effectively express their thoughts and analytical conclusions in written or oral form (*Skill 2*).

The third type of problem included applying production functions. Namely, calculating the conditional extrema of bivariate functions, finding the global extrema of a bivariate function in the domain, and finding the function's partial elasticity. These types of problems were used for testing the ability to create mathematical models, as well as the ability to adapt the results obtained by the conditions of the problem, the ability to make a specific conclusion, the ability to analyze a situation in a broad context, concerning all the factors that may affect it, the ability to build logical arguments, develop consistent proof and exclude unfounded assumptions, the ability to evaluate alternatives, collect enough information for a reasonable choice and draw conclusions, the ability to clearly and effectively express one's thoughts in written or oral form (*Skill 3*).

The percentage of correctly solved problems of the first, second, and third types, that is, the level of formation of the corresponding skill in each group, is presented in the table (table 2) and the diagram (figure 1).

The table and diagram show that the percentage of skills formation 1, 2, and 3 in the experimental group is higher than in the control group, for the first skill by 9.6%, for the

Table 2. Percentage of formation of the corresponding skill in the CG and EG.

	Skill 1	**Skill 2**	**Skill 3**
Percentage in CG	81.3	68.1	55.2
Percentage in EG	90.9	84.1	79.2

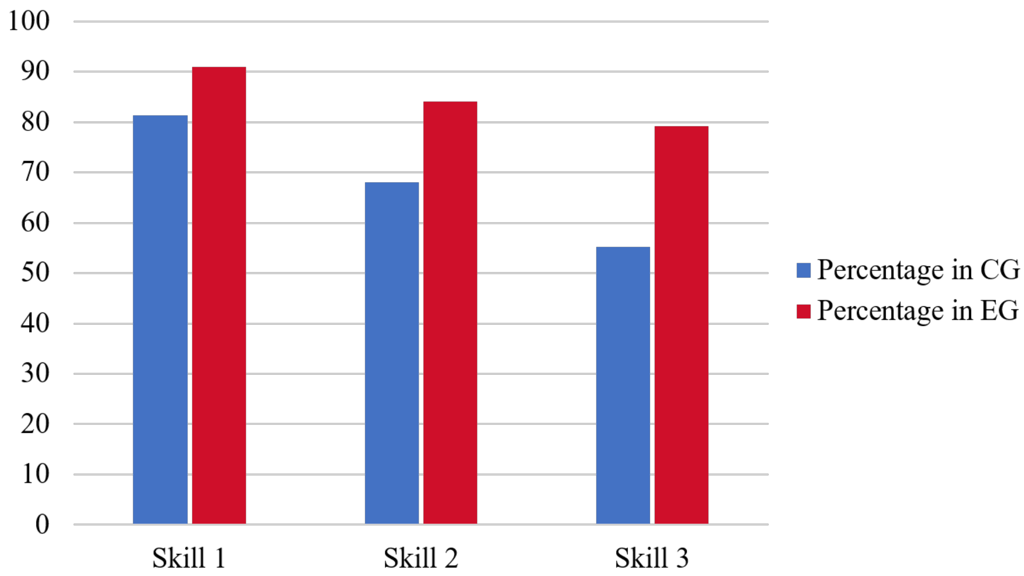

Figure 1. Results of the control test at the final stage of the experiment.

second skill – by 16.0%, and for the third skill – by 24.0%, which confirms the practicality of the problems system where production functions are used.

4. Discussion

Economic instability constantly accompanies as developed as developing countries, so the issue of training high-quality economic specialists is always relevant. While studying would-be economists, attention should be paid to their mathematical training because it allows students to acquire those skills and abilities that help them to master, research, and analyze various economic processes and phenomena in the future. The works of researchers Vlasenko et al. [25] and Zhelavskyi [26] confirm that mastery of mathematical modeling skills ensures the development of analytical skills of economic education students through involvement in real production processes.

It is offered that problems with Cobb-Douglas production functions be picked up while training students who major in economics. Solving such tasks contributes to developing students' analytical skills through their mathematical modeling. They study to understand the context, analyze data, make decisions, solve issues, and communicate results. Such skills allow would-be economists to train in the competencies needed in their future professional activities.

The problems' systematization on the base of the Cobb-Douglas function was carried out with the help of a combination of table and clustering methods. Individual clusters were to contribute to the development of the ability to analyze a situation or problem in a broad context, taking into account all the factors that may affect it, the formation of the ability to effectively process and interpret large amounts of information using various methods of data interpretation, improving the ability to evaluate alternatives, collect enough information for a reasonable choice and concluding, etc.

The developed system of problems was involved while learning bivariate functions. It took three practical classes. Before presenting the tasks at the lectures, the versatility of applying the Cobb-Douglas function. There are such areas as forecasting trends in industrial development, analyzing the impact of certain climatic variables on productivity, assessing and analyzing the sustainability of the metallurgical sector, forecasting the development of farming in certain areas, etc. In addition, problem-solving was accompanied by the presentation of economic terms considered in the tasks. Working with this terminology introduces students to it and lays the ground for mastering professional disciplines. The problem-solving contributes to the immersion of its participants in real business situations keeping the proper level of motivation and interest in learning, ensuring mastery of the topic and the development of analytical skills.

5. Conclusions

Stabilization and improvement of the country's economic processes require qualified specialists. The training of high-quality economists should begin with their mathematical preparation and the development of their analytical skills. The level of such skills is ensured via mathematical modeling and training, which depends on problem-solving.

The widespread application of the Cobb-Douglas production function in many branches of the economy explains its involvement as the basis for developing the problems system. The problems' systematization can be done by combining the table method with the clustering one. The first method simplifies the problems' submission; the second ensures the filling of the cluster through the problems' correspondence to certain abilities, the development of which results in forming analytical skills of its participants.

The developed system of problems can be offered to students who majored in economic specialties while mastering Multivariate Functions. The problems' presentation should be preceded by a lecture that describes the Cobb-Douglas function application in various areas of economics and business. Economic terminology and its connection with mathematics should be presented and explained when solving problems in three practical classes. Such practice improves the students' motivation, demonstrates the connection of Mathematics with Economics, and develops students' analytical skills. The problems' systematization, which considered these conditions, ensured the study's novelty.

The system of problems can be divided into three types. The first type of problem contributes to developing skills in computing partial derivatives of Multivariate Functions, constructing logical arguments, identifying issues, setting specific goals for their solution and finding optimal ways to achieve them, and evaluating ideas, proofs, or solutions.

The problem system can be divided into three types. The first type introduced into Mathematics learning contributes to developing skills in calculating partial derivatives of Multivariate Functions, constructing logical arguments, identifying problems, setting specific goals for their solution and finding optimal ways to achieve them, and evaluating ideas, proofs, or solutions. The second type of problem can help work out higher-order derivatives, contribute to the development of consistent proofs and the exclusion of unfounded assumptions, and process and interpret significant amounts of information, clearly expressing one's thoughts and analytical conclusions in various forms. The problems of the third type for calculating the conditional extremum of Multivariate Functions, finding their largest and smallest values, make it possible to develop the skills of adapting the obtained results concerning the condition of the task, analyzing the situation or issue in a broad context, taking into account the factors that may affect it, and evaluating alternatives.

The running experiment confirmed the practicality of the implemented problem system in the experimental groups.

Future research aims to systematize the problems to introduce students to mathematical modelling while learning the first-order approximation theorem for univariate functions and their multivariate version.

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