NON-CLASSICAL BOUNDARY VALUE PROBLEM FOR THE HEAT CONDUCTION EQUATION

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The first boundary value problem for the heat conduction equation was studied in [1]. We provide the first proof of a formula for solving the non-classical boundary value problem, where the temperature is specified at the left end of a homogeneous rod and its flux at the right end.

Let $\varphi(x): R \to R$, $\mu_1, \mu_2: [0, T] \to R$ be continuous and bounded functions, $u(x,t) \in C_{x,t}^{2,1}(R \times [0,T])$. For the problem

$$\frac{\partial u(x,t)}{\partial t} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2}, \ \ t > 0, \ \ x \in R,$$

$$u(x,0) = \varphi(x), \ \ x \in R,$$

$$u(0,t) = \mu_1(t), \ \ t > 0,$$

$$\frac{\partial u(l,t)}{\partial x} = \mu_2(t), \ \ t > 0$$

a formula for the solution is established

$$u(x,t) = \sum_{k=1}^{\infty} \frac{\int_0^l \hat{\varphi}(x) \sin\frac{\pi}{l}\left(k-\frac{1}{2}\right) x\, dx}{\int_0^l \sin^2\frac{\pi}{l}\left(k-\frac{1}{2}\right) x\, dx} \exp\left\{-a^2 \left(\frac{\pi}{l}\left(k-\frac{1}{2}\right)\right)^2 t\right\} \times$$

$$\times \sin\frac{\pi}{l}\left(k-\frac{1}{2}\right) x + \ \mu_1(t) + x\mu_2(t),$$

where $\hat{\varphi}(x) = \varphi(x) - \mu_1(0) - x\mu_2(0), \ t > 0, 0 < x < l.$

References

1. A.N. Tikhonov, A.A. Samarskiy. Equations of Mathematical Physics. – Nauka, 1972. – 735 p.