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USE OF VECTOR LOGISTIC SMOOTH TRANSITION AUTO-REGRESSION MODEL IN FINANCIAL MODELING

In various areas of financial modeling, the data may exhibit nonlinear or regime switching behavior, also called asymmetries. Generally, there may be three approaches to model such cases:

1. Model the regimes using separate models: extract the periods of parameter stability and estimate separate models for each of these periods;
2. Use threshold approach: define a regime-switching indicator and condition the model parameters on a certain value threshold of this indicator. Essentially, this is a reiteration of the first approach above, with the difference being that the regime periods are auto-selected rather than extracted manually. However, in order to estimate every regime, each state has to be represented by a sufficient number of observations;
3. Use smooth transition approach: instead of splitting the sample in pieces, assume that at every point of time the underlying process is a weighted sum of two or more equations, and the weights may vary in range (0; 1), defining the "mix" of regimes at every single point in time.

The above is valid for both univariate and multivariate setups. In particular, the Vector Auto-regression family of models, widely used for monetary policy analysis, also has a threshold (TVAR) and smooth-transition (VLSTAR) variations.

The Vector Logistic Smooth Transition Auto-regression (VLSTAR) model is an extension of the Smooth Transition Auto-regression (STAR) framework, applied in a multivariate context. It is used for capturing nonlinear dynamics in time series data, which makes it particularly useful in financial modeling where data often exhibit regime-switching behavior. Unlike traditional linear models, the VLSTAR model allows for gradual transitions between different regimes or states, enabling a more realistic representation of economic or financial systems that shift over time.

The essence of the VLSTAR model lies in its ability to model variables whose relationships change depending on an underlying process. In a standard Vector Auto-regression (VAR) model, the parameters remain constant across time. However, in many real-world situations, financial variables exhibit different dynamics under various conditions, such as during periods of economic expansion versus recession. The VLSTAR model introduces smooth transitions between these different regimes, controlled by a logistic function that depends on an observable transition variable. This setup allows the model to accommodate gradual shifts in behavior rather than abrupt changes, making it more flexible and capable of capturing complex non-linear patterns.

Mathematically, the VLSTAR model can be represented as:

$$\mathbf{Y_t} = \Phi_1\mathbf{X_{t-1}} + \Phi_2\mathbf{X_{t-1}}G\left(\mathbf{Z_{t-1}};\gamma, c\right) + \boldsymbol{\varepsilon_t}$$

where $\mathbf{Y_t}$ is the vector of endogenous variables, $\mathbf{X_{t-1}}$ represents lagged explanatory variables,Φ_1 and Φ_2 are parameter matrices, and $\boldsymbol{\varepsilon_t}$ is a vector of error terms. The key component is the logistic transition function $G\left(\mathbf{Z_{t-1}};\gamma, c\right)$, which depends on a transition variable $\mathbf{Z_{t-1}}$, a smoothness parameter γ, and a threshold parameter c. The logistic function G controls the degree to which the model transitions between regimes, ranging from zero to one, thus smoothly altering the influence of Φ_2. Studies such as those by Teräsvirta (1994) and Granger & Teräsvirta (1993) have laid the theoretical foundations for STAR models, while more recent works have extended these concepts to the vectorized, multivariate context seen in the VLSTAR model.

In particular, the smooth-transition approach is valuable in the cases, where time series data is scarce, for instance the aggregated macroeconomic indicators for Ukraine. This kind of data, on one hand, represents a likely nonlinear dynamic, which make the application of simpler linear models challenging or unlikely. On the other hand, the observation counts of even twenty years result in 240 monthly observations, which might not be a large enough dataset for alternative methods, including widely-used Threshold Vector Autoregression (TVAR) approach. Figure 1 illustrates VLSTAR modeling of the dynamic relation between financial stress index (FSI), key policy rate (KPR), consumer price index (CPI) and real GDP (RGDP_M) of Ukraine from July 2008 to June 2023, where the regime is defined by the level of FSI.

As seen from the Figure 1, the VLSTAR model has two advantages over the same TVAR setup. Firstly, TVAR will have to estimate the high-stress periods based on only ~44 observations of 180, which imposes restrictions on the model complexity. With the same setup, VLSTAR uses all 180 observations to estimate both regime. Secondly, the “critical” regimes have different levels of impact for different indicators, adding another layer of analysis of the phenomenon.

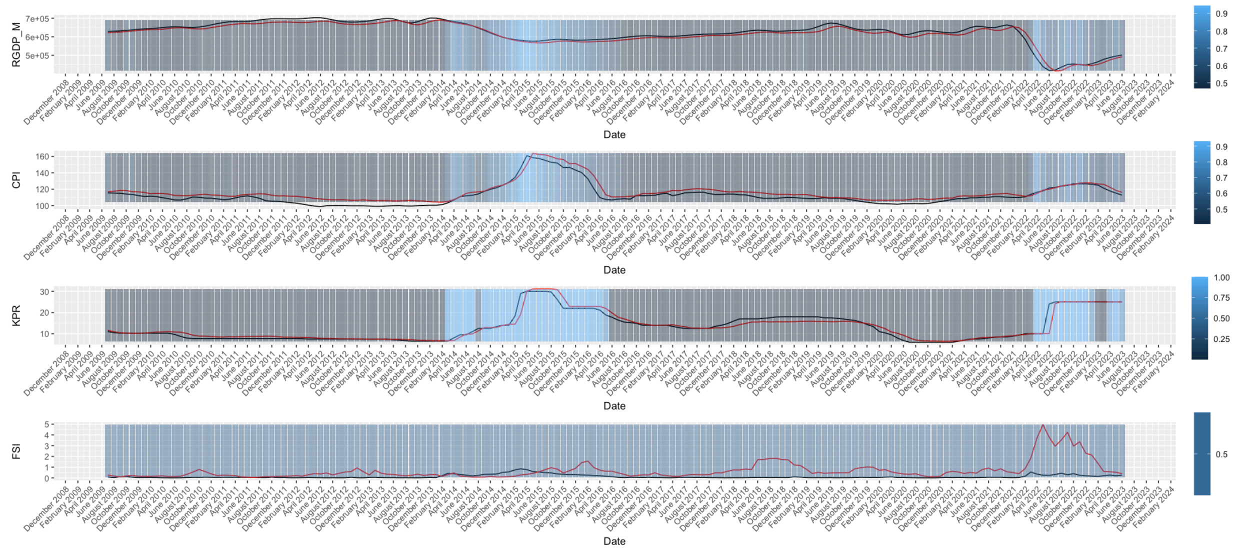

Figure 1. VLSTAR model of FSI-stress-related asymmetry in KPR pass-through

Recent works have demonstrated the application of the Vector Logistic Smooth Transition Auto-regression (VLSTAR) model in various aspects of financial modeling, primarily due to its ability to capture non-linear dynamics and regime shifts effectively.

1. Exchange Rate Pass-Through (ERPT): Recent studies have utilized the VLSTAR model to analyze how exchange rate fluctuations influence inflation across different pricing stages. For example, a study by Kwon & Shin (2023) focused on Sweden, where they used a logistic smooth transition VAR model to capture regime-dependent dynamics in ERPT, revealing that the pass-through effect varies significantly between high and low inflation periods. This application underscores the model's ability to adapt to changes in inflation regimes, making it particularly valuable for policymakers who need to understand non-linear relationships in exchange rate economics.
2. Market Regime Detection: Another recent study compared the use of VLSTAR with other models like TVAR (Threshold VAR) and unsupervised learning techniques for detecting market regimes. The study analyzed the realized covariances of financial instruments, applying VLSTAR to identify structural changes in market conditions that standard linear models might miss. This highlighted the model's strength in financial time series that exhibit volatility clustering or regime shifts.
3. Forecasting and Risk Management: VLSTAR models have been used for improving forecasts in financial contexts, such as predicting asset prices or volatility. For instance, recent methodological advancements have focused on

enhancing prediction accuracy through approaches like Monte Carlo simulations and bootstrap methods to generate interval forecasts from VLSTAR models. These tools help in robust risk management by providing clearer insights into future trends under different scenarios.

Summing up, vector logistic smooth transition models are an intuitive and efficient instrument to model nonlinear dynamics in a multivariate setup, and it is especially useful in experiments with limited samples or low availability of statistical data.

References

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