STUDY OF NUMERICAL AND ANALYTICAL SOLUTIONS OF A GENERALIZED BOUNDARY VALUE PROBLEM FOR THE HEAT CONDUCTION EQUATION

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In the boundary value problem from [1] let $\varphi(x) = \cos x,\ x \in R$, $\mu_1(t) = \mu_2(t) = 1$. Then, its solution

$$u(x,t) = \sum_{k=1}^{\infty} 2B_k \exp\left\{-\left(\pi\left(k-\frac{1}{2}\right)\right)^2 t\right\} \cdot \sin \pi\left(k-\frac{1}{2}\right)x + x + 1,$$

where

$$B_k = \frac{\pi\left(k-\frac{1}{2}\right) + (-1)^k \sin(1)}{\pi^2\left(k-\frac{1}{2}\right)^2 - 1} - \frac{1}{\pi\left(k-\frac{1}{2}\right)} - \frac{(-1)^{k+1}}{\left(\pi\left(k-\frac{1}{2}\right)\right)^2},$$

we approximately compute it using the Python language and plot it (Fig.1).

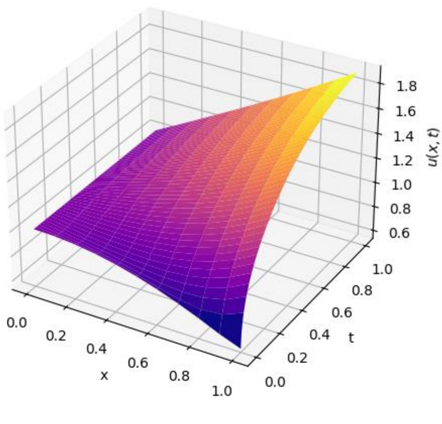

Fig. 1

We compare the values of the analytically found solution with those obtained by the finite difference method. We compute the relative change expressed in percentages relative to the value found by the analytical method:

$$\Delta u = \frac{u_{FD} - u_A}{u_A} \cdot 100\%,$$

where u_{FD} is the value found by the finite difference method and u_A is the value found by the analytical solution. Let's compare the graphs (Fig. 2).

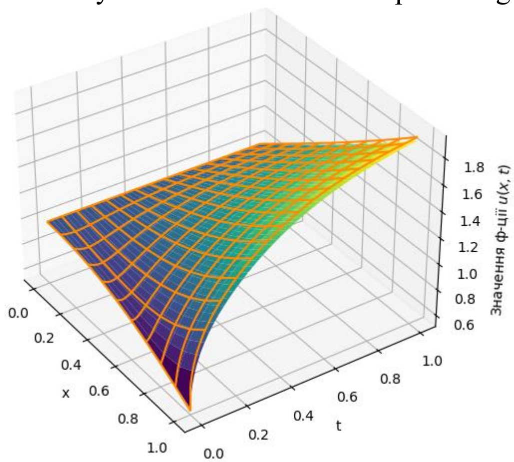

Fig. 2

Thus, the computed values of the solution obtained by the finite difference method and the results of the numerical investigation of the analytical solution of this problem match with maximum and average relative errors of +7.03% and ±1.82%, respectively.

The graphs of the numerical and analytical solutions coincide over the entire range of investigated time and space values.

Further improvements in the accuracy of the numerical solution can be achieved by adjusting grid parameters – reducing spatial step size and increasing the number of computational iterations.

References

1. I. Drin, S. Drin, Y. Drin, M. Lutskiv. Non-classical boundary value problem for the heat conduction equation // International Scientific Conference "Current Issues in Mathematical Modeling and Computational Methods." Kyiv. — 2024.