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A search for regular K_3-irregular graphs

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For a given graph F, the *F-degree* of a vertex v in G is the number of subgraphs of G, isomorphic to F, to which v belongs. A graph G is called *F-irregular* if all vertices of G have distinct F-degrees. In [1], the existence of regular K_3-irregular graphs was posed as an open question. Examples of such graphs for regularities $r \in \{10, 11, 12\}$ were constructed in [2]. We analytically prove that no such graphs exist for $r \leq 7$, present such a graph for $r = 9$, and establish bounds on the order for $r = 8$. We will use $t(v)$ to denote the K_3-degree.

Lemma 1. *For any vertex v of a r-regular K_3-irregular graph, it holds* $\lceil \frac{n-r-1}{2} \rceil - \frac{nr}{2} + r^2 \leq t(v) \leq \binom{r}{2} - \lceil \frac{r}{2} \rceil$.

Theorem 2. *No regular K_3-irregular graph exist for $r \leq 7$.*

Using evolutionary algorithms, we found examples of regular K_3-graphs for various values of r. Notably, $r = 9$ is the smallest regularity for which we discovered such graphs. We present an example of a 9-regular K_3-irregular graph on 24 vertices. The adjacency list of this graph is provided in the following table:

v	$t(v)$	$N(v)$
0	20	$\{1, 2, 3, 4, 5, 6, 7, 11, 12\}$
1	23	$\{0, 2, 3, 4, 5, 6, 7, 9, 10\}$
2	24	$\{0, 1, 3, 4, 5, 6, 7, 13, 23\}$
3	19	$\{0, 1, 2, 4, 5, 7, 8, 19, 23\}$
4	22	$\{0, 1, 2, 3, 5, 6, 7, 8, 14\}$
5	26	$\{0, 1, 2, 3, 4, 6, 7, 9, 23\}$
6	21	$\{0, 1, 2, 4, 5, 7, 8, 9, 23\}$
7	25	$\{0, 1, 2, 3, 4, 5, 6, 10, 23\}$
8	3	$\{3, 4, 6, 11, 12, 16, 20, 21, 22\}$
9	6	$\{1, 5, 6, 11, 12, 13, 20, 21, 22\}$
10	5	$\{1, 7, 11, 12, 14, 16, 20, 21, 22\}$
11	10	$\{0, 8, 9, 10, 14, 15, 16, 17, 18\}$

v	$t(v)$	$N(v)$
12	4	$\{0, 8, 9, 10, 15, 17, 18, 19, 23\}$
13	14	$\{2, 9, 14, 15, 16, 18, 20, 21, 22\}$
14	15	$\{4, 10, 11, 13, 15, 17, 19, 21, 22\}$
15	18	$\{11, 12, 13, 14, 16, 18, 19, 20, 21\}$
16	11	$\{8, 10, 11, 13, 15, 17, 18, 19, 23\}$
17	16	$\{11, 12, 14, 16, 18, 19, 20, 21, 22\}$
18	17	$\{11, 12, 13, 15, 16, 17, 20, 21, 22\}$
19	13	$\{3, 12, 14, 15, 16, 17, 20, 21, 22\}$
20	7	$\{8, 9, 10, 13, 15, 17, 18, 19, 23\}$
21	12	$\{8, 9, 10, 13, 14, 15, 17, 18, 19\}$
22	8	$\{8, 9, 10, 13, 14, 17, 18, 19, 23\}$
23	9	$\{2, 3, 5, 6, 7, 12, 16, 20, 22\}$

Proposition 3. *If an 8-regular K_3-irregular graph exists, its order is in range* $18 \leq n \leq 22$.

Conjecture 1. *No 8-regular K_3-irregular graph exists.*

[1] Chartrand G., Erdős P., Oellermann O. R. How to define an irregular graph. Coll. Math. J., 1988, **19**, 36–42.

[2] Stevanović D., Ghebleh M., Caporossi G., et al. On regular triangle-distinct graphs. Comp. Appl. Math., 2024, **43**, article number 336.

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