Attempts at Computing Gröbner Bases without S-polynomials whenever Possible

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Abstract

In this note we lay down some thoughts on computing Gröbner bases using subresultant polynomial remainder sequences (prs's) to eliminate variables.

In this way we try to minimise S-polynomial computations and, if possible, to completely avoid them.

A personal note to us by Bruno Buchberger — at the Polynomial Computer Algebra conference (PCA-2015) in St. Petersburg, Russia — served as the motivation for our effort.

Outline of our Algorithm

In order to understand our method, we first give a brief presentation of Buchberger's original algorithm for computing the Gröbner basis of a system of polynomials equations in many variables with rational coefficients.

Buchberger's Original Algorithm

- for *each* pair of polynomials compute the S-polynomial,

- reduce *each* polynomial by all others,

- the process terminates when no new S-polynomials appear.

Our Algorithm — working version

1. for *each* pair of polynomials A, B compute C, the last member of the subresultant prs of A, B,

    (a) if $\text{lc}(B) = 1$, then we replace the pair A, B by the pair B, C,

    (b) if $\text{lc}(B) \neq 1$, then we replace the pair A, B by the triplet A, B, C,

2. reduce *each* polynomial by all others,

3. the process terminates when no new polynomials C appear.

Acknowledgments

References

[1] Bruno Buchberger: private communication. Tue, 14 Apr 2015 10:39:56 +0200