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MODELLING OF NATURAL AND ECONOMIC SYSTEMS` SUSTAINABILITY

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The monograph considers theoretical and practical issues of sustainability modelling of natural and economic systems. The modelling of mineral resources extraction processes, namely minerals for construction, power and coking coal has been made. Particular attention has been paid to ensuring sustainable development of territorial complexes aiming at rational balance of environmental costs, attracting environmental investment, optimizing emissions of harmful substances and improving waste management. The sustainability statistical modelling of aquatic complexes` ecosystems has been fulfilled.

Monograph will be useful to scholars, entrepreneurs, experts in the field of economics, management and administration, educators, graduate students, students and all those who wish to improve their command in English.

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INTRODUCTION OR HOW TO MAKE A CORRECT AND EFFECTIVE MODEL OF NATURAL AND ECONOMIC SYSTEMS` SUSTAINABILITY

Sustainable development has been recently considered not as a general paradigm of balanced and stable growth, but as a specific guide, a system of relationships between certain elements and players in the socio-economic environment at the basic decision-making level. United territorial communities form the baseline in Ukraine, as well as their development amid wider territorial and administrative reforms, acquiring specific features and qualities.

Modelling of sustainable development potential for basic territorial entities is part of the general progress to enhance the rational use of natural resources, involving active and harmonious application of economic and social components. It is not efficient and promising to consider united territorial communities without taking into account their general and specific regional characteristics, therefore, in the proposed monographs, the issue of sustainability, as a general and local growth process, is studied through the methodology of natural and economic systems. It is possible to distinguish features and factors having the biggest effect on the sustainable management at the level of natural and economic systems. This type of economic activity has variable and changing characteristics, and therefore the issue of sustainability modelling aimed at further use of modelling results for the specific managerial decision-making is a vital task. One of the important and, traditionally, determining economic factors at the local business level is application of current natural resources. In particular, mineral resources. Therefore, effective models of ecological and economic balancing for mineral resources and the corresponding stock have traditionally attracted scholars` attention. Indeed, if one notes that the mineral resources exploitation is one of the main sources of filling local budgets, then the issue of the process` modelling – from the direct extraction to income distribution for different level budgets taking into account ecological and economic components and determinants, is a task of particular importance.

Considerable attention was paid to the dynamic balance of environmental management and its practical part aiming at finding out and realizing why this kind of balancing should answer the question of sustainable interests and opportunities coordination within the general system of regional environmental management. The general system involves not only local or regional development trends, but also global interactions, their impact on dynamic characteristics that are of particular importance for sustainable economy and environmental management. The environmental Kuznets curve and its interpretation for different regimes of management have been taken into consideration. Ukrainian scholars do not pay attention to the environmental Kuznets curve application for solving various ecological and economic problems, especially if one pays attention to the results practical application. Therefore, the priority of the authors is to implement study results of the processes and factors to model the Kuznets curve for basic territorial entities.

Based on the conducted studies, general models have been proposed as well as their interpretation for the territorial development`s decision-making systems. Sustainable development of local areas having regional specificity based on environmental development factors and their economic interpretation have been proposed and substantiated. The interpretation takes into account the "turning point" – a point of non-return to the desired development options due to the variable-based circumstances or determinants. For the first time, this "non-return point" has been studied precisely for basic territorial entities, which gives grounds to consider very promising results` application to form strategies, policies and programmes of various taxonomic and economic characteristics of the regions and united territorial communities of Ukraine.

Being scholars we sincerely and professionally hope for a broad interest of colleagues, managers and public figures in studying the proposed research. We are ready for fruitful cooperation for the further research of Ukrainian territories` sustainable development.

1 MODELLING OF MINERAL RESOURCES MANAGEMENT'S SUSTAINABLE DEVELOPMENT INDICATORS IN UKRAINE

1.1 Theoretical and methodological principles of the complex systems' development modelling

In general, the behavior of the complex system is described by the set of integro-differential equations of different orders.

Ordinary differential equations are equations containing functions of one variable and its derivatives or differentials. This attribute in general mathematical form may be written as:

$$F\ (x,\ y(x),\ y'(x), y''(x) \ldots . y^{(n)}(x)) = 0 \qquad (1.1)$$

The higher-order derivative containing the differential equation are called the order of a differential equation.

Solution of n-th order differential equations which is function y depending on argument x and arbitrary constants $C_1, C_2, \ldots . C_n$ is given by:

$$y = y(x,\ C_1,\ C_2, \ldots . C_n) \qquad (1.2)$$

Collocating it at the equation, it is transformed into the identity.

The general differential equation solution may not be satisfied to y, so it becomes:

$$F\ (x,\ y(x), C_1,\ C_2, \ldots . C_n) = 0.$$

In this case, it is called the general integral of differential equation.

If in the general solution (integral) of a differential equation instead of arbitrary constants we assign specific values to the arbitrary constant, then the resulting solution is called a particular solution of this equation.

Often, constants $C_1, C_2, \ldots . C_n$ are not chosen arbitrarily, but so that the solution of the equation satisfies some initial conditions.

The integral task of the differential equation and the corresponding number of initial conditions is called the Cauchy problem.

Often, economic phenomena are described by differential equations of the first and second order.

The Cauchy problem for the differential equation of the first order is:

$$F\,(x,\, y(x),\, y'(x)) = 0, \qquad y\,/_{x=xo} = y_0$$

Among the differential equations used to describe dynamic processes there is so called exponential equation, which has the form:

$$\frac{dy}{dx} = ky\,, \qquad (1.3)$$

where k – constant of proportionality.

This equation means that the rate of y change to the value of x is proportional to the current value of y. If y grows, $k > 0$, if y decays, $k < 0$.

The variables are divided in the equation, so

$$\frac{dy}{y} = k\,dx$$

$$ln\ /y/ = kx + ln\ C$$

$$y = Ce^{kx}$$

If the initial condition is

$$y\,/_{x=xo} = y_0,$$

we obtain

$$y_0 = C e^{kx_0},\ C = y_0\ e^{-kx_0},\ i.e.\ y = y_0\ e^{k(x-x_0)}.$$

Thus, the solution of equation (1.3) is exponential. If x takes values that form the arithmetic progression with difference Δx, then the relevant values form the geometric progression with the variable $e^{k\Delta x}$.

The equation (1.3) with $k > 0$ is called the law of natural growth (population, living cells, and crystals) and with $k < 0$ it is the law of radioactive decay.

Let us describe mathematically periodic nature of the complex system development in time.

Increment in the value of the subsystem dN quantitative characteristics in a certain interval will be proportional to the value of the subsystem N quantitative characteristics. The increment is proportional to the length of the small interval. Assuming this function`s property, which is considered as continuous, we obtain

$$dN = \varepsilon N\, dt\,, \qquad (1.4)$$

where ε is constant proportionality coefficient, reflecting the correlation of the increment rate of the subsystem $\frac{dN}{dt}$ quantitative characteristics to N. We define it as increment coefficient of the subsystem quantitative characteristic.

From the equation

$$\frac{dN}{dt} = \varepsilon N$$

by integrating we obtain

$$N = N_0\, e^{\varepsilon(t - t_0)}, \qquad (1.5)$$

where $T = t - t_0$.

This is the formula of the exponential law, which states that if the time increases in the arithmetic progression, then the quantitative characteristics of the subsystem change in the geometric progression.

It is possible to identify the figure ε, which characterizes the subsystem development rate. Actually, during the period $T = t - t_0$ the qualitative characteristics increase $e^{\varepsilon T}$ times, which is equal, less or greater that one depending on growth, decay or invariability of the subsystem quantitative characteristics. Herewith ε does not depend on the initial time period. Never the less the case is the ideal one, as we considered the external environment to be invariable.

If we consider the environment to be slowly changing, then in the short-run we assume the problem to be the same. In the case, the increment rate $\frac{1}{N}\frac{dN}{dt}$ is slowly changing simultaneously with the external environment. If this adjustment law would have been known, the differential equation becomes

$$\frac{dN}{dt} = \varepsilon(t)\, N\,(t). \qquad (1.6)$$

Its integration would give the desired function N.

Analyzing a few subsystems of one complex system, we find out the dependence of their increment rates $\frac{1}{N}\frac{dN}{dt}$ from the quantitative characteristics of the complex system and time period, if the external impact exists and has to be considered.

Thus, let us formulate hypotheses about increment rates dependence from the function *N* and time period. These hypotheses are written in the form of functions of *N*, from which we obtain differential equations:

$$\frac{1}{N_i}\frac{dN_i}{dt} = f(N_1, N_2, \dots N_p).$$

If the increment rates depend not only from the current value N_1, but also on the previous periods values, they cannot be regarded as functions of N_1. In this case, we study complex integro-differential equations.

The proposed model describes the ideal situation, i.e. the complex system is isolated from the external environment. In fact, it is not proved. That is why let us examine the general case of two subsystems of complex system functioning, which have increment rates λ_1 i λ_2 respectively. The subsystems` mutual influence affect their quantitative characteristics. We write the influence algebraically as the increment of subsystems` β_1 i β_2 quantitative characteristics corresponding to the number of interaction facts *n* (*n* – constant, significant enough).

We assume the increments currently occur, just in time. Then during the period *dt* subsystems ‘quantitative characteristics are increased by:

$$d_1 = \lambda_1 N_1 dt + \alpha N_1 N_2 \frac{\beta_1}{n} dt;$$

$$d_2 = \lambda_2 N_2 dt + \alpha N_1 N_2 \frac{\beta_2}{n} dt$$

respectively.

Hence, we get the system of differential equations:

$$\begin{cases} \dfrac{dN_1}{dt} = N_1(\lambda_1 + \mu_1 N_2), \\ \dfrac{dN_2}{dt} = N_2(\lambda_2 + \mu_2 N_1), \end{cases} \quad (1.7)$$

where $\mu_1 = \frac{\alpha\beta_1}{n}$ and $\mu_2 = \frac{\alpha\beta_2}{n}$.

From equations (1.7) we obtain for any case (with $N_1 > 0$, $N_2 > 0$)

$$\mu_2 \frac{dN_1}{dt} - \mu_1 \frac{dN_2}{dt} = \mu_2\lambda_1 N_1 - \mu_1\lambda_2 N_2,$$

$$\lambda_2 \frac{\frac{dN_1}{dt}}{N_1} - \lambda_1 \frac{\frac{dN_2}{dt}}{N_2} = \mu_1\lambda_2 N_2 - \mu_2\lambda_1 N_1.$$

Then we have

$$\mu_2 \frac{dN_1}{dt} + \lambda_2 \frac{1}{N_1}\frac{dN_2}{dt} - \mu_1 \frac{dN_2}{dt} - \lambda_1 \frac{1}{N_2}\frac{dN_2}{dt} = 0.$$

Integrating, we obtain

$$\mu_2 N_1 + \lambda_2 lnN_1 – (\mu_1 N_2 + \lambda_1 lnN_2) = const.$$

or

$$N_1^{\lambda 2} e^{\mu 2N1} = C\, N_2^{\lambda 1} e^{\mu 1N2}. \tag{1.8}$$

Taking into consideration that any limitations for the increment rate λ_1, λ_2, μ_1, μ_2 signs were imposed, we obtain large signs` probability set for these values. That is why we chose one case. Suppose that one subsystem is characterized by the increment rate $\varepsilon_1$, which we take as constant and positive ($\varepsilon_1 > 0$) in terms of the subsystem`s isolation. The other subsystem is also isolated; its increment rate is ε_2, which is constants and negative ($\varepsilon_2 < 0$).

When these subsystems interact, the quantitative characteristics of the first subsystem are less, the more intensively the other subsystem functions, and the second subsystem on the contrary is more intensive, the slower the first one operates.

The increment rates are identical:

$$\varepsilon_1 - \gamma_1 N_2 \text{ i } - \varepsilon_2 + \gamma_2 N_1,$$

where γ_1 i γ_2 are positive, constant values. These coefficients reflect the needs of both subsystems` quantitative characteristics respectively.

Now we obtain the system of differential equations, which illustrate the subsystems` quantitative characteristics:

$$\begin{cases} \frac{dN_1}{dt} = N_1(\varepsilon_1 - \gamma_1 N_2), \\ \frac{dN_2}{dt} = - N_2(\varepsilon_2 - \gamma_2 N_1), \end{cases} \quad (1.9)$$

$\varepsilon_1, \varepsilon_2, \gamma_1, \gamma_2 > 0.$

Let us check the system (1.9) for the compliance with the initial conditions, namely whether it is a system in which all flows or go to infinity, or tend to equilibrium (conservative system).

We multiply the first equation in (1.9) by γ_2, and the second by γ_1 and adding them we obtain:

$$\gamma_2 \frac{dN_1}{dt} + \gamma_1 \frac{dN_2}{dt} = \varepsilon_1\gamma_2 N_1 - \varepsilon_2\gamma_1 N_2.$$

We multiply the first equation by ε_2/N_1 the second by ε_1/N_2, add and obtain:

$$\varepsilon_2 \frac{1}{N_1}\frac{dN_1}{dt} + \varepsilon_1 \frac{1}{N_2}\frac{dN_2}{dt} = - \varepsilon_2\gamma_1 N_2 + \varepsilon_1\gamma_2 N_1.$$

So,

$$\gamma_2 \frac{dN_1}{dt} + \gamma_1 \frac{dN_2}{dt} - \varepsilon_2 \frac{d\ln N_1}{dt} + \varepsilon_1 \frac{d\ln N_2}{dt} = 0.$$

The last equation is integrated and we obtain an unambiguous integral:

$$\gamma_2 N_1 + \gamma_1 N_2 - \varepsilon_2 \ln_1 - \varepsilon_1 \ln_2 = const.$$

This integral may be written in the form:

$$F(N_1, N_2) = e^{-\gamma 2N1}\, e^{-\gamma 1N2}\, N_1^{\varepsilon 2}\, N_2^{\varepsilon 1} = const.$$

The expression $\iint \frac{d N_1 d N_2}{N_1 N_2}$ is the integral invariant.

Thus, the examined system is conservative.

Values N_1 i N_2 are periodic, so the equation (1.7) may be rewritten as:

$$\frac{d\ln N_1}{dt} = \varepsilon_1 - \gamma_1 N_2,$$

$$-\frac{d\ln N_2}{dt} = \varepsilon_2 - \gamma_2 N_1.$$

Integrating by the periods, we obtain:

$$0 = \varepsilon_1 T - \gamma_1 \int_{t_0}^{t_0+T} N_2 dt\,,$$

$$0 = \varepsilon_2 T - \gamma_2. \int_{t_0}^{t_0+T} N_1 dt\,.$$

As a result, we obtain:

$$K_1 = \frac{1}{T} \int_{t_0}^{t_0+T} N_1 dt\,,$$

$$K_2 = \frac{1}{T} \int_{t_0}^{t_0+T} N_2 dt\,.$$

Thus, K_1 i K_2 are average meanings of N_1 i N_2 during the period T.

Thus, average (during the period T) meanings of the subsystem`s quantitative characteristics do not depend on the initial conditions and equal to the values corresponding to the stationary state of increment rates $\varepsilon_1$, $\varepsilon_2$ and the coefficients of the need to provide certain subsystem`s quantitative characteristics γ_1 та γ_2 respectively.

We introduce functions:

$$n_1 = \frac{N_1}{K_1},$$

$$n_2 = \frac{N_2}{K_2},$$

that satisfy differential equations of the system (1.7):

$$\frac{dn_1}{dt} = \varepsilon_1 n_1 (1 - n_2),$$

$$\frac{dn_2}{dt} = - \varepsilon_2 n_2 (1 - n_1). \qquad (1.10)$$

The system research (1.10) is similar to the system research (1.9). It represents the special case of the system, when

$$\gamma_1 = \varepsilon_1,\ \gamma_2 = \varepsilon_2.$$

Let us use the obtained equations (1.10) to study insignificant changes.

Let us use the obtained equations (1.10) to study insignificant changes. Insignificant changes (fluctuations) exist for the case when initial n_1 i n_2 are close to 1.

Consider that

$$v_1 = n_1 - 1,\ v_2 = n_2 - 1,$$

then obtain, disregarding the product $v_1 v_2$. Equations (1.10) become:

$$\frac{dv_1}{dt} = -\varepsilon_1 v_2,$$

$$\frac{dv_2}{dt} = \varepsilon_2 v_1. \qquad (1.11)$$

Then by integration we have this system`s solution

$$v_1 = A\sqrt{\varepsilon_1}\ cos\ (\sqrt{\varepsilon_1\varepsilon_2}\ t + a),$$

$$v_2 = A\sqrt{\varepsilon_2}\ sin\ (\sqrt{\varepsilon_1\varepsilon_2}\ t + a),$$

where A, a – constants.

The solutions are obtained regarding that v_1 i v_2 satisfy $\frac{d^2y}{dt^2} + \varepsilon_1\varepsilon_2 y = 0$ from the general integral $y = \alpha\ cos\ (\sqrt{\varepsilon_1\varepsilon_2}\ t + \beta)$. If we assume $v_1 = \alpha\ cos\ (\sqrt{\varepsilon_1\varepsilon_2}\ t + \beta)$, the first equation of the system (1.11) demands that $v_2 = \alpha\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\ sin\ (\sqrt{\varepsilon_1\varepsilon_2}\ t + \beta)$.

Then, if $E = A\frac{\varepsilon_1\varepsilon_2}{\gamma_1\gamma_2}$, we have

$$N_1 = \frac{\varepsilon_2}{\gamma_2} + \frac{\gamma_1}{\sqrt{\varepsilon_1}} E\cos(\sqrt{\varepsilon_1\varepsilon_2}\, t + a),$$

$$N_2 = \frac{\varepsilon_1}{\gamma_1} + \frac{\gamma_2}{\sqrt{\varepsilon_2}} E\sin(\sqrt{\varepsilon_1\varepsilon_2}\, t + a).$$

So, the point (N_1, N_2) describes the ellipse with a center at Ωi with semi-axis $\frac{E\gamma_1}{\sqrt{\varepsilon_1}}$, $\frac{E\gamma_2}{\sqrt{\varepsilon_2}}$ with the help of periodic motion (in the positive direction from the vector ON_1 to the vector ON_2) in the period $\frac{2\pi}{\sqrt{\varepsilon_1\varepsilon_2}}$.

The period of insignificant changes (fluctuations) correlates only with individual increment rates and does not depend on interconnected increment rates, but the ratio of amplitudes $\frac{\gamma_1}{\gamma_2}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$ correlates with all four coefficients.

As a result we it is possible to draw functions $n_1(t)$ and $n_2(t)$ by plotting points (Fig. 1.1). Let us determine the economic essence of the curve in Fig. 1.1. As $n_1 = \frac{N_1}{K_1}$, and $n_2 = \frac{N_2}{K_2}$, where K_1 i K_2 are average meanings of N_1 i N_2 during the period T. Hence, we assume value n (n_1 and n_2) as a basic growth rate, when the average meaning of the subsystem`s quantitative characteristics is used as the base.

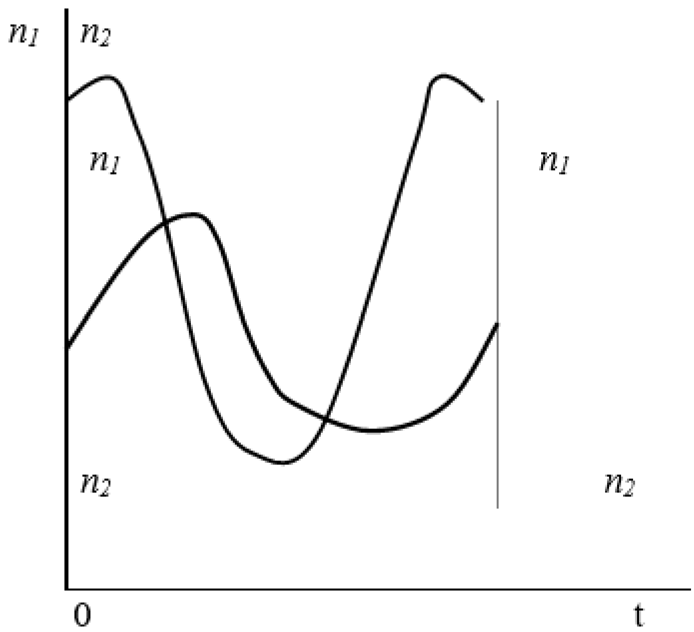

Fig. 1.1 Functions graphs $n_1(t)$ and $n_2(t)$

Thus, solving the system of equations (1.4) we used the terms with the following economic interpretation: increment rate of the subsystem`s quantitative characteristics $\lambda = \varepsilon$ (proportionality coefficient, which indicates correlation between the increment rate of the quantitative characteristic $\frac{dN}{dt}$ and $N$), increment rate of the need to provide the subsystem`s $\mu = \gamma$ quantitative characteristic, basic growth rate (base as average meaning) $n = \frac{N}{K}$, average meaning of the subsystem`s quantitative characteristic $K = \frac{\varepsilon}{\gamma}$; $\varepsilon = \frac{\ln \frac{N}{N_0}}{t - t_0}$.

As $K_1 = \frac{\varepsilon_2}{\gamma_2}$, and $K_2 = \frac{\varepsilon_1}{\gamma_1}$, as $\varepsilon = \frac{\ln \frac{N}{N_0}}{t - t_0}$, we may form dependencies $\varepsilon_1(t)$ and $\varepsilon_2(t)$, and find the values of coefficients $\gamma_1 = \frac{n_1 \varepsilon_2}{N_1}$ and $\gamma_2 = \frac{n_2 \varepsilon_1}{N_2}$, built their dependence on time t.

The average meanings of the subsystems` quantitative characteristics do not depend on the initial conditions and equal to values correlating with stationary state of coefficients.

The easiest way to make ε_1 and ε_2 variables not changing γ_1 and γ_2, is to contract values uniformly in time and proportionally to the subsystems` quantitative characteristics. If during the period $dt$ the first subsystem`s quantitative characteristics contract by $\alpha\lambda N_1 dt$ $(\alpha\lambda \geq 0)$ and the second subsystem`s quantitative characteristics contract by $\beta\lambda N_2 dt$ ($\beta \geq 0$), the equations (1.7) must be substituted for equations:

$$d_1 = (\varepsilon_1 - \alpha\lambda - \gamma_1 N_2)\, N_1 dt, \qquad (1.12)$$

$$d_2 = -\,(\varepsilon_2 + \beta\lambda - \gamma_2 N_1)\, N_2 dt,$$

which we obtain by substitution of ε_1 and ε_2 for $\varepsilon_1 - \alpha\lambda$ and $\varepsilon_2 + \beta\lambda$.

Numbers α and $\beta \geq 0$ characterize the ways of subsystem`s quantitative characteristics contraction, $\lambda \geq 0$ – contraction intensity.

Previously obtained results are used for the case where $\varepsilon_1 - \alpha\lambda > 0$. If there is the way of contraction, and the intensity λ is less than ε_1/λ, changes (fluctuations) take place. They are stopped, if λ accesses these values, because in this case there is the differential system (1.8) with $\lambda_1 < 0$, $\lambda_2 > 0$ so Both subsystems will contract quantitative values to zero. If λ is quiet insignificant, then the average meaning of the quantitative indicator, which contracts per a unit of time, becomes

$$\frac{1}{T}\int_{t_0}^{t_0+T}\alpha\lambda N_1 dt = \frac{\alpha\lambda(\varepsilon_2+\beta\lambda)}{\gamma_2}.$$

The contraction should not reach the magnitude $\dfrac{\varepsilon_1(\varepsilon_2+\frac{\beta}{\alpha}\varepsilon_1)}{\gamma_2}$.

Let us compare these changes (fluctuations) with the case without quantitative characteristics contraction. Average meanings for N_1 and N_2 зinstead of $\frac{\varepsilon_2}{\gamma_2}$ and $\frac{\varepsilon_1}{\gamma_1}$ will equal to $\frac{\varepsilon_2+\beta\lambda}{\gamma_2}$ and $\frac{\varepsilon_1+\alpha\lambda}{\gamma_1}$.

If a complex system consists of some subsystems, the system of linear differential equations is:

$$\begin{cases} \dfrac{dN_1}{dt} = N_1(\varepsilon_1 + \gamma_1 N_2), \\ \dfrac{dN_2}{dt} = N_2(\varepsilon_2 + \gamma_2 N_1), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dN_i}{dt} = N_i(\varepsilon_i + \gamma_i N_{i-1}), \end{cases} \qquad (1.13)$$

where i – number of subsystems in a complex system; N_i – quantitative characteristic of i-subsystem; ε_i – increment rate of the subsystem`s quantitative characteristic, operates independently, dimensionless quantity; $\gamma_i = \frac{\alpha\beta}{n}$ – increment rate of the subsystem`s internal characteristics, dimensionless quantity; – constant value determined by the subsystem`s internal characteristics; $\beta_i$ – increment of the subsystem`s quantitative characteristic which correlates with the number of their interactions.

1.2 Applying the proposed model (patent of Ukraine) to forecast the development of complex systems allows to incorporate the cyclic nature of economic process in the long-run period.

Ukraine occupies one of the leading places in the world in terms of mineral resources per capita and their production. It mines up to 4% of the world mineral raw materials each year. Taking into account the importance of the resource provision for the country's development, as well as of the mineral and raw material base (MMB) reproduction and the creation of conditions for the growth of minerals, National Program for the Development of MMB in Ukraine up to 2030 was adopted. Difficult economic situation in the country and significant reduction in budget funding of exploration made impossible to achieve the expected rates of MMB reproduction by 2017. To ensure program's implementation, it is important to find the ways to increase the effectiveness of Ukrainian MMB exploitation and build the system of minerals` balanced mining, production, export and import management.

Ukraine has significant mineral resources of non-metallic minerals, which can be used in various branches of building materials industry due to their composition and technological properties: cement, wall materials of natural stones, non-metallic rubble-stone and facing materials, porous fillers, glass, brick-tiling, building ceramics, etc. The study is devoted to non-metallic minerals for construction, namely: building stone, construction sand and expanded clay.

Findings` analysis revealed that the dependencies reach their peak in 2006 – 2007s, 2011–2012s; minimum in 2009 and 2014–2015s (Fig.1.2).

Moreover, fluctuations of studied indices of non-metallic mineral resources extraction for construction needs have periodic, cyclic nature. We consider that these fluctuations reflect short cycles of economic development lasting 4 – 5 years. The comparisons of the obtained results and overall economic situation in the country show that maximum of volumes of extraction and the growth rates coincide with the years when Ukraine's economy grew.

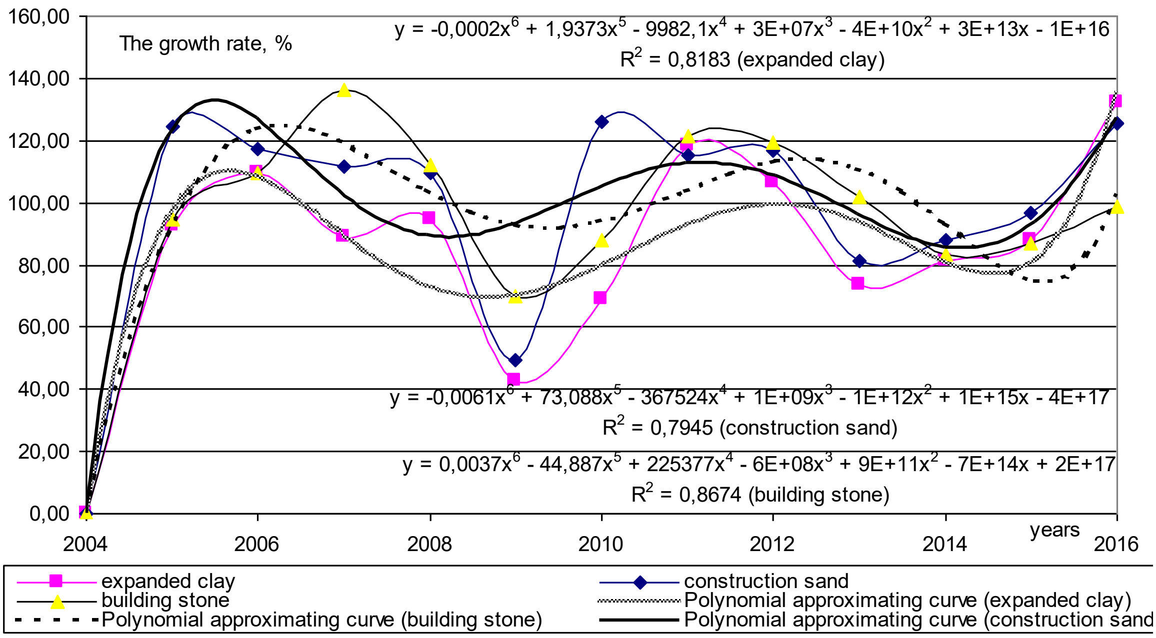

Fig. 1.2 Growth rates of building stone, construction sand and expanded clay mining during 2004–2016 and the appropriate approximation curves

It is possible to simulate the situation for each individual mineral, without taking into account interconnections applying traditional methods of approximation and smoothing. Therefore, in order to predict the development of the industry, it has been proposed to use the author's multifactorial economic and mathematical model for forecasting the development of complex systems.

To describe interdependent processes of mineral resources extraction`s business cycles` development taking into account interconnections between them, the system of differential equations (1.13) : where i – number of subsystems in a complex system; N – volumes of mineral resources extraction; ε – increment rate of volumes of mineral resources extraction, if there is no correlation with other volumes (it is constant of proportionality which states growth rates of volumes of extraction $\frac{dN}{dt}$ to N ratio), γ - increment rate of need in mineral resources.

The system (1.13) is comprised of three equations (number of resources) as three kinds of mineral resources are selected for analysis, alike: building stone, mason sand and expanded clay. The further analysis and forecasting will be based on the following indices: base growth rate (*n*) and average of volumes of extraction (*K*). The correlation is: $n = N/K$, $K = \varepsilon / \gamma$; $\varepsilon = \frac{\ln \frac{N}{N_0}}{t - t_0}$.

Fig. 1.3 shows the obtained dependencies of the growth rates of construction sand $\varepsilon_1(t)$, expanded clay $\varepsilon'_1(t)$ and building stone $\varepsilon_2(t)$ mining. Dependences of the growth rates $\varepsilon_1(t)$, $\varepsilon'_1(t)$ and $\varepsilon_2(t)$ characterize the dynamics of each individual mineral mining independently.

The curves of the obtained growth rates indexes of the named minerals demand have been built:

$\gamma_1 = \frac{n_1 \varepsilon_2}{N_1}$, $\gamma'_1 = \frac{n'_1 \varepsilon'_2}{N'_1}$, $\gamma_2 = \frac{n_2 \varepsilon_1}{N_2}$ and $\gamma'_2 = \frac{n'_2 \varepsilon'_1}{N'_2}$ and their dependence on time (Fig. 1.4).

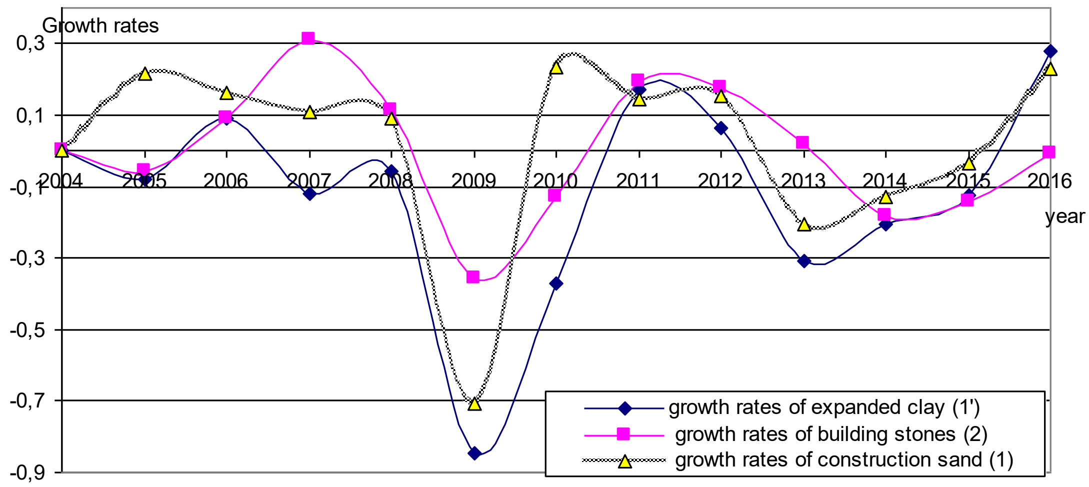

Fig. 1.3 Dynamics of the growth rates of construction sand, expanded clay and building stone without dependencies

Dynamics of dependencies for the coefficients $\gamma_1(t)$, $\gamma'_1(t)$, $\gamma_2(t)$ and $\gamma_2'(t)$ presented in Fig. 1.4 shows that they correlate with each other on time. This is quite logical, since when building the model it was considered that the coefficients γ_1, γ'_1, γ_2 and γ_2' are values that reflect the needs of construction for sand extraction, the need of expanded clay production for expanded clay mining, the need for stone extraction depending on the needs for the production of sand and expanded clay raw materials respectively. These values should be fully correlated, since during the studied period there was no change in the production technology.

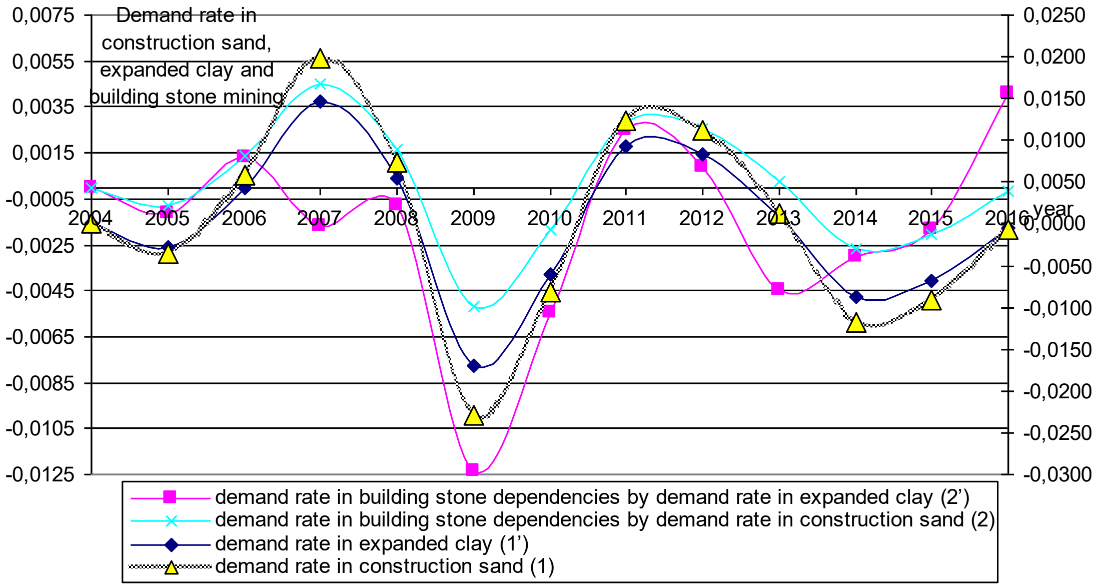

Fig. 1.4 Dynamics of the demand rate in construction sand, expanded clay and building stone mining

The system of equations (1.1) reveals the dependence of the growth rate of the expanded clay extraction $\lambda_1'(t) = \varepsilon'_1(t) + \gamma'_1(t)\ N_2(t)$, the growth rate of building stone mining depending on the amount of expanded clay extraction $\lambda_2'(t) = \varepsilon'_2(t) + +\gamma'_2(t)\ N_1(t)$, the growth rate of construction sand production $\lambda_1(t) = \varepsilon_1(t) + \gamma_1(t)\ N_2(t)$, the growth rate of construction stone mining depending on the volumes of construction sand production $\lambda_2(t) = \varepsilon_2(t) + \gamma_2(t)\ N_1(t)$, if there is a correlation between them (Fig. 1.5).

Correlations $\lambda_1'(t) = \varepsilon'_1(t) + \gamma'_1(t)\ N_2(t)$, $\lambda_2'(t) = \varepsilon'_2(t) + \gamma'_2(t)\ N_1(t)$, $\lambda_1(t) = \varepsilon_1(t) + \gamma_1(t)\ N_2(t)$, $\lambda_2(t) = \varepsilon_2(t) + \gamma_2(t)\ N_1(t)$ (Fig. 1.5) correlate with each other on time to a greater extent than the corresponding coefficients $\varepsilon_1(t)$, $\varepsilon'_1(t)$, $\varepsilon_2(t)$ i $\varepsilon_2'(t)$ (Fig. 1.3). This is due to the fact that growth rates` time changes shown in Fig. 1.5 are based on interconnections between three minerals extraction.

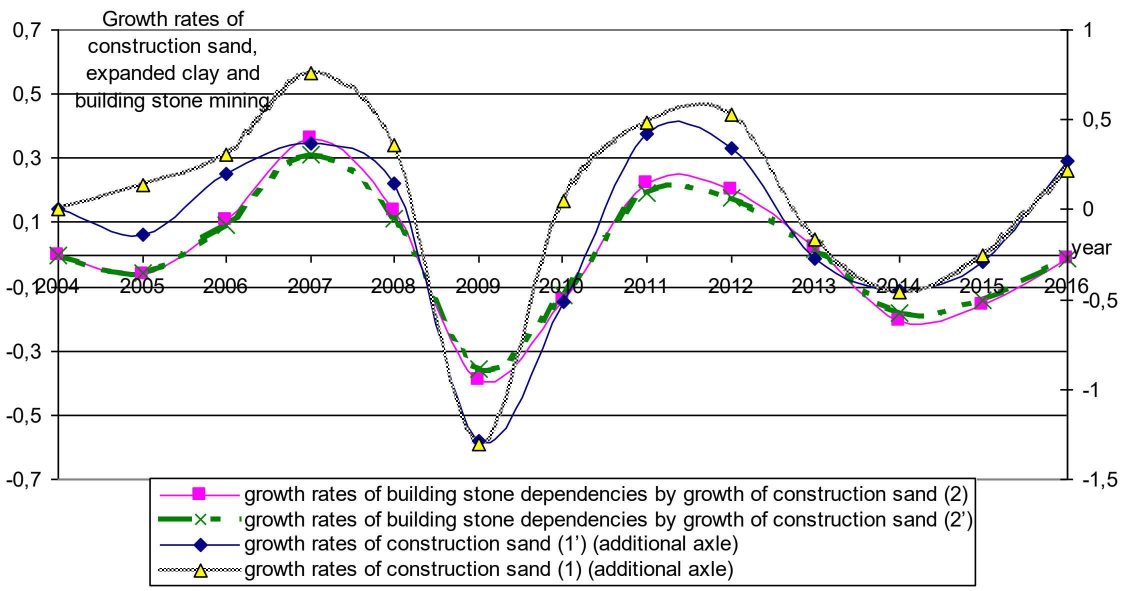

Fig. 1.5 Dynamics of the growth rates of construction sand, expanded clay and building stone mining if there is a correlation between them

It is proved that such a model will allow controlling the volumes of mining of technologically related minerals in interdependence on the volumes of their use.

The work develops a systematic approach to the management of mining, using, export and import non-metallic minerals for construction considering the economic parameters of the development of the world market situation and the domestic market as well.

The analysis showed that the classification of minerals used by the Public Service Geology and Mineral Resources of Ukraine, the State Statistics Committee of Ukraine and Statistics of foreign economic activity – codes UKTZED – do not coincide. This complicates the study process using these official sources.

According to the results of the conducted researches, the indicators of the dynamics of minerals production in their natural and costly dimensions have been found to be inadequate.

It is proved, that total import and export volumes decreased during 2011 – 2017s. Natural sand export prices dropped from 27,81 USD per Ton in 2011 to 18,91 USD per Ton in 2017. Import prices decreased as well from 423,02 USD per Ton in

2011 to 248,78 USD per Ton in 2017. We have found out key differences in export-import price scan. We think that is because the code of 2505 item includes a few kinds of sand, which prices significantly differ. It is possible to predict that high import prices are caused by the fact of mason sand export (cheap) and quartz sand import (more expensive). The obtained results prove that share of natural sand exports is 3-5% of the volume of its extraction in the country.

Pebble stone, gravel and crushed stone export prices fell from 13,582 USD per Ton in 2011 to 5,783 USD per Ton in 2016. Export prices grew insufficiently to 6,027 USD per Ton in 2017. Import prices rose during 2011 – 2013s to 90,437 USD per Ton in 2013. During 2014 – 2017 prices declined to 58,383 USD per Ton in 2017.

The comparison of pebble stone, gravel, crushed stone export and import prices show that import prices change is insignificant, while exports decreased almost twice. Besides, it should be noted that export prices are almost 10 times lower than of import. It is determined that the share of pebble stone, gravel, crushed stone export is 20 – 30% of the volume of their extraction in the country. Thus, significant part of the resources is exported in the context of negative external market price situation (high import prices and low export prices). Foreign and domestic market price comparison shows that alike export prices decline during 2013 – 2015s, domestic prices of stone, sand and clay producers increased substantially (almost 20%). This approach to the formation of export flows of mineral resources we consider unacceptable, since it poses threat to the financial situation in mining industry and the country as a whole.

Analysis of the geographical structure of natural sand, pebbles, gravel and crush stone export and import allowed proposing a systematic approach to rational extraction, the use of the country's mineral resources base, while taking into account export-import flows and the price conditions of the external and internal markets and taking into account the resource component of state security.

The obtained results indicate that the domestic export of the analyzed mineral resources for construction is not sufficiently cost-effective, since the price for exported resources is too low. Moreover, it should be pointed out that there is a real need for pebble stone, gravel and crushed stone import to meet their own needs in the Republic of Belarus, and the Russian Federation has its own substantial reserves of these minerals. We believe that the position of the Russian Federation is explained by the attempt to use other countries` mineral resources and to save its own. A similar situation regarding raw materials is known in steelmaking industry. For over 20 years, the world steelmaking leaders have preferred to import raw materials, namely ore and metallurgical coal, not exploiting their own deposits.

Taking into account the fact that the implementation of the National Program for the Development of Ukrainian raw material base by 2030 is under threat as a result of the permanent lack of funding for exploration works, the funds received from export operations can be used for its financing. However, one should pay attention to the fact that the main tasks of the program are to reproduce and increase the reserves of mineral resources. It is possible to solve these important tasks not only conducting geological prospecting work, but above all by creating conditions for more rational extraction and exploitation of the country`s mineral raw material base. In our opinion, resources exhaustibility and ineffective export operations, endanger the resource component of national security.

1.3 Multifactor mathematical modelling of ecological and economic systems (the example of coal mining development)

The model for planning and forecasting of coal resources as a component of the management system of the national raw material base has been developed as a result of the study.

The model of national potential for coking coal`s rational use management based on balancing flows of coal extraction, coke and semi-coke production according to the needs of the country's metallurgical industry (ironmaking) has been proposed. The model is confirmed by the official statistics.

The model of anthracite's and other coals` rational extraction management has been proposed based on balancing flows of coal extraction, its production according to the energy sector needs (heat production). The model is confirmed by the official statistics.

In order to solve the problem of system management of national coal mining, it has been proposed to balance the volumes of coal mining with the volumes of its consumption in metallurgy and energy sectors. The necessity of planning and forecasting the extraction volumes has been proved taking into account cyclical nature of economic processes` development. It has been proposed to use a multi-factor economic and mathematical model for forecasting the development of complex systems as a sample frame.

Coal industry is one of the basic sectors of the national economy. It is also the key to the national energy security. Armed hostilities in the East of Ukraine which began in 2014 led to the destruction of the infrastructure, damage and destruction of coal mines, so domestic coal sector is in the clutch. Today, the coal deficit in Ukraine, offset by forced imports, is 27%. That is why there is a need to build system for managing national needs in coal under the condition of saving approach to its extraction and rational use.

We consider that it is possible to ensure rational use of Ukraine's coal resources by balancing mining, production of feed coal and its further use in the subsequent production processes. Coke is produced from coke coal, which is then used in metallurgical production. Anthracite is used by thermal power plants and thermal heating and power plants, as well as in the industrial production. It makes sence to set relations between coal mining and its use (consumption).

In order to be able to determine the optimal development of coal mining, it is possible to use a multifactor economic and mathematical model for predicting the development of complex systems. The model will be used for the following systems: 1) cast iron production; coking of coal and production of coke and semi-coke; 2) thermal energy production; extraction of anthracite and other coal and production of anthracite (including loss of production).

In order to determine the optimal development of coal mining, it is possible to use a multifactor economic and mathematical model for predicting the development of complex systems. The model will be used for the following systems: 1) cast iron production; coking coal mining, and coke and semi-coke production; 2) thermal energy production; anthracite and other coal mining, and production of anthracite (including mining losses).

To describe the interdependent processes of cyclical development of the complex system`s mentioned elements, taking into account interconnections between them, system of differential equations will be applied (13): where *i* – number of subsystems in complex system; *N* – cast iron production; coking coal mining, coke and semi-coke production (1st model); thermal energy production; anthracite and other coal mining , anthracite production (2nd model); ε – increment rate of the listed values, if there is no correlation with other amounts (coefficient of proportionality, which demonstrates the ratio of mining growth rate $\frac{dN}{dt}$ to *N*), γ – need increment rate.

Taking into consideration that each model has three components, system (13) contains three equations. Further analysis and forecasting will apply the following indicators, namely basic growth rate (*n*) and mean value (*K*). Correlation between them is: $n = N/K$, $K = \varepsilon/\gamma$; $\varepsilon = \frac{\ln \frac{N}{N_0}}{t - t_0}$.

The model was verified on the basis of official statistics on coking coal, anthracite and other coal mining data disseminated by the State Scientific Geological Fund of Ukraine; on coke, semi-coke and anthracite production; cast iron and thermal energy production data presented by the State Statistics Service of Ukraine (“Statistical Yearbook of Ukraine”)

Fig. 1.6 – 1.8 illustrate calculations` results for the 1st model: cast iron production, coking coal mining, coke and semi-coke production. Fig.1.6 shows basic growth rates. Cast iron increment rates $\varepsilon_1(t)$, coking coal mining $\varepsilon'_1(t)$ coke and semi-coke production $\varepsilon_2(t)$. Dependencies of increment rates $\varepsilon_1(t)$, $\varepsilon'_1(t)$ i $\varepsilon_2(t)$ characterize each models` value autonomously (Fig. 1.7).

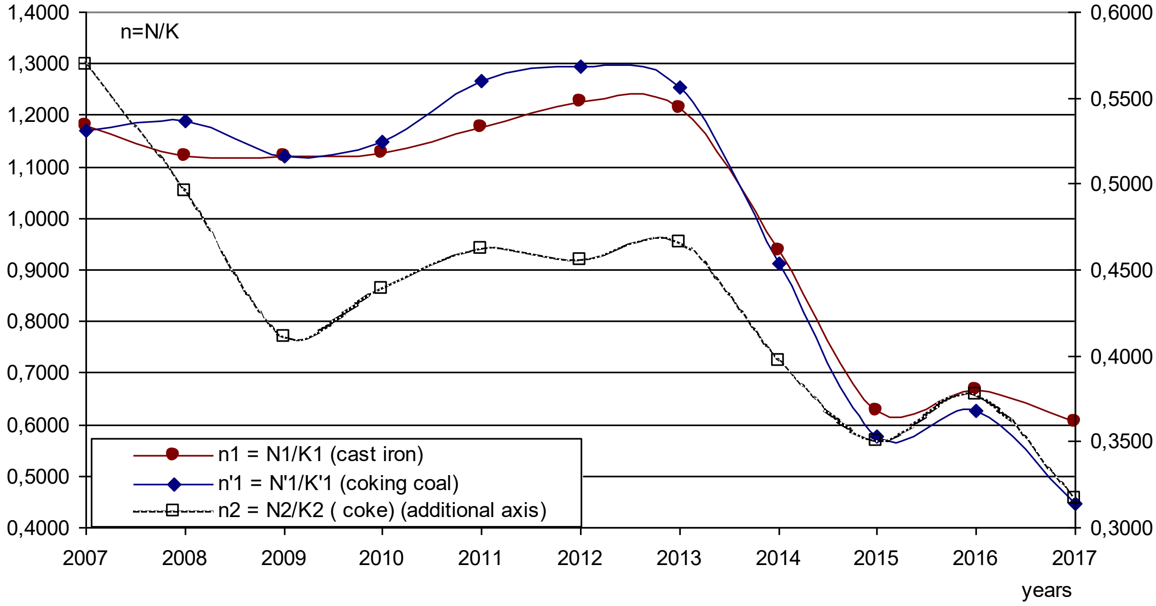

Fig. 1.6 Dynamics of basic cast iron production growth rates, coking coal mining, coke and semi-coke production during 2007–2017

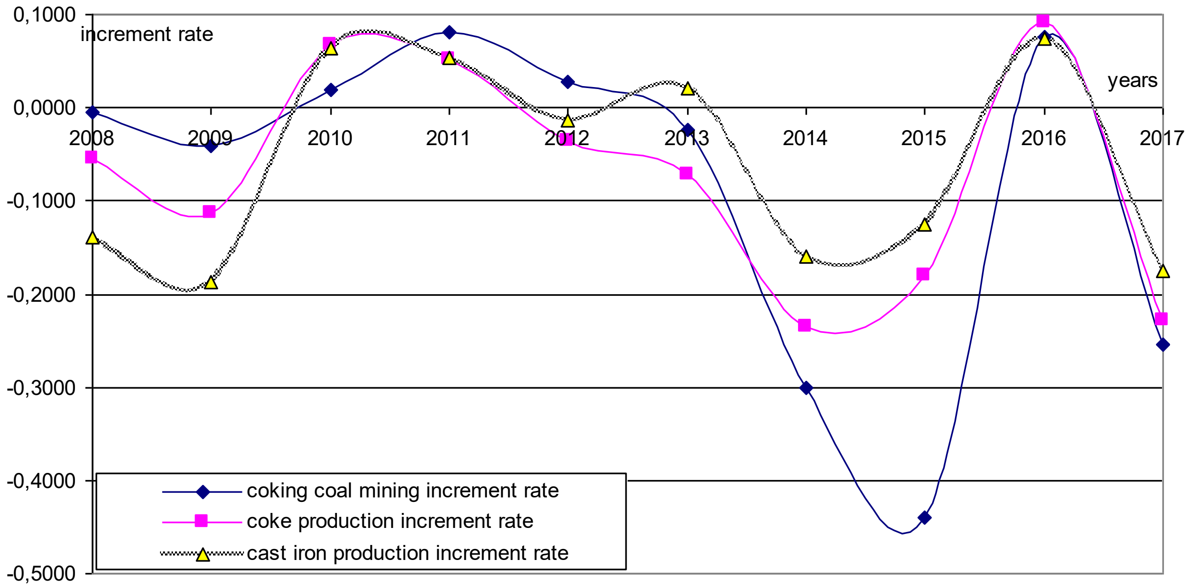

Fig. 1.7 Dynamics of increment rates of coking coal mining, coke and semi-coke production, cast iron production during 2007–2017 without correlation

Solution of the system of equations (1.1) reveals the dependence of the increment rate of coking coal mining $\lambda_1{}'(t) = \varepsilon'_1(t) + \gamma'_1(t)\, N_2(t)$, coke and semi-coke production`s increment rate depending on coking coal mining $\lambda_2{}'(t) = \varepsilon'_2(t) +$

$+\gamma'_2(t)\ N_1(t)$, cast iron increment rate $\lambda_1(t) = \varepsilon_1(t) + \gamma_1(t)\ N_2(t)$, coke and semi-coke production`s increment rate depending on cast iron production $\lambda_2(t) = \varepsilon_2(t) + \gamma_2(t)\ N_1(t)$ if there is correlation between indices (Fig.1.8).

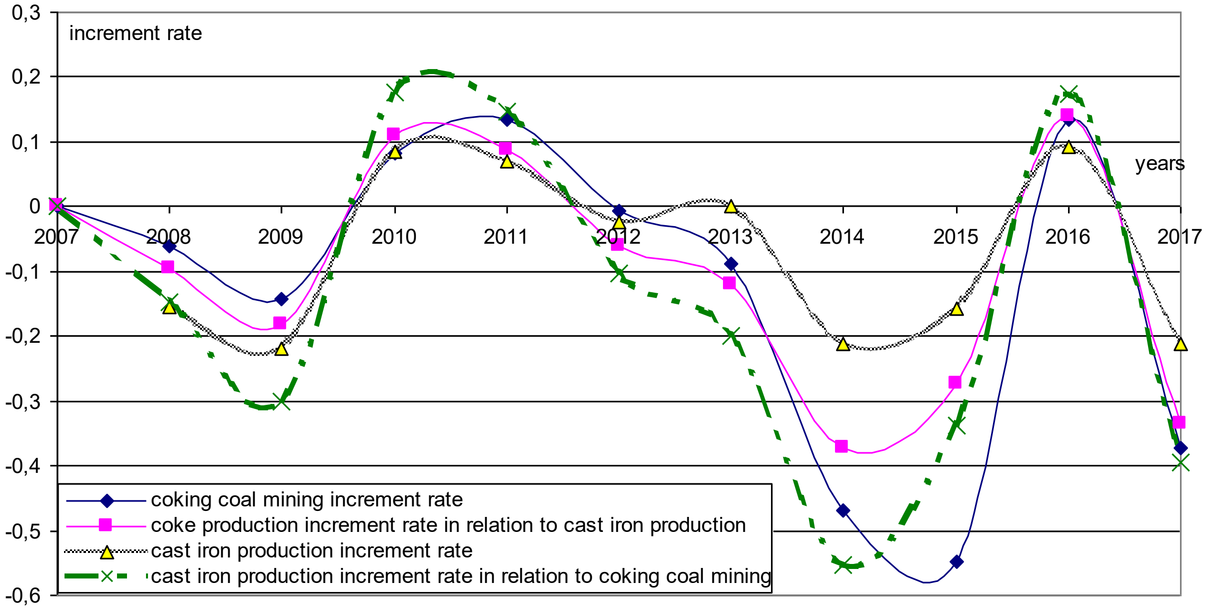

Fig. 1.8 Dynamics of increment rates of coking coal mining, cast iron production, coke and semi-coke production in relation to cast iron production and cast iron production in relation to coking coal mining during 2007-2017 with correlation

Thus, the proposed model allows to forecast coking coal mining correlating to coke and semi-coke production and metallurgical industry demand (cast iron production), taking into account its cyclical development.

Fig. 1.9 – 1.11 give information about calculations` results for the 2nd model: thermal energy production; anthracite and other coal mining, anthracite production. Fig.1.9 illustrates basic growth rates. Fig.1.10 shows increment rates of thermal energy production $\varepsilon_1(t)$, anthracite and other coal mining $\varepsilon'_1(t)$ and anthracite production $\varepsilon_2(t)$. Dependencies of increment rates $\varepsilon_1(t)$, $\varepsilon'_1(t)$ i $\varepsilon_2(t)$ characterize each index`s dynamics autonomously.

Solution of the system of equations (1.1) makes it possible to find out correlation between anthracite and other hard coal mining`s increment rate, anthracite and other hard coal production`s increment rate depending on anthracite and other hard coal mining, thermal energy production`s increment rate, anthracite and other hard coal production`s increment rate depending on thermal energy production, if there is correlation between indices (Fig.1.9).

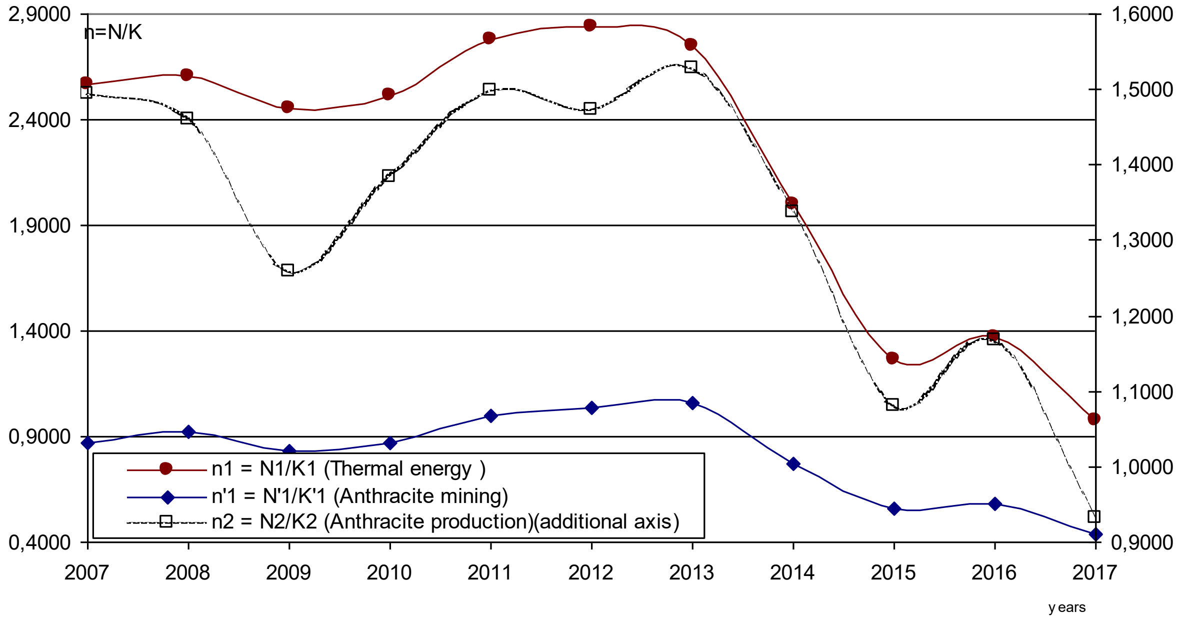

Fig. 1.9 Dynamics of basic thermal energy production growth rates, anthracite and other coal mining, anthracite production during 2007 – 2017

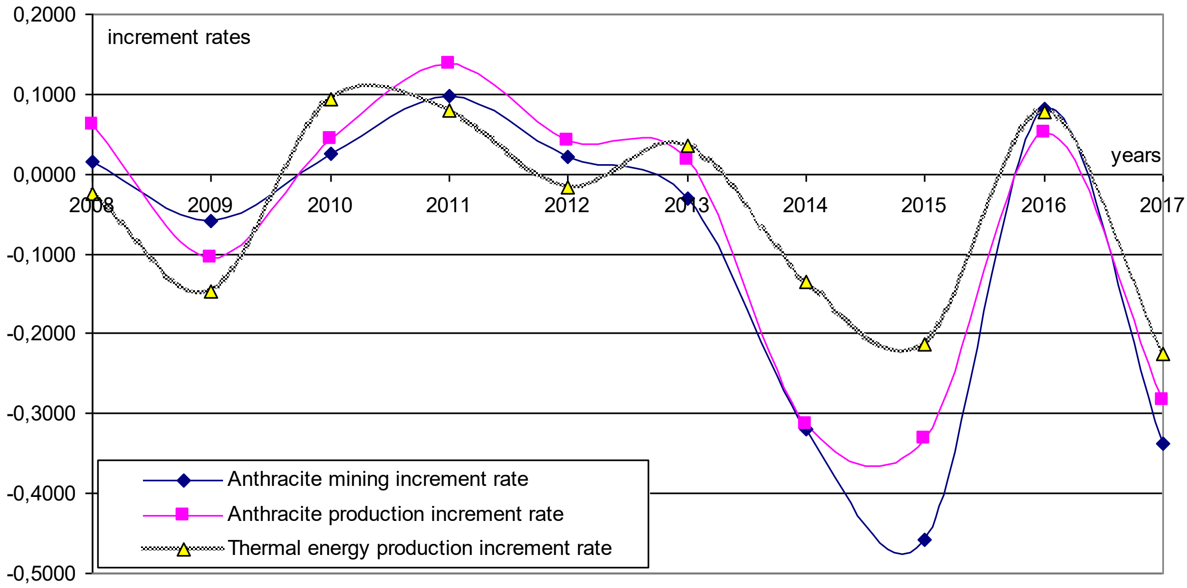

Fig. 1.10 Dynamics of increment rates of anthracite and other coal mining, production and thermal energy production during 2007 – 2017 without correlation

With this result, the proposed model allows to predict anthracite and other coal production, depending on their and thermal energy production, taking into account the cyclical nature of power industry development.

National coking coal sustainability managerial model for balancing flows of coal and semi-coal mining depending on metallurgy needs (cast iron production) has been proposed. The model of sustainable anthracite and other coal mining management for balancing coal mining and production depending on electric power energy needs (thermal energy production). The correctness and adequacy of the developed models` application is confirmed by official statistics.

Thus, it is possible to solve the problem of systematic rational management of the country's coal resources by balancing coal mining and its consumption in metallurgy and energy. In order to take into account cyclical nature of economic processes when planning and forecasting coal mining and production in Ukraine, it is proposed to use a multifactor economic and mathematical model for predicting the development of complex systems.

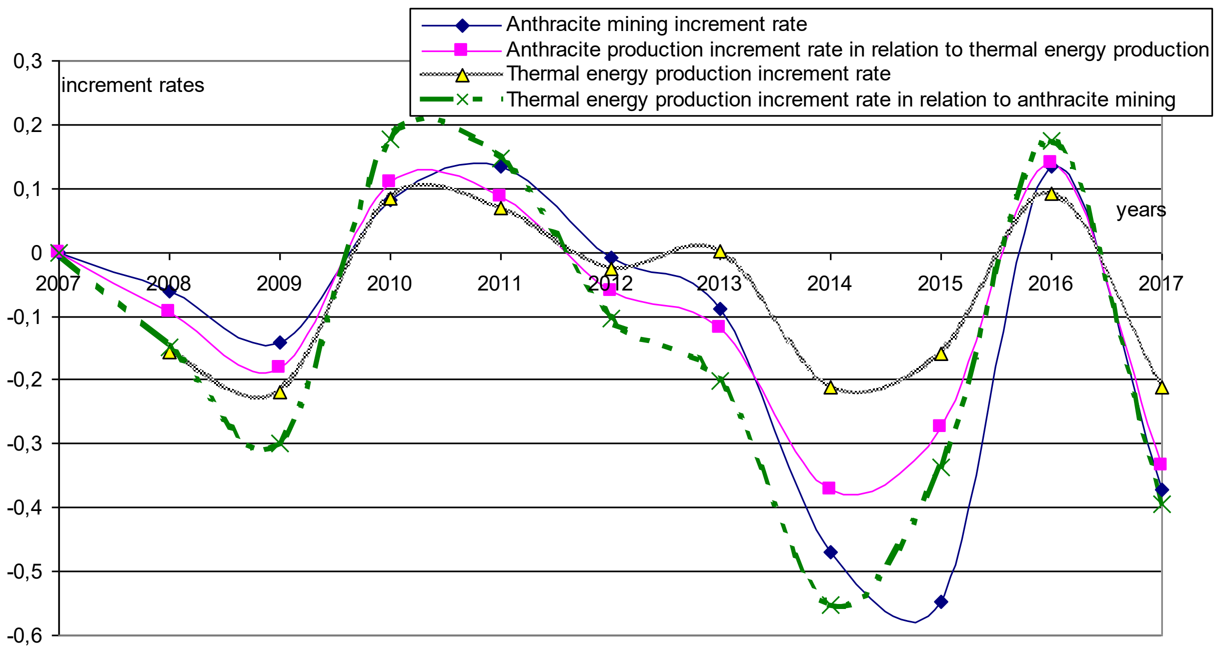

Fig. 1.11 Dynamics of increment rates of coal, coking coal, anthracite and other coal mining during 2007 – 2017 if correlated to extraction

The application of the proposed model will allow to form complex of managerial decisions and project proposals for ensuring mining industry`s sustainable development, taking into account strict requirements for environmental safety of the entire product life cycle and territories balanced development where mineral extraction, in particular, coal mining is carried out. Current and strategic tasks of rational coal mining can be solved only by taking into account alternative resource strategies and forecasting industry`s market needs for the mid- and long term.

1.4 Modelling of resource flows in the coal industry of Ukraine

The mechanism for Ukrainian coal mining industry`s sustainable development management has been formed as a result of the study. It has been proved that balancing mining, production of coal raw materials and their subsequent use in the production processes are the ways to ensure rational use of coal resources of Ukraine. The processes for coking coal is metal production, and for anthracite – thermal energy production. Relationship between growth rates of coking coal mining and production, coke mining, exports and imports of coke and ironmaking during 2008 – 2017s have been studied. It has been demonstrated that imports of coking coal essentially depends on the market pricing and has little to do with the needs of the metallurgical industry of Ukraine. It has been shown that the system for rational use of coking coal`s capacity in Ukraine should include balancing flows of coal mining, coke and semi-coke mining according to the needs of national metallurgical industry, taking into account cyclical nature of its development. The necessity of coordinating the volumes of anthracite extraction, production and consumption and taking into account short cycles of heat power development has been defined.

Total resources of coal in Ukraine are 112,3 billion tonnes, proved coal reserves – 51,9 billion tonnes, including 17,1 billion tonnes of coking coal (30,5 %) and 7,6 billion tonnes of anthracite (13,5 %). Reserves of coking coal and anthracite amount to 31,5 % and 14,3 % of total resources of coal in Ukraine. However, there is a need to develop system of rational treatment for this important mineral. Therefore, the problem of developing effective coal management system in Ukraine based on optimization of resource flows for mining, use, export and import of coal in Ukraine based on their interconnection needs to be solved.

The study of correlation between coking coal mining and production growth rates, coke production, export and import of coke, cast iron production during 2008–2017s has been carried out. The results are presented in Fig.1.12.

During 2008 – 2013s the dynamics of coking coal mining did not correspond with the relative indices of coal, coke, coal semi-coke production and metallurgy (cast iron production).

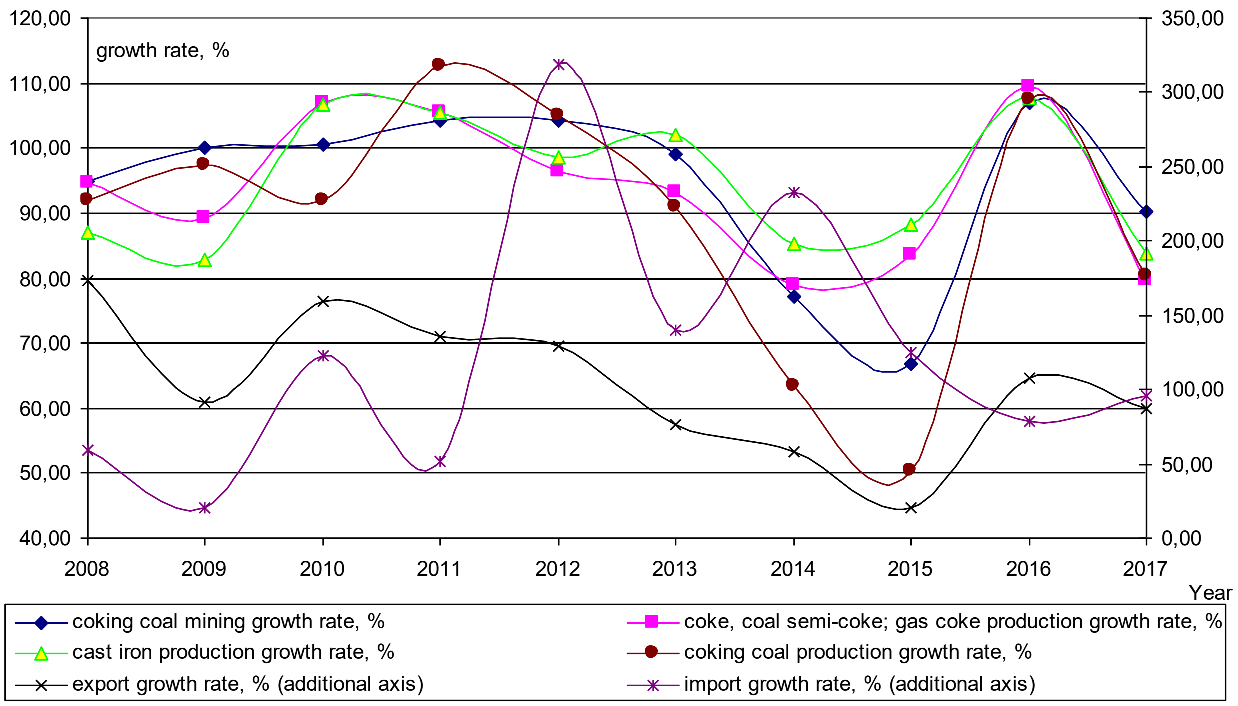

Fig. 1.12 Dynamics of coking coal mining and production growth rates, coke production, coke exports and imports, cast iron production during 2008 – 2017

The discrepancy in the dynamics of mining and production of coal can be explained by the different level of its losses in the process of mining. Coking coal production increased by 2013, but other indices changed on a periodic basis. The significant drop in production in 2015 can be explained by the beginning of armed hostilities in Donbass, as a result part of mines came to be in temporarily uncontrolled territories. The dynamics of coke production since 2013 coincides with the dynamics of coal production. During the reviewed period, the dynamics of exports of coke were fully consistent with trends in coke production. By 2011 Ukraine imported coke during the period, when domestic coal production grew. Since 2021 the pattern has changed, import rose when production declined. Since 2013, all the named indices fit the trend – cyclical, as the small cycle period is three years.

One to one correlation coefficient was used to identify relationship between coking coal mining, coke production, its export and import. The function characterizes density of connections between each element of time series of dependent (resultant) y_t and explanatory x_t variables` values relatively shifted to one time lag τ. One to one correlation coefficient is determined by the formula:

$$r_\tau = \frac{(n-\tau)\sum_{t-1}^{n-\tau} y_t x_{t-\tau} - \sum_{t-1}^{n-\tau} y_t \sum_{t-1}^{n-\tau} x_{t-\tau}}{\sqrt{[(n-\tau)\sum_{t-1}^{n-\tau} y_t^2 - (\sum_{t=1}^{n-\tau} y_t)^2][(n-\tau)\sum_{t-1}^{n-\tau} x_{t-1}^2 - (\sum_{t=1}^{n-\tau} x_{t-\tau})^2]}} \qquad (1.14)$$

where y_t and x_t – elements of vector of dependent (resultant) and explanatory variables relatively shifted to one time lag τ; n – number of quantitative r_τ. values. Calculations are presented graphically in Fig. 1.13.

As we can see, the process of coke production and its export has the highest level of interconnection. Processes of coking coal production, coke production and its application in metallurgical production are characterized by almost the same interconnection level. The correlation between import of coke and its production is low, but the level has increased in recent years.

To specify export-import transactions` peculiarities we compared coke and semi-coke exports and imports growth rates; as well as of retort carbon`s (code 2704 UCGEED) and corresponding price indexes. The obtained results are presented in Fig. 1.14.

The analysis of Fig. 1.14 shows that the frequency of export and import prices` growth rate changes coincides. Thus, we have small cycles lasting 3 – 4 years. Comparison of import growth rates and corresponding price growth rates proves that these indices change in counter-phase, that is, import prices increase, if imports decrease and vice versa.

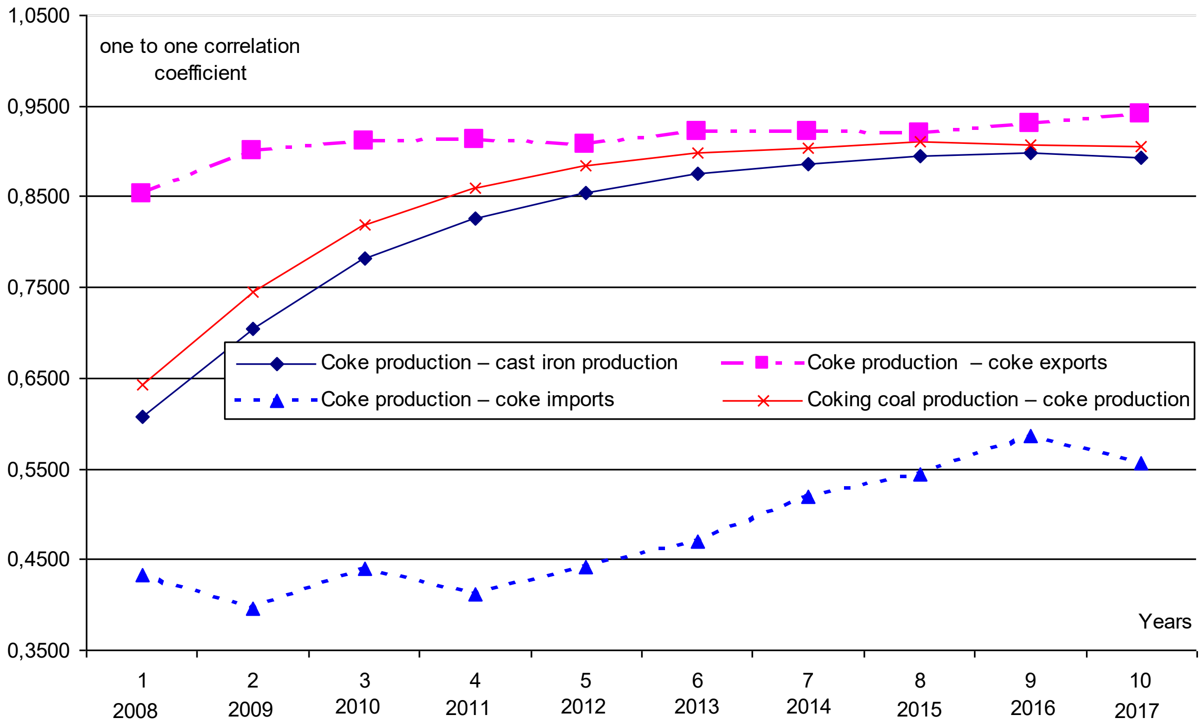

Fig. 1.13 One to one correlation functions for coking coal mining, coke production, exports, imports and application in metallurgy

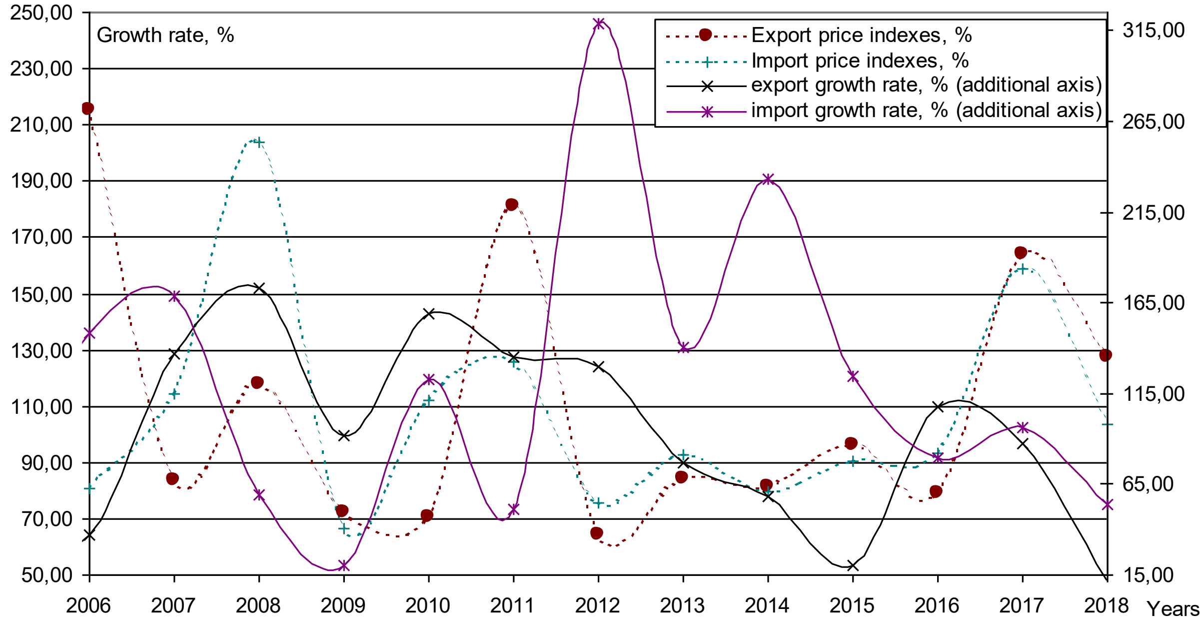

Fig. 1.14 Dynamics of coke and semi-coke exports and imports growth rates; retort carbon (code 2704 UCGEED) and corresponding price indexes during 2006 – 2017

Interdependence between export prices and export reveals matching trends, i.e. export prices go up with export volumes, except the period of 2013 – 2015s, which is explained by the production drop as a result of armed hostilities in Donetsk and Luhansk regions.

Thus, coking coal imports are significantly dependent on price trends and are almost unrelated to the needs of Ukraine's metallurgical sector. Therefore, in our opinion, management system of the country's coking coal potential should include balanced flows of coal mining, coke and semi-coke production depending on the needs of national metallurgical sector. Taking into account cyclical nature of metallurgical sector development, coking coal mining and coke, semi-coke production have to be cyclic too. The small cycles` period is 3 – 4 years.

Let us make similar assessment of steam coal – anthracite. Analysis` results are shown in Fig. 1.15.

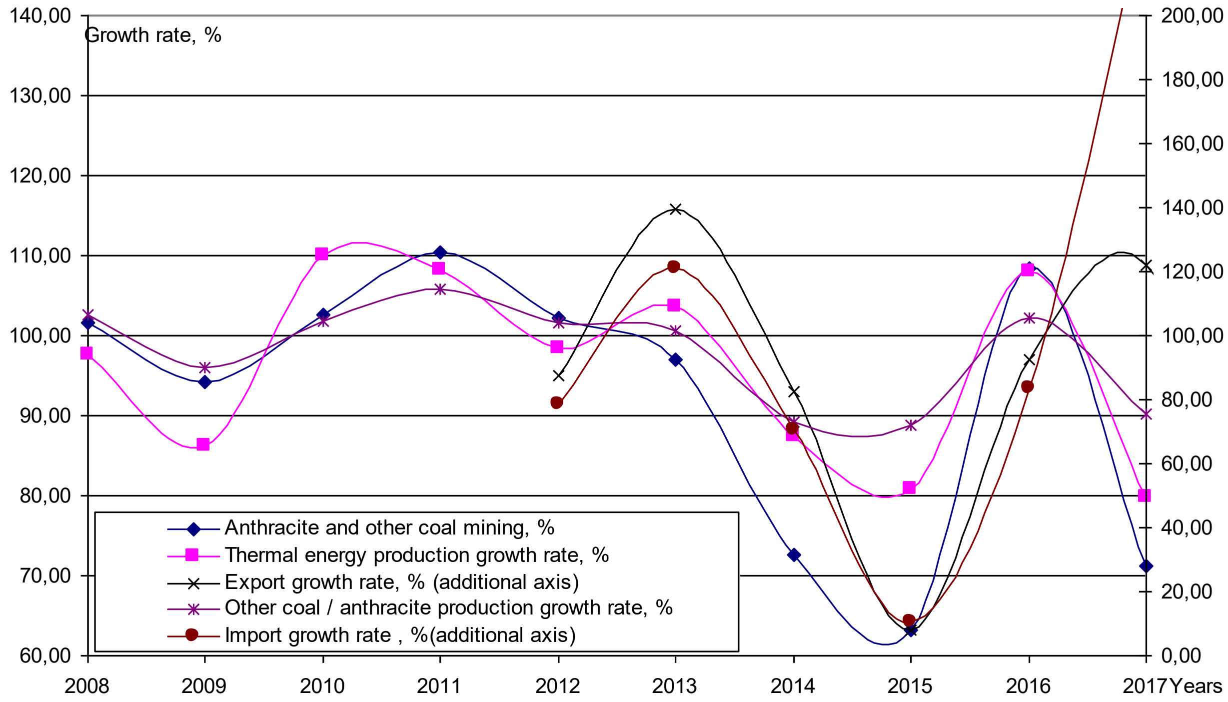

Fig. 1.15 Dynamics of anthracite mining, other coal / anthracite production, coal exports and imports, anthracite (code 2701 UCGEED) and thermal energy production during 2008 – 2017

As we can see, there is correlation between anthracite mining growth rates, other coal / anthracite production, coal exports and imports, anthracite (code 2701 UCGEED) and thermal energy production during 2008 – 2017s. Dependencies have periodic nature, indicating small cycles lasting 3 – 4 years. 2017 was the exception for exports and imports.

Fig. 1.16 illustrates the results of correlation analysis on anthracite mining and production, exports and imports, its application in the process of thermal energy production. As we can observe, unlike the situation with coking coal, there is high level of correlation between the analyzed values regarding anthracite. Besides, since 2014, correlation between production and exports has become even more significant.

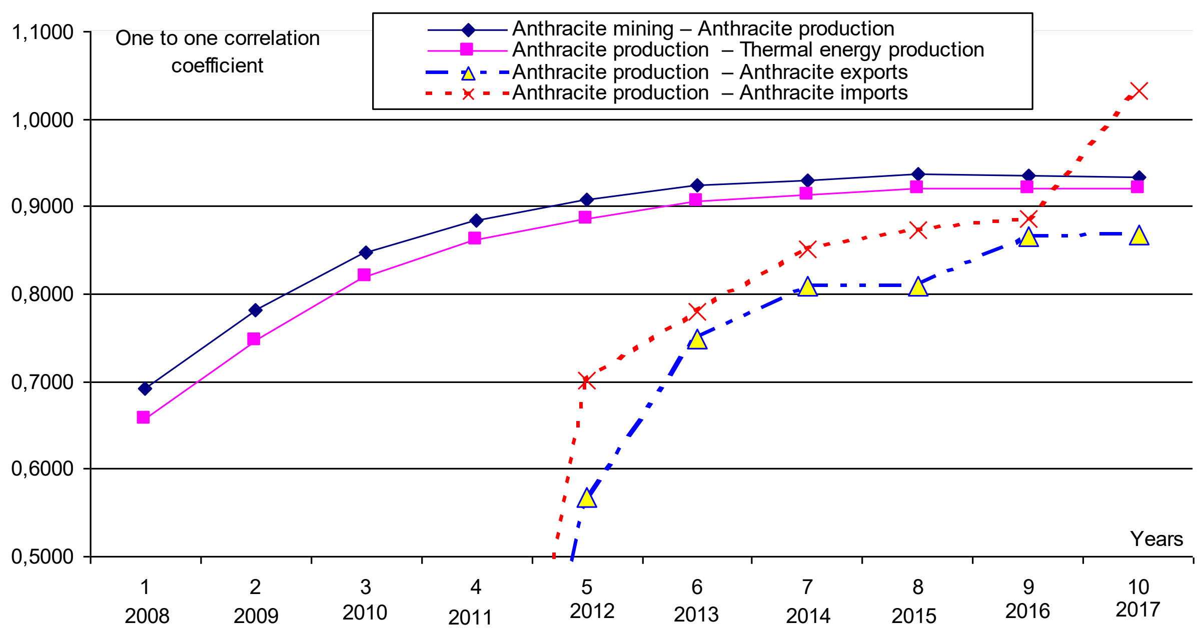

Fig. 1.16 Functions of one to one correlation for anthracite mining and production, exports and imports, its application for thermal energy production

We will analyze the import growth rates and corresponding rates and prices (Fig. 1.17). Exports and imports growth rates are identical as the same are price indexes. The trend for imports is economically justified, as when import prices drop down, its volume goes up. Application of the same approach for exports is not economically justified, because it brings losses. The situation has changed since

2015, both export and import prices, as well as volumes began to rise. We consider, that the increase in anthracite imports is consequence of lower production caused by the armed hostilities in Donetsk and Luhansk regions, where most of the mines are concentrated. In the situation, higher exports of the deficit resource having strategical importance for the national energy security may be unjustified.

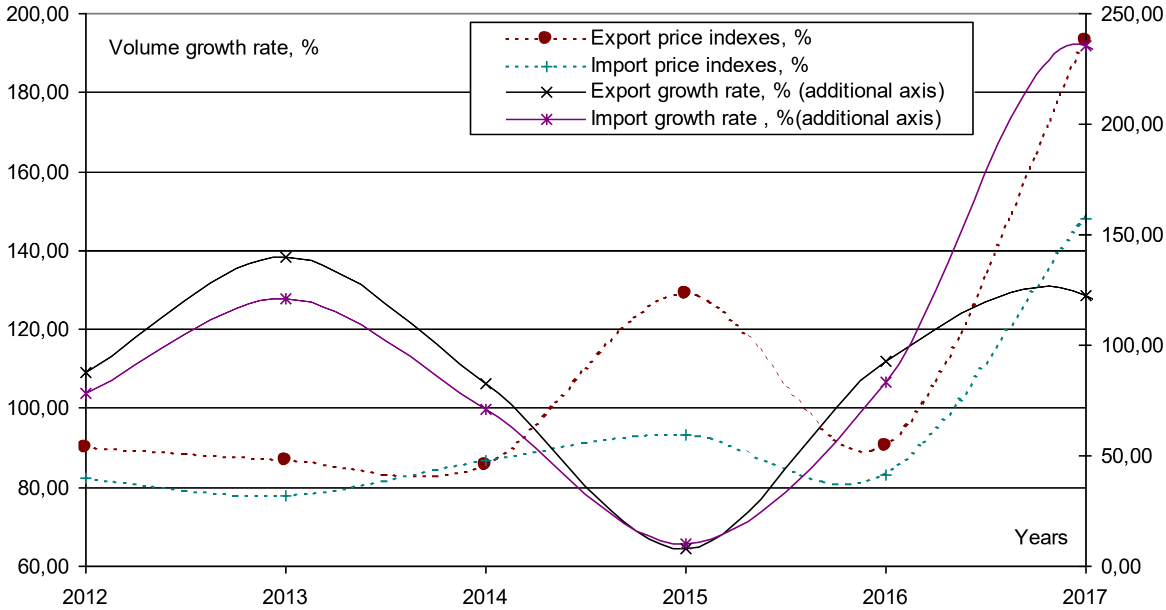

Fig. 1.17 Dynamics of coal exports, imports growth rates, anthracite (code 2701 UCGEED) and corresponding price indexes during 2012 – 2017

To set priorities in the managerial system of coal industry, analysis of the relationship between coal mining, production, use, exports and imports in Ukraine have been made.

It has been illustrated that managerial system of coking coal potential`s substantial use should include balancing flows of coal mining, coke and semi-coke production depending on the needs of the country's metallurgical industry (cast iron production), taking into account its cyclical development nature. Small cycles presence in the development of all subsystems has been proved. Moreover, correlation between anthracite mining and production, export and import growth rates and thermal energy production have been proved if there are small cycles lasting 3 – 4 years. The results obtained indicate that it is advisable to reconcile anthracite mining, production and consumption amounts and to take into account small cycles of heat power industry development.

Thus, finite nature of minerals and inefficient export transactions pose a threat to the national resource security. In addition, optimization activities may include the following: exploration of mineral deposits, funding of mines` reconstruction, which will reduce the amount of mining coal losses and increase the efficiency of industry performance.

1.5 Economic and mathematical modelling of sustainable extraction of natural resources

To determine the framework to ensure conditions for Ukrainian mining industry`s sustainable development, the methodological principles to determine the level of sustainability and to model the parameters and limits of sustainable development applying the case of coking coal and iron ore mining have been proposed. The method to measure the extractive sector`s level of sustainability, i.e. the sustainability index, which is determined on the basis of the linear and quadratic deviation of production volumes, has been proposed. The method to measure coking coal and iron ore mining level of sustainability has been developed using statistical indicators of relative and accumulated frequencies, which made it possible to obtain absolute indices of sustainability level in kind. It has been recommended to measure the extractive sector`s limit of sustainability finding the extremum of functions that describe the regression relationships between coke and ore production from the extracted raw materials, taking into account export-import flows and their use in metal production. The obtained modelling results provide an opportunity to forecast the optimal volumes of mining as a basis for sustainable development management of the Ukrainian extractive sector.

Ukraine is one of the world leaders by the explored reserves of coal, iron, manganese, titanium and zirconium ores, as well as graphite, kaolin, potassium salts, sulfur, refractory clays, and facing stone. For instance, it has 7,5% of the world coal reserves, 15% of iron ore reserves. At present, Ukraine extracts significant amounts of hard coal (1,5% of the world production) and commercial iron ore (4,5%). Nowadays, the pace and scale of its own mineral resources base reproduction do not meet the country`s needs. The lack of funds reduced geologic exploration by 3 – 4 times. Therefore, since 1994, the explored reserves growth of most important minerals does not offset their extraction. Therefore, there is a need to form the system of rational minerals extraction. That is why, in our opinion, it is worth to model and forecast their production in the long run.

We consider, that system parameters of coking coal and iron ore production should be modelled in the context of the domestic metallurgy needs. The approach will, on the one hand, ensure domestic economy`s sustainable development, as 30% of its GDP and 27% of foreign exchange earnings are provided by the metallurgy. On the other hand, it will allow a more rational approach to the national resource use.

Total coal resources of Ukraine include 112,3 billion tons, 51,9 billion tons of explored reserve, including coking coal – 17,1 billion tons (30,5%) and anthracite – 7,6 billion tons (13,5%). Balance hard coal reserves are explored to the depths of 1200 – 1400 m, sometimes 1600 – 1700 m. Forecast and prospective hard coal resources are explored to the depths of 1800 – 2000 m. Today in coal production of Ukraine, 90 mines are subordinated to the Ministry of Energy and Coal Industry of Ukraine, but only 33 mines are located in the territory controlled by Ukraine. Moreover, of these mines, 24 operate (extracting about 21 thousand tons per day) and 2 operate in the mode of maintenance (in the drainage mode). Thus, the study shows a decrease in coal production in Ukraine in recent years. The needs for coal are met through import flows.

By the explored iron ore reserves, Ukraine is one of the world leaders. As of January 1st, 2020, the State Balance of Mineral Reserves of Ukraine takes into account 60 iron ore deposits, of which 25 are under development. Total balance reserves of iron ore are 18836,4 million tons, C2 – 7584,6 million tons; off-balanced – 4958,8 million tons. The development of 25 iron ore deposits by 12 mining companies continued during in 2019. Operational work was carried out at 7 mines and 13 quarries. In addition, ores that had previously been lost were mined too. In 2019, 167,9 thousand tons of previously lost saleable ores were mined in the field of the Ternivska mine. In 2019, 157,4 million tons of ore were mined in Ukraine (without impoverishment). Ore production compared to 2018 (152,6 million tons) rose by 4,8 million tons, which is 3,05%.

To optimize iron ore and coking coal production with simultaneous adjustment of import and export flows, it is necessary to determine the level and limits of sustainable development of such industries as metallurgy, coke and iron ore. As a measure of sustainability, let us use the stability factor (E,%), determined by the formula: $E = 100 – V$, where V is the coefficient of variation, which gives a relative estimate of variation and can be obtained by comparing the linear or standard deviation from the mean level phenomena, %. Production data in the industries were used for a period of 20 years. The formulas for the linear and quadratic mean deviation use the mean value determined by the theoretical trend equation as the mean value. The standard deviation is always greater than the linear deviation due to different methods of calculation. The linear deviation has the same units of measurement as the variant or the mean value, it gives an absolute measure of variation. The standard deviation is the deviation from the mean and it also has the same units as the variants or the mean. The calculation results are summarized in Table 1.1. The iron ore industry has the highest stability factor – 84,41%, the stability factor for ferrous metallurgy and coke industry is approximately the same – 81,70% and 81,12%, respectively. Thus, this is fully corresponding to the economic situation regarding the dynamics of iron ore and coking coal mining in Ukraine.

The obtained results could be interpreted as follows. All industries have the cyclical nature of development during the study period, but the intensity of fluctuations in production volumes was not the same. Due to the lowest fluctuations in the iron ore industry`s growth rates, it has the highest stability factor.

To determine the absolute value of stability factor, we used statistical indicators of relative and cumulative frequency. These data in terms of mathematical statistics are a sample of 20 variants for each industry. A positive number that indicates how many times a variant has occurred in the data is called a frequency. Relative frequencies (the ratio of variant frequency to sample size) are often applied instead of frequencies. Relative frequencies illustrate how often over the past 20 years' certain production volumes have been identified in the analyzed industries. The results of the calculations are shown in Fig. 1.18 – 1.19.

Table 1.1 Stability factor evaluation for the ferrous metallurgy and technologically related industries of Ukraine during 1991 – 2011s

Industry	Mean linear deviation, million tons	Mean standard deviation, million tons	Linear coefficient of variation, %	Quadratic coefficient of variation, %	Stability factor (linear), %	Stability factor (quadratic), %
Ferrous metallurgy	15,88	20,85	18,30	24,03	81,70	75,97
Coke industry	4,15	4,93	18,88	22,41	81,12	77,59
Iron ore industry	9,66	13,40	15,59	21,63	84,41	78,37

We consider 93 millions of tons of ferrous metals production, 56 millions of tons of iron ore and 20 millions of tons of coke, respectively as optimal. These values are those production volumes that provide the required level of industry sustainability. Thus, the relative frequency of production is the absolute value of stability in kind. The obtained values can be considered as economic indicators of the stability factor.

An important task in forecasting the dynamics of coking coal and iron ore mining is to model the limits of changes in raw materials demand for the metallurgy, coke and iron ore industries. To do this, we have determined marginal values of the factor characteristic (extremum point). Its further growth will cause decline in metal production. It is determined by finding the extremum of functions that describe the regression relationships between coke and metal production, and ore and metal production. Let us draw in one figure the correlation between ore and coke production, and metal production (Fig. 1.20).

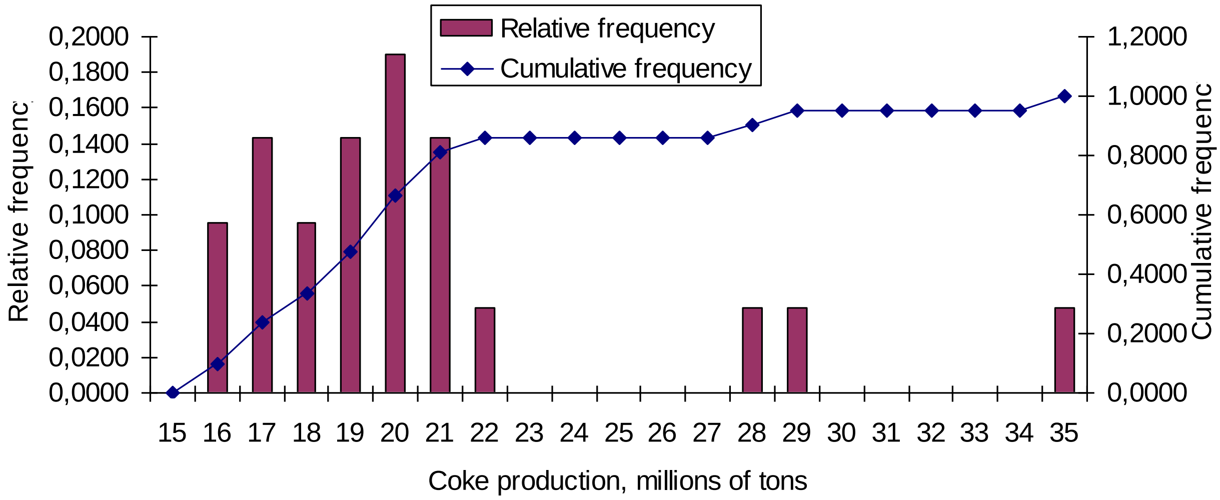

Fig. 1.18 Relative and cumulative frequency of coke production in Ukraine during 20 years

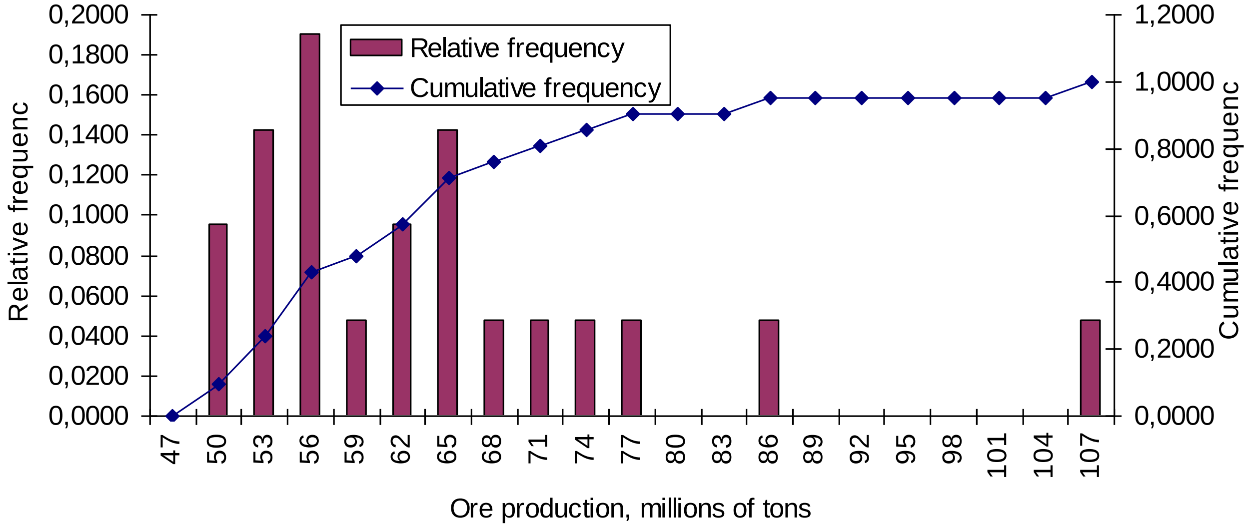

Fig. 1.19 Relative and cumulative frequency of ore production in Ukraine during 20 years

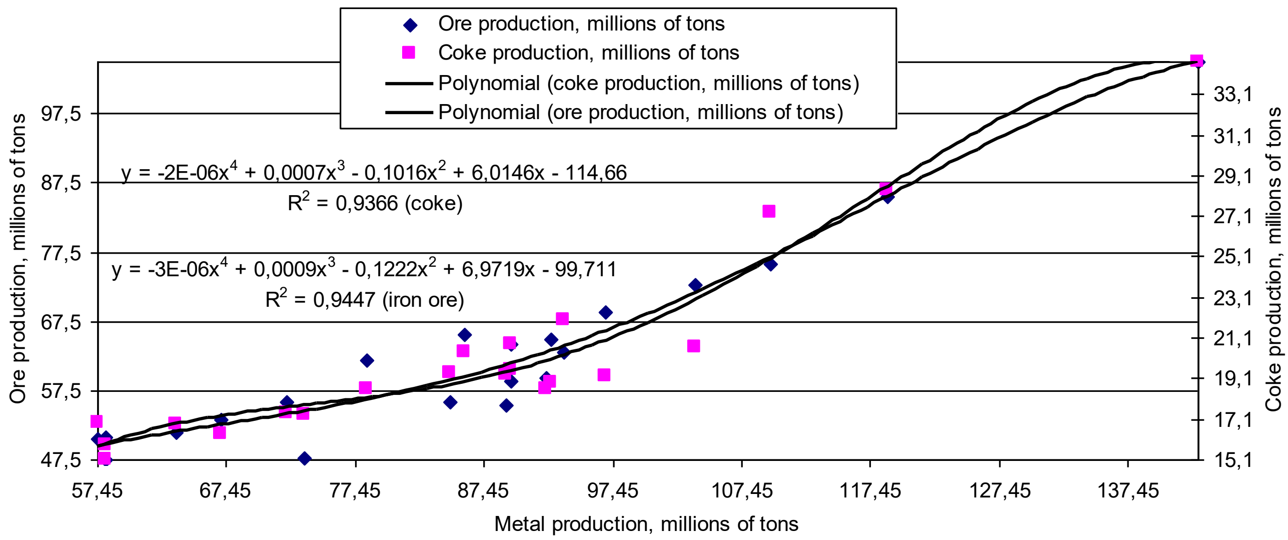

Fig. 1.20 Actual correlation of ore and coke production, and metal production in Ukraine

Thus, the stability limits are 78 – 112 million tons for metal, 56 – 77 million tons for iron ore, and 18 – 25 million tons for coke. Based on the actual data on ore and coke production volume, we determine the equation of correlation between these values and ferrous metals production volume in Ukraine. We use the functions of MS Excel, i.e. a special LOGEST function (for the power law), which allows y in case when z is approximated variable, to depend on several independent variables – *x* and *y*. We obtain the following coefficients: a_2 = 1,0115, a_1 = 1,0135, a_0 = 31,4309. Standard errors of coefficients: 0,0055; 0,0160 and 0,1184, respectively; coefficient of determination is R^2 = 0,8023 and standard error is y = 0,1135; the F-test (Fisher's criterion) is 36,5178; the degrees of freedom are 18; regression sum of squares is 0,9413; the final sum of squares is 0,2320. That is, there is a sufficient degree of accuracy approximation (R^2 = 0,8023). Thus, the equation of correlation between ferrous metals production volumes (z) and coke (x) and ore (y) production volumes is: $z = 31{,}4309 \times 1{,}0135^x \times 1{,}0115^y$. Using these equations, we determine the theoretical ferrous metals production volumes in Ukraine and make the correlation between ore and coke production volumes and this value (Fig. 1.21).

The stability limits are 76 – 111 million tons for metal, 57 – 82 million tons for iron ore and 18 – 26 million tons for coke. Thus, the theoretical curve coincides with the curve built on the basis of actual statistics.

Ukraine exports and imports not only metal, but also ore and coke, so we will determine the safety factor taking into account these flows. That is, we are evaluating the stability limits of metallurgy and its supporting industries, based on the needs of ferrous metallurgy in coking and iron ore. To do this, when making the graph, we use data on ore and coke volume for the metallurgical industry (equal to the production volume in the country minus exports plus imports). We get another dependence, which, in our opinion, allows a more accurate measurement of the national economy`s stability factor (Fig. 1.22).

Thus, the stability limits could be narrowed to 88 – 105 million tons for metal, 41 – 61 million tons for iron ore and 18 – 22 million tons for coke. The absolute optimums of production volumes obtained above, namely 93, 56 and 20 million tons, respectively, are within the stability limits of industries.

The results of coking coal and iron ore development modelling in Ukraine have proved that the relationship between trends and cyclical development of ferrous metallurgy, coke and iron ore industries have to be taken into account.

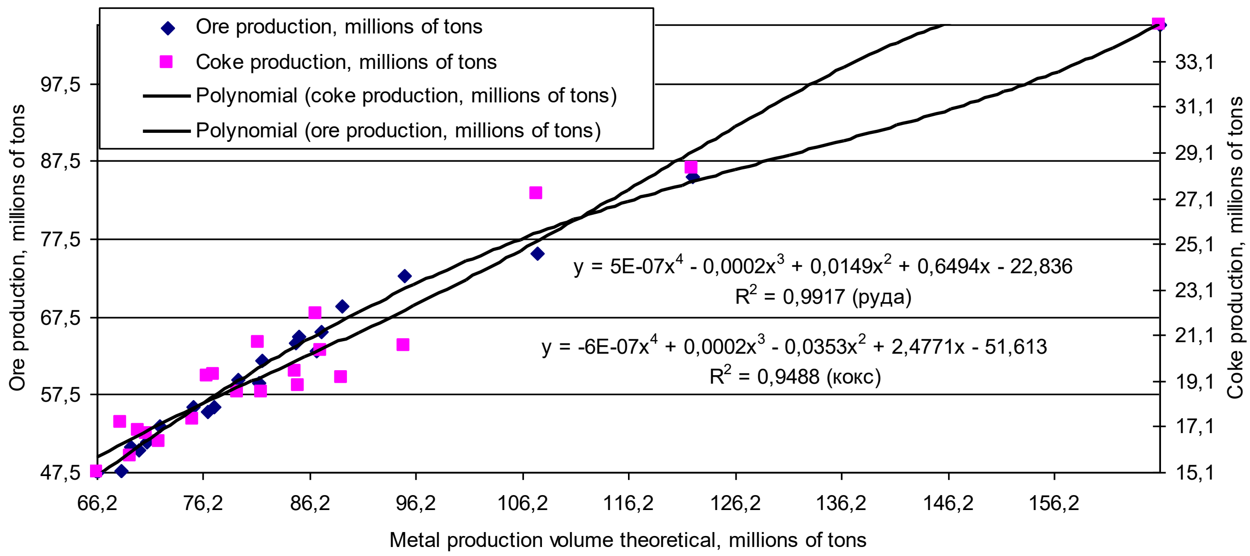

Fig. 1.21 Correlation between theoretical ore and coke production volumes, and metal production volumes in Ukraine

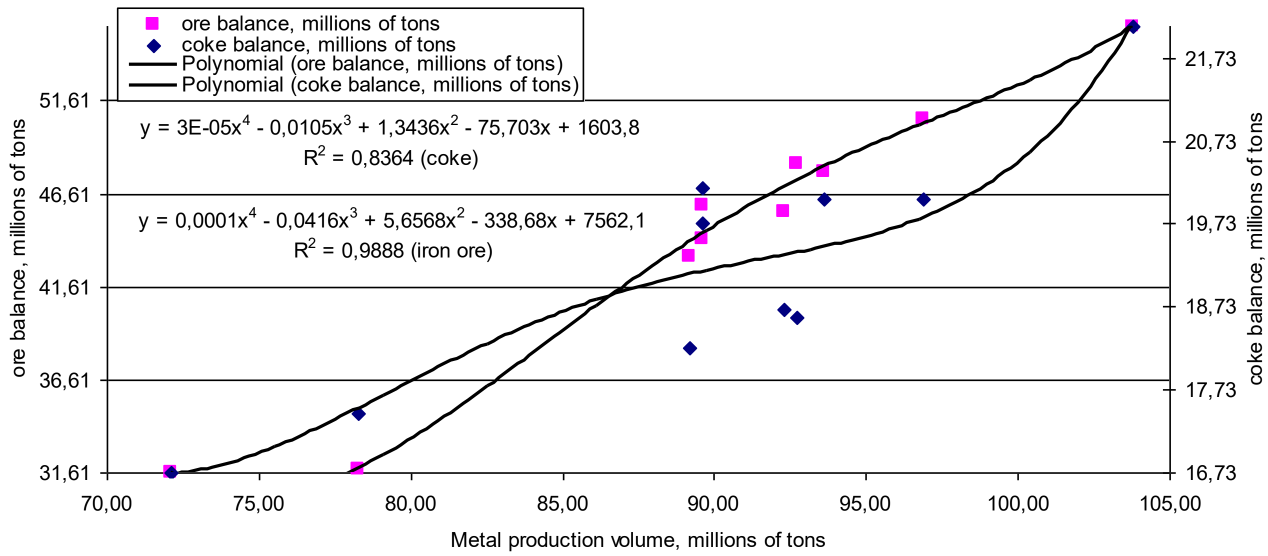

Fig. 1.22 Correlation between ore and coke balance, and metal production volumes in Ukraine

The method to measure sustainability of the extractive industry on the basis of stability factor has been proposed. It has been found out that the iron ore industry (84,41%) has the highest stability factor, and the ferrous metallurgy and coke industry have lower values – 81,70% and 81,12%, respectively. This is explained by the fact that ore production during the study period gradually grew and had lower (within 35%) fluctuations compared to other industries. The ferrous metallurgy and coke industry had fluctuations of 50 – 70%.

The model has been built applying data on ferrous metallurgy, coke and iron ore industries` production volumes and statistical indicators of relative and cumulative frequency. This made it possible to obtain absolute indicators of the industries` stability factor in kind.

The stability limits of an economic system have been determined by finding the extremum of functions that describe the regression relationship between coke and ore production from the extracted raw materials, taking into account export-import flows and their use in metal production. A satisfactory correlation of the evaluation results of the absolute stability factor and its limits has been obtained.

1.6 Dynamic balance of natural resource use aimed at national economy`s sustainable development (the case of coal mining adaptive modelling)

It has been proposed to improve the coal mining management system in Ukraine applying a bifurcation and adaptation mechanism for the development of natural resource use in Ukraine. The economic and mathematical modelling was used to prove the necessity of taking into account small fluctuations and coking coal and anthracite export-import flows to rise the efficiency of the resource base of Ukraine exploitation. The methods of efficient consideration of the world market price`s environment as a mechanism of own resource base preservation, long-term forecasting of its development parameters and paving the way for safe development of metallurgy and energy production in Ukraine have been argued.

Ukraine ranks 7th in the world of proven coal reserves. It has 834 years of coal left at current production level. Never the less, the National Programme for the Mineral Resources Base (MRB) Development of Ukraine by 2030, which takes into account the reproduction of mineral reserves, outlines geological research aimed at coal production growth. Meeting economy's needs for coal resources and their efficient application is one of the important factors to overcome the crisis in Ukraine's economy. That is, anthracite is the only type of energy raw materials, which reserves are potentially sufficient to ensure the country's energy security. At the same time, the metallurgy`s efficiency depends on production capacity of coking coal. The field provides up to 30% of national foreign exchange earnings. That is why there is a need to model and forecast the volume of coal production in Ukraine.

The development of coal mining and related sectors of the national economy can be or crisis or adaptive. Crisis processes correspond to the bifurcation mechanism of complex systems development; adaptive, on the contrary, supports high rates of economic growth. Therefore, the dynamic equilibrium of a complex system could be defined as unstable or stable equilibrium, respectively. Studying non-equilibrium states of open, dissipative, self-organized systems, for example mining industry, Stengers I., Prigogine I. and Moiseev N. concluded that each system in the process of its development could rise the resistance level to environmental impact, and pass through non-equilibrium states, i.e. bifurcation points. System behaviour and development trajectories cannot be forecasted here. One should mention that according to the law of conservation, the development trajectories retain the development trend. Besides, the system acquires more stable structure affected by self-organization processes in non-equilibrium state. The system is developing to increase its stability and accumulate efficiency. After reaching marginal stability, it gets bifurcation state, in which of all possible system states, those that meet the principle of minimum resource dissipation amid their scarcity are realized. A new system structure is formed to preserve the accumulated amount of efficiency being the basis for further development. According to the dynamical systems theory by A.M. Lyapunov and A. Poincaré, their evolution is described by a system of differential equations. This is the approach we have proposed when building economic and mathematical model to describe the cyclical nature of coal mining development, coal production and its use in metallurgy and energy production. In the economic and mathematical model it was assumed that the increment rates of electricity and cast iron production in relation to anthracite and coking coal mining, and anthracite and coke production, respectively, depend only on instantaneous value N_1, so they were considered as functions of N_1. This allowed to use linear differential equations:

$$\begin{cases} \frac{dN_1}{dt} = N_1(\varepsilon_1 + \gamma_1 N_2), \\ \frac{dN_2}{dt} = N_2(\varepsilon_2 + \gamma_2 N_1), \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dN_i}{dt} = N_i(\varepsilon_i + \gamma_i N_{i-1}) \end{cases} \tag{1.15}$$

where i – number of subsystems in complex system; N – cast iron production; coking coal mining, coke and semi-coke production (1st model); thermal energy production; anthracite and other coal mining , anthracite production (2nd model); ε – increment rate of the listed values, if there is no correlation with other amounts (coefficient of proportionality, which demonstrates the ratio of mining growth rate $\frac{dN}{dt}$ to *N*), γ - need increment rate. Taking into consideration that each model has three components, system (1.15) contains three equations. Further analysis and forecasting will apply the following indicators, namely basic growth rate (*n*) and mean value (*K*). Correlation between them is: $n = N/K$, $K = \varepsilon / \gamma$; $\varepsilon = \frac{\ln \frac{N}{N_0}}{t - t_0}$.

The studied system – coal mining, will be in a steady equilibrium state, if $N_1 = K_1$, where K_1 is mean value for the period. The system stability is associated with its tendency towards equilibrium. The equilibrium is the state of maximum efficiency. However, on the one hand, a coal mining as a complex system cannot reach full equilibrium because there are industries, which differently process and consume its products, and have dynamic interaction with each other. On the other hand, the system efficiency is being accumulated in the process of system`s development. After reaching maximum for a certain development period (Pareto efficiency), there is redistribution to the disorganization of a new state. Efficiency or the ratio of production and consumption indicators change from maximum to minimum. The process of economic system efficiency accumulation has its own limits. It ends with a crisis. The state of dynamic equilibrium between marginal maximum and minimum values is stable. When some equilibrium is reached, the system tends to leave it. Thus, a stable system fluctuates around the equilibrium state (passes from one equilibrium to another), i.e. it is in a dynamic equilibrium, which ensures its development. In our opinion, one can describe coal mining`s state of dynamic equilibrium taking into consideration small fluctuations.

Our research has shown that there is correlation between the growth rates of anthracite mining, other coal / anthracite production, exports, imports of coal, anthracite and heat production during 2008 – 2017s. Correlations are cyclic, indicating the presence of small cycles lasting 3 – 4 years. Correlation analysis has shown a high level of relationship between anthracite mining, imports and its use in energy production. Moreover, no lags have been found out between those resource flows.

Thus, to model the parameters of the equilibrium development of coal mining in Ukraine, we studied small fluctuations in the development of anthracite mining (v_1) in relation to heat production (v) taking into account import flows (v_2). To do this, we use the system of equations (1.15). Small fluctuations also exist when the value of n is close to 1. Let us consider the correlation between fluctuations in anthracite mining and its import. Let $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$, then neglecting the product v_1 , v_2, we obtain the solution of system (1):

$$\begin{cases} v_1 = A\sqrt{\varepsilon_1}\cos(\sqrt{\varepsilon_1\varepsilon_2}\,t + a) \\ v_2 = A\sqrt{\varepsilon_2}\sin(\sqrt{\varepsilon_1\varepsilon_2}\,t + a), \end{cases} \tag{1.16}$$

where A and a – constants.

Similarly, the equation describing fluctuations in heat production could be added to the system. Fig. 1.23 shows graphical interpretation of the functions $v_1(t)$, $v_2(t)$ and $v_3(t)$ with the subsequent approximation and curves smoothing by polynomial functions.

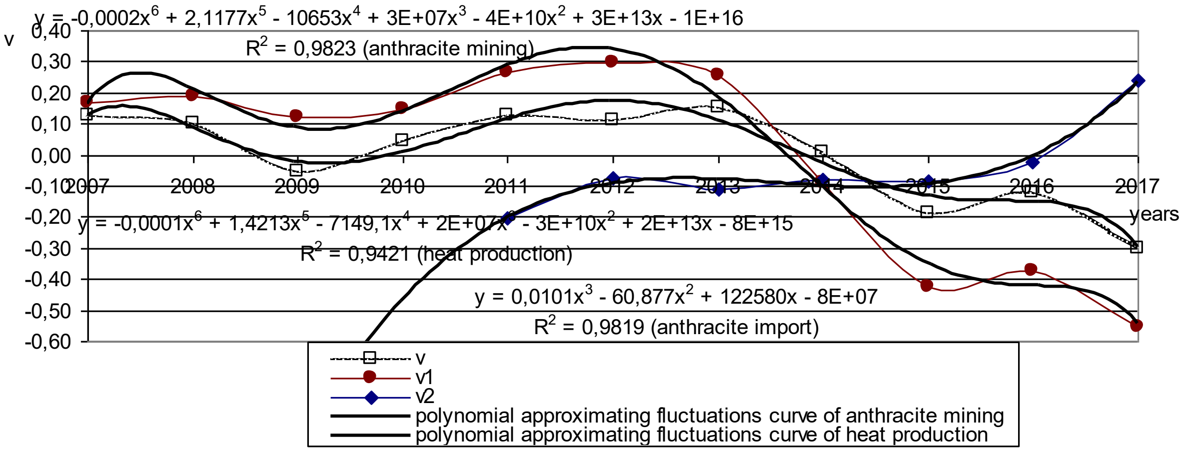

Fig. 1.23 Dynamics of small fluctuations of heat engineering development and thermal coal production taking into account anthracite imported flows during 2007 – 2017s

Thus, the obtained results indicate fluctuations' dependence on heat engineering development, anthracite production and import during 2007 – 2014s.

Due to a significant reduction (almost 27%) in domestic thermal coal production, energy sector needs have been offset by anthracite import flows since 2014. Therefore, 2014 should be considered as bifurcation point for anthracite mining in Ukraine.

In the future, aiming at shifting system to a qualitatively new state without efficiency loss, there is a need for mutual regulation of heat engineering development to meet its needs in anthracite through import flows.

We have studied the correlation between coking coal mining and production growth rates, coke production, export and import, and cast iron production during 2008 – 2017s. It has been found out that during 2008 – 2013s the dynamics of coking coal mining did not coincide with the corresponding indicators of coal mining, coke and semi-coke production and metallurgical industry (cast iron production). This one can explain by the extraction losses. Total coking coal production rose until 2013 and significantly dropped since 2015. The rest of indicators changed cyclically. A noticeable decline in production in 2015 one can explain by the hostilities in Donbass. As a result, some mines found themselves in an uncontrolled territory. During the study period, the dynamics of coke exports completely coincided with its production trends. As for the imports, Ukraine imported coke when its own production was growing until 2011. The trend has changed since 2012.

Thus, production downturn was accompanied by higher imports. The cycle is identical to the anthracite case and lasts three years. Correlation analysis has shown a high correlation between coke mining and import, and its use in metallurgy. Besides, the lags between the resource flows have been identified.

Thus, there is a need to consider lags in small fluctuations modelling for metallurgy development (cast iron production) and coking coal mining, export and import. The model for the correlation between metallurgy and coking coal mining is described by equations (1.15).

Considering time lags, the system of differential equations (1.16) must be replaced by a system of integro-differential equations:

$$\begin{cases} \frac{dN_1}{dt} = [\, \varepsilon_1 - \gamma_1 N_2(t) + \int_{t-T_0}^{t} F_1(t-\tau) N_2(\tau) d\tau \,]\, N_1(t) \\ \frac{dN_2}{dt} = -[\, \varepsilon_2 - \gamma_2 N_1(t) + \int_{t-T_0}^{t} F_2(t-\tau) N_1(\tau) d\tau \,]\, N_2(t), \end{cases} \quad (1.17)$$

where N_1 and N_2 – total production in metallurgy and coking coal mining, respectively, $F_1(t-\tau)$ and $F_2(t-\tau)$ – non-negative continuous functions, taking into account the lag in industries development, T_0 – lag period.

Let us study small fluctuations for the case of the distributed time lag.

Similar to the previous case without lag, let us suppose that $q_1 = n_1 - 1$ и $q_2 = n_2 - 1$, where $n_1 = \frac{N_1}{K_1}$ and $n_2 = \frac{N_2}{K_2}$; $N_1 = \frac{\varepsilon_2}{\gamma_2 - \Gamma_2}$ and $N_2 = \frac{\varepsilon_1}{\gamma_1 - \Gamma_1}$; $\Gamma_1 = \int_0^{T_0} F_1(\tau) d\tau$ and $\Gamma_2 = \int_0^{T_0} F_2(\tau) d\tau$.

Then q_1 and q_2 are defined from the system:

$$\begin{cases} Q_1 = \frac{dq_1}{dt} + \alpha_1 q_2 + \int_0^{T_0} F_1(\tau) q_2(t-\tau) d\tau, \\ Q_2 = \frac{dq_2}{dt} + \alpha_2 q_1 + \int_0^{T_0} F_2(\tau) q_1(t-\tau) d\tau, \end{cases} \quad (1.18)$$

where Q_1 and Q_2 – externalities, $Q = aq'' + bq$.

Coking coal export and import one can consider as the externalities. Fig. 1.24 shows graphical interpretation of functions $q(t)$ and $q_1(t)$ for cast iron production (metallurgy development) and coking coal mining, as well as for the model with the time lag.

Considering the time lag between coking coal export-import flows and cast iron production, and coking coal mining in Ukraine, the system of integro-differential equations has been applied. It has been followed by graphical interpretation of the functions $q_2(t)$ and $q_3(t)$ for coal imports and exports, respectively (Fig. 1.25).

The obtained results reveal that fluctuations in metallurgy development and coking coal mining, and heat engineering development and anthracite mining are similar. The limits are of 40%. Fluctuations of anthracite import flows and coking coal export-import flows are much more significant. This can be explained by the world market`s price environment. Therefore, we consider it as a balanced approach, when Ukraine during 2017 – 2013s had reserves amid falling world prices on coking coal. It also positively effected the resource component of the coal industry, i.e. a frugal attitude to own resources.

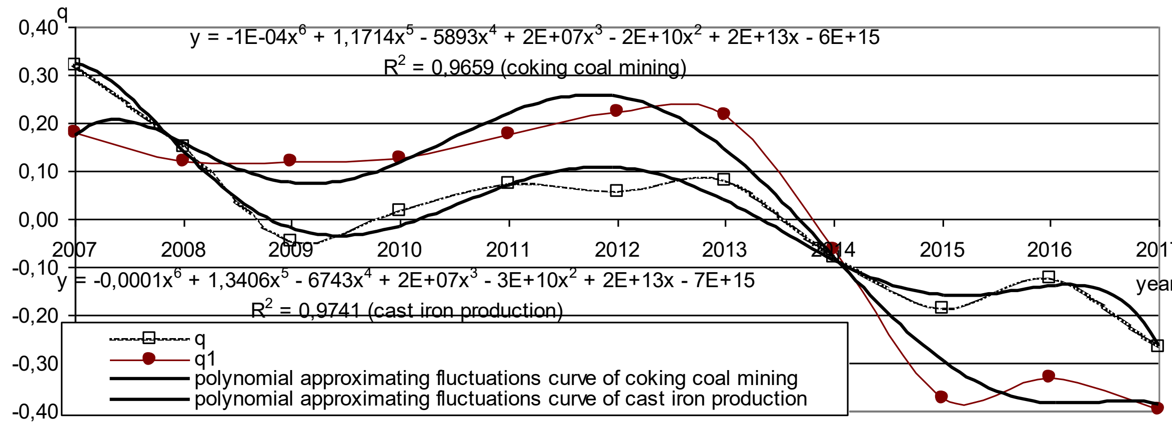

Fig. 1.24 Dynamics of small fluctuations in metallurgy, coking coal mining development, taking into account anthracite import flows during 2007 – 2017s

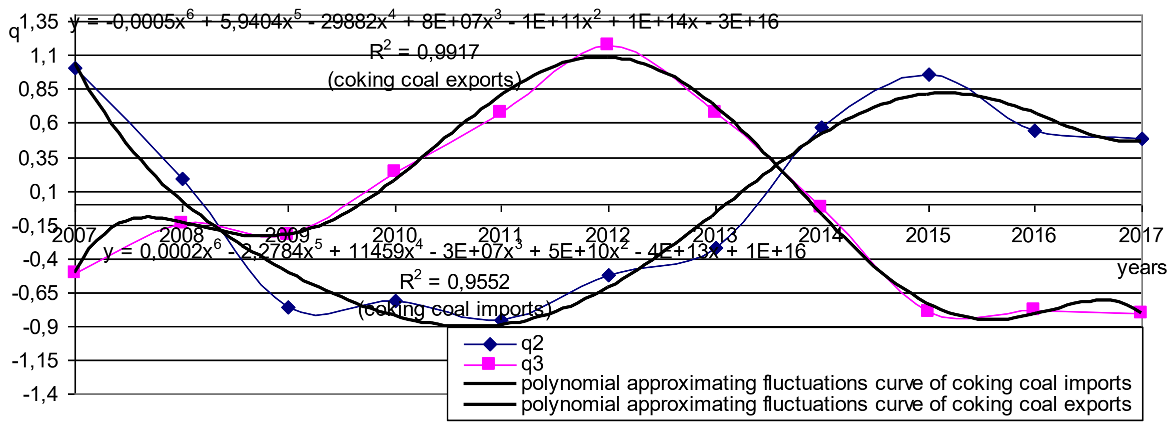

Fig. 1.25 Dynamics of small fluctuations of coking coal export-import flows during 2007 – 2017s

The bifurcation point for coking coal mining, export and import is also considered to be in 2014. The reason is the Donbass armed conflict, which prohibited deposits development in the uncontrolled territories of Ukraine. In the context of paving the way for domestic metallurgy`s safe development, one should consider more important fluctuations (the level of 80%) of import flows in the long-term forecasting of domestic coal mining.

In order to determine the priorities of efficient coal industry management in Ukraine, the system parameters of the correlation between coal mining, consumption, export and import in Ukraine have been modelled in the study. It has been proved that to form a system of long-term forecasting, it is expedient to take into account small fluctuations and determine bifurcation points. These provide the framework for stable development of both mining and related industries – metallurgy and energy production. It has been found out that there are opportunities to effectively use world market`s price environment as a mechanism to preserve own resource base and pave the way for the safe development of domestic metallurgy and energy production.

2 MODELLING OF ENVIRONMENTAL INDICATORS OF NATURAL AND ECONOMIC SYSTEMS' DEVELOPMENT

2.1 Environmental expenditure modelling for sustainable development strategies creation and implementation

The level of development of the economic mechanism of regulation and stimulation in environmental management on the basis of nature protection funding indicators has been assessed. The correlation between environmental tax revenues in the budget and the dynamics of environmental protection expenditures has been studied. It has been proved that the dynamics of environmental tax revenue in budget does not coincide with the dynamics of environmental protection expenditures. The necessity of methodological approaches development to substantiate effective lines of national policy implementation in the field of environmental protection, as well as recommendations on their scientific support based on the proposed economic and mathematical model providing the balance between environmental revenues in the budget and environmental protection expenditures has been proved. It has been confirmed that it is possible to achieve environmental policy sustainability both at the national and local level by using differential stabilization, in which government environmental demand management (needs for environmental expenditures or budget environmental protection expenditures) is associated with the rate of change of revenue (budget revenues from environmental tax and other fees).

In the context of comprehensive reform in Ukraine, decentralizing governance, public administration reform and regional policy are important. The Strategy for Sustainable Development "Ukraine 2020" contains the list of planned reforms and national development programmes. The document defines key prerequisites for the progressive national economy development, namely the tasks of ensuring sustainable, dynamic economic growth in environmentally friendly and safe way.

Territorial development, as well as of branches of the national economy and all economic entities can ensure national sustainable development only if material and spiritual needs of the population are met in accordance with world quality standards, effective application of environmental, economic and natural capacity, its preservation and reproduction. That is why the issue of sustainable development framework design focusing on its environmental component needs theoretical research and practical substantiation. The assessment criterion for the efficiency level of the economic mechanism for environmental management regulation and fostering can indirectly be the indicators of environmental funding. The basic assessment criterion for the environmental expenditures in Ukraine is primarily their share on state expenditure, which during 2006 – 2009 was 0,91 – 0,98%, during 2010 – 2014 – 0,67 – 1,11%, during 2015–2017 – 0,75 – 0,81%. Compared to the indicators of 2002 – 2004 (1,2 – 1,3%), there is noticeable contraction of environmental funding. The dynamics of environmental protection expenditures in consolidated budget of Ukraine indicates their annual growth (except for 2014), but the growth rate during 2016 – 2017 slowed down amounting to 113,12% and 117,49%, respectively. The growth rates of expenditures are insignificant, if the inflation impact is offset. That is the sign of environmental expenditures stagnation. The situation is typical for the state budget too. Besides, expenditures in 2017 decreased. As for the local budgets, their growth rates have accelerated since 2015, in 2015 and 2017 they boosted significantly. In our opinion, this is due to the processes of decentralizing governance. There is reason to believe that the established communities have more balanced approach to environmental policy and environmental matters funding.

The study results are presented in Fig. 2.1. Environmental tax rates increased during 2014 – 2018; they were risen on 01.04.2014, 01.01.2016, 01.01.2017 and 01.01.2018. The maxima on the curve of dynamics of environmental tax growth rates correspond to these periods.

As one can see from the figure, the dynamics of environmental tax revenues to the budget do not coincide with the dynamics of environmental protection expenditures budget. It is illogical situation, because according to the analysis, about 75% of the tax is paid to the state budget by big businesses, which discharge pollutants. For instance by metallurgical plants like PJSC " ArcelorMittal Kryvyi

Rih", PJSC " ILYICH IRON AND STEEL WORKS", PJSC " Dneprovsky Integrated Iron&Steel Works named after Dzershinsky", enterprise of PJSC "Cherkasykhimvolokno", the DTEK Group enterprises and others. Thus, government receives the taxpayers' money and directs it to environmental protection with a delay of at least one year. This indicates the environmental policy`s regulatory imperfections at the national and territorial levels. Unfortunately, in the context of decentralizing governance (since 2015), the situation has not improved, which illustrates low efficiency of the national environmental policy as for its territories. Current mechanism of redistributive environmental taxation for the territories allows only certain united territorial communities (UTCs) to have the opportunity to receive their share of the tax (those having polluting companies registered on their territories). Given this, there is a need to develop methodological approaches to determine the rationale for effective implementation of national environmental policy and recommendations for its scientific support, development of the set of measures for their implementation and improvement of current legal framework. It is clear that precautionary approach application enables significant savings due to even small environmental investments. Continuous investment support of environmental issue requires governmental guarantees of environmental protection expenditures, implementation of regional preventive measures with high environmental risk, support of businesses implementing innovations. We believe that synergetic effect during environmental policy implementation could be achieved, if the dynamics of environmental tax revenues in budgets will coincide with the corresponding dynamics of environmental protection expenditures. Thus, there is a need to build economic-mathematical model that would ensure higher environmental policy efficiency in the country and its regions, including UTCs. To our mind, the model should be based on the possibility of coherence between environmental taxes revenues and environmental protection fees and expenditures. That is, these flows must be balanced. Thus, the need to design territorial communities` expenditure management ecological policy is of particular importance.

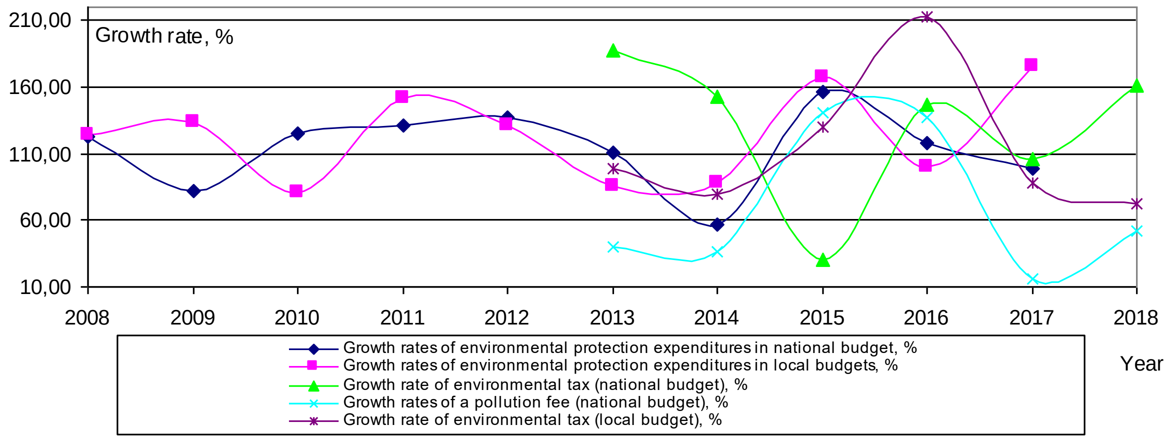

Fig. 2.1 Growth rates of expenditures in national and local budgets, a pollution fee, environmental tax transferred to national and local budgets

In our opinion, taking into consideration previous positive experience, the process of policy-making has to be grounded on mathematical tools known as Phillips' stabilization model.

Assume as input data three basic options of governmental ecological expenditure policy (environmental management expediture) $G^{*}(t)$:

a) Proportional stabilization policy with government ecological expenditure. In this case, the government ecological expenditure is described by equation:

$$G^{*}(t) = -\gamma_{p} \cdot U(t), \tag{2.1}$$

where $U(t)$ – income, generated by the cumulative money transferred from environmental protection fees and expenditures;

b) Derivative stabilization ecological policy: government ecological expenditure adjustments correlate with the rate of ecological income change (environmental protection fees and expenditures) (the derivative is used). In this case the government ecological expenditure is:

$$G^{*}(t) = -\gamma_{d} \cdot \frac{dY}{dt}. \tag{2.2}$$

c) Integral stabilization ecological policy: government ecological expenditure is proportional to the ecological income deficit generated and accumulated. In this case the government ecological expenditure is:

$$G^*(t) = -\gamma_i \int_0^t U(\tau) \cdot d\tau\,, \tag{2.3}$$

In all three cases, the coefficients of proportionality $\gamma_p, \gamma_d, \gamma_i$ – are coefficients with given values (coefficients of proportionality). There is difference $G^*(t) - G(t)$, between potential public ecological expenditure $G^*(t)$ and actual $G(t)$, which affects the economic system and should be eliminated for the sake of stabilization. We assume $\beta < 0$ – is a coefficient, which indicates the speed of response to state decision-making (reaction coefficient). Then decisions about state ecological expenditure $G(t)$ are determined as solution of first order linear differential equation:

$$\frac{dG(t)}{dt} + \beta \cdot G(t) = \beta \cdot G^*(t), \tag{2.4}$$

where $G^*(t)$ is determined by one of the policy options (a – c) or their combination aiming to balance demand and supply of ecological income.

The use of the Phillips model only with the multiplier, without the accelerator, i.e. the coefficient of proportionality of the induced ecological investment to the rate of ecological income change $\frac{dY}{dt}$, aggregated ecological demand $Y_c(t)$,with government ecological expenditure becomes:

$$Y_c(t) = cY(t) + G(t) + C_0 + I_0\,, \tag{2.5}$$

where $0 < c < 1$ – propensity to ecological consume; C_0 – ecological consumption; I_0 – autonomous ecological investment.

The supply ecological resources in this case is made of equation:

$$\frac{dY}{dt} = \alpha(Y_c(t) - Y(t)) \cdot \tag{2.6}$$

We make the mathematical model of economic stabilization ecological policy and calculate its effect.

Substituting into equation (2.6) for $Y_c(t)$ its values from correlation (2.5), we obtain:

$$\frac{dY}{dt} = -\alpha s Y(t) + \alpha G(t) + \alpha A, \tag{2.7}$$

where $s = 1 - c$ – propensity to ecological save (rate of accumulation) ($1/s = \mu$ – the Keynesian multiplier); $A = C_0 + I_0$ (investment is const.).

Differentiating (2.5) we obtain:

$$\frac{dYc}{dt} = c\frac{dY}{dt} + \frac{dG}{dt}. \tag{2.8}$$

We add equation (2.8) to equation (2.5) and multiply the sum by β (reaction coefficient), then we obtain:

$$\frac{dYc}{dt} + \beta Yc = c\frac{dY}{dt} + \beta cY(t) + \beta G(t) + \beta A \cdot \tag{2.9}$$

Substituting into (2.8) for $\frac{dG}{dt}$ its value from (2.4), we obtain:

$$\frac{dY_c}{dt} + \beta Y_c(t) = c\frac{dY}{dt} + \beta cY(t) + \beta G^*(t) + \beta A \cdot \tag{2.10}$$

From equation (2.6)

$$Y_c(t) = \frac{1}{\alpha}\left(\frac{dY}{dt} + \alpha Y(t)\right), \tag{2.11}$$

obtain

$$\frac{dY_c}{dt} = \frac{1}{\alpha}\left(\frac{d^2Y}{dt^2} + \alpha\frac{dY}{dt}\right) \cdot \tag{2.12}$$

Substitute the expressions (2.11), (2.12) into the ratio (2.10):

$$\frac{1}{\alpha}\left(\frac{d^2Y}{dt^2} + \alpha\frac{dY}{dt}\right) + \frac{\beta}{\alpha}\left(\frac{dY}{dt} + \alpha Y(t)\right) = c\frac{dY}{dt}\beta G^*(t) + \beta A \cdot \tag{2.13}$$

or

$$\frac{d^2Y}{dt^2} + (\beta + \alpha s)\frac{dY}{dt} + \alpha\beta_s Y(t) = \beta\alpha G^*(t) + \beta\alpha A \cdot \tag{2.14}$$

Relevant characteristic equation for the equation (2.14) can be shown to be:

$$\lambda^2 + (\beta + \alpha s)\lambda + \alpha\beta s = 0. \tag{2.15}$$

And has the roots $\lambda_1 = -\beta$, $\lambda_2 = -\alpha s$.

Thus, general solution of the relevant homogeneous equation becomes:

$$Y^0(t) = C_{1e}^{-\beta_t} + C_{2e}^{-\alpha st}, \tag{2.16}$$

where C_1, C_2 – arbitrary constants. We have $Y^0(t) \to 0$ with $t \to \infty$.

We analyze peculiarities of the policy of derivative stability.

Its equation (2.5) becomes:

$$\frac{d^2Y}{dt^2} + (\beta + \alpha s)\frac{dY}{dt} + \alpha\beta(s + \gamma_p)Y(t) = \alpha\beta A \cdot \tag{2.17}$$

The equation (2.17) general solution is obtained as $Y(t) = Y^0(t) + Y^*$.

We use trial and error method to find partial solution:

$$\mathrm{Y}^{*} = \frac{\mathrm{A}}{s + \gamma_{\rho}}. \qquad (2.18)$$

To find general solution $\mathrm{Y}^{0}(t)$ of the relevant homogeneous equation we make the characteristic equation:

$$\lambda^{2} + (\beta + \alpha s)\lambda + \alpha\beta(s + \gamma_{\rho}) = 0. \qquad (2.19)$$

To find the roots of the equation (2.19) we find the discriminant:

$$D = (\beta + \alpha s)^{2} - 4\alpha\chi(s + \gamma_{\rho}) = (\beta - \alpha s)^{2} - 4\alpha\beta\gamma_{\rho}. \qquad (2.20)$$

If D = 0, then $\gamma_{\rho} = \frac{(\beta - \alpha s)^{2}}{4\alpha\beta}$ and $\lambda_{1} = \gamma_{2} = \frac{\beta + \alpha s}{2}$, where $\mathrm{Y}^{0}(t) = (C_{1}t + C_{2})e^{\frac{\beta+\alpha s}{2}e}$, C_{1}, C_{2} are arbitrary constants. If $t \to \infty$ we obtain: $\mathrm{Y}^{0}(t) \to 0$ *a* $\mathrm{Y}(t) \to \mathrm{Y}^{*} = \frac{\mathrm{A}}{s + \gamma_{\rho}}$, so the development process is being stabilized. If $D < 0$, then $\gamma_{\rho} > \frac{(\beta - \alpha s)^{2}}{4\alpha\beta}$, and roots $\lambda_{1,2}$ are complex-conjugate, also oscillations take place. In this case $\mathrm{Y}\ (t) = e - \frac{(\beta + \alpha s)}{2}(C_{1}\cos\varphi\, t) + C_{2}s\ \alpha\ t)$, and if $t \to \infty$ we obtain: $\mathrm{Y}^{0}(t) \to 0$.

There are damped oscillations and the process is tend to stabilize. Thus, application of differential stabilization policy means that government environmental demand management (needs for environmental expenditures or budget environmental protection expenditures) is associated with the rate of change of revenue (budget revenues from environmental taxes and other fees), i.e. the use of a derivative.

The level of development of the economic mechanism of regulation and stimulation in environmental management on the basis of nature protection funding indicators has been assessed. The analysis of dynamics of total environmental protection expenditure change has been made. The correlation between environmental tax revenues in the budget and the dynamics of environmental protection expenditures has been studied. It has been proved that the dynamics of environmental tax revenue in budget does not coincide with the dynamics of environmental protection expenditures. This indicates the imperfection of environmental policy`s regulatory mechanisms both at the state and territorial level.

The necessity of methodological approaches development to substantiate effective lines of national policy implementation in the field of environmental protection, as well as recommendations on their scientific support based on the proposed economic and mathematical model providing the balance between environmental revenues in the budget and environmental protection expenditures has been proved. It has been confirmed that it is possible to achieve environmental policy sustainability both at the national and local level by using differential stabilization, in which government environmental demand management (needs for environmental expenditures or budget environmental protection expenditures) is associated with the rate of change of revenue (budget revenues from environmental tax and other fees).

2.2 Economic and mathematical modelling of ecosystems` territorial sustainability

The study is relevant, because nowadays the growth of anthropogenic loads and air pollution require special attention to the modelling of determinants of pollutant emissions` dynamics

Economic and statistical modelling of pollutant emissions` mass and growth rates during 1991 – 2017 applying statistical indicators of relative and cumulative frequency has been made in the article. The following substances have been studied: sulfur dioxide, nitrogen dioxide, carbon monoxide and carbon dioxide. Statistical indicators of relative and cumulative frequency have been used when modelling. It has been taken into consideration in the model that values of emissions during the studied period are marginal ones, which do not pose a threat to the environment in the country. It has been proposed to consider the reliability degree equal to half of the confidence interval for the general arithmetic average growth rate of emissions and their growth rates as the limits of Ukrainian ecosystem`s sustainable development. The obtained results have been tested by the estimation based on the confidence interval for the mean.

The study results can be used to predict environmental pollution parameters in Ukraine, which will not cause an environmental disaster.

The atmosphere plays essential role in global, regional and local transporting of pollutants and environmental pollution. Growing anthropogenic load caused by the accumulation of harmful impurities affecting air pollution, slows down the process of atmosphere`s natural self-cleaning. Today, global effects of air pollution exist, i.e. accumulation of greenhouse gases and ozone depletion. Thus, the issue of the determinants of air pollution dynamics modelling in Ukraine needs to be solved.

The qualitative composition of atmospheric air directly depends on the anthropogenic loads on the air. Atmospheric pollutant emissions in Ukraine both by stationary and mobile sources generally tend to decrease (except for 2012). However, in 2016 there was acceleration in growth rates of pollutant emissions, except nitrogen dioxide (Fig. 2.2).

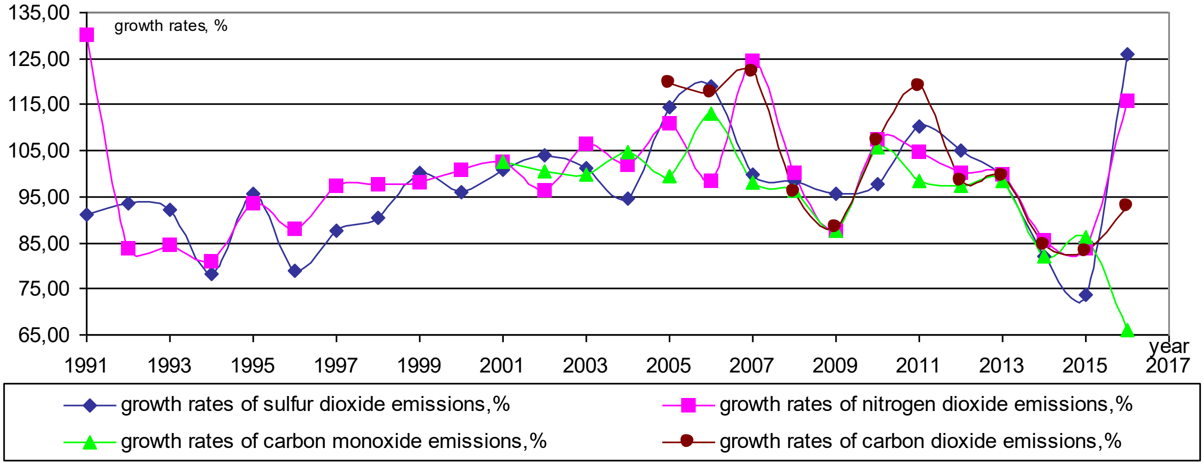

Fig. 2.2 Dynamics of growth rates of pollutant emissions during 1991 – 2016

Economic and statistical modelling of the dynamics of pollutant emissions` determinants in Ukraine for the following substances: sulfur dioxide, nitrogen dioxide, carbon monoxide and carbon dioxide have been made. Data on the sum of emissions during 1991 – 2017 and their growth rates have been used. The model is to determine the safe limits of changes in emissions and their growth rates. The approach is explained by the fact that during the studied period emissions fluctuated within certain limits, but the environmental disaster did not happen. These data can be used as a guide for the target (expected) emission values (minimum emission degree) and critical ones (maximum emission degree).

Economic and statistical modelling was made based on statistical indicators of relative and cumulative frequency. These data according to mathematical statistics are the sample of each substance emissions` values. A positive number that indicates how many times a variant happens within the data is called a frequency.

Relative frequencies (the variant frequency ratio to the sample) are often used instead of frequency values, called ratios. Relative frequencies indicate how often certain pollutant emissions were received during the studied period.

We assume that emission values most frequently met during the studied period are the marginal emissions that do not pose a threat to the national environmental situation.

On the other hand, the limits can be estimated by the confidence interval for mean values. For the accepted 0,95 confidence level presented in most studies, the confidence interval for mean values will be determined using Microsoft Excel (Service, Data Analysis, Descriptive Statistics).

Thus, we propose the reliability degree equal to half of the confidence interval for the general arithmetic mean of pollutant emissions` growth rate. It is considered as the limits of present ecosystem sustainable development.

Let us make calculations in two ways and compare them. The calculation results are shown in Fig. 2.3 – 2.6.

Fig. 2.3 demonstrates the following masses of sulfur dioxide emissions: 1077,5 and 1375,5 thousand tons as optimal values. That is, these values do not pose a threat to the ecosystem`s sustainability.

When applying the second method, we obtain the following results: 0,95 confidence level provide the limits of allowable emissions of 1397,6 ± 200,0689 thousand tons.

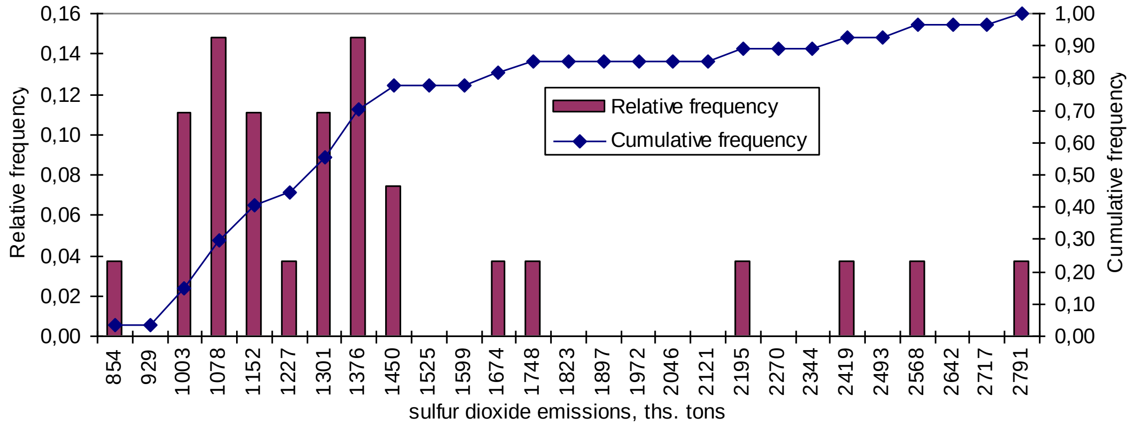

Fig. 2.3 Relative and cumulative frequencies of sulfur dioxide emissions during 1991 – 2016

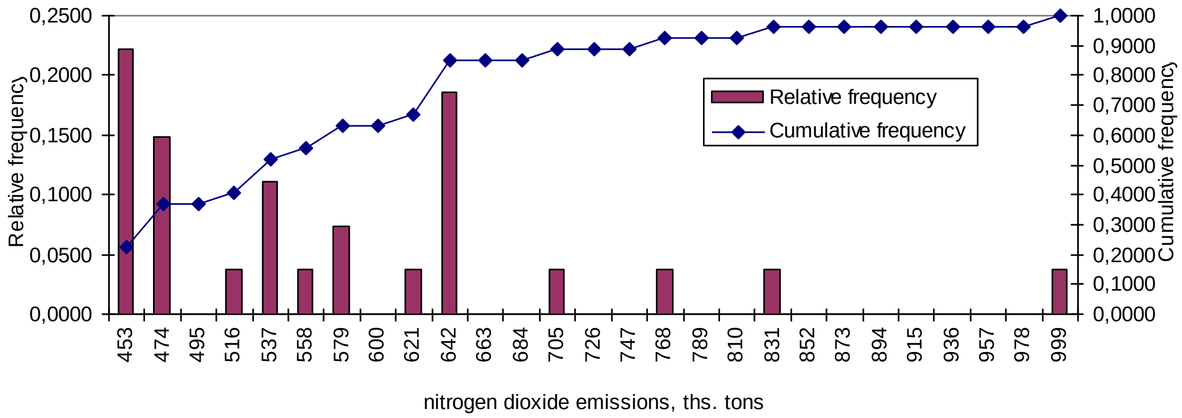

Fig. 2.4 Relative and cumulative frequencies of nitrogen dioxide emissions during 1991 – 2016

Fig. 2.4 illustrates the following masses of nitrogen dioxide emissions: 453 and 642 thousand tons as optimal values. These values do not threaten the ecosystem`s sustainability.

When applying the second method, we obtain the following results: 0,95 confidence level provide the limits of allowable emissions of 568,6852 ± 53,2569 thousand tons.

Fig. 2.5 shows the following masses of carbon monoxide emissions: 2871,2 and 2975,8 thousand tons as optimal. That is, these values do not pose a threat to the ecosystem`s sustainability.

The second method application allows to obtain the following results: 0,95 confidence level provide the limits of allowable emissions of 2763,62 ± 257,0544 thousand tons.

Fig. 2.6 illustrates the following masses of 132,2; 156,2 and 236,2 million tons as optimal values. That is, these values do not pose a threat to the ecosystem`s sustainability.

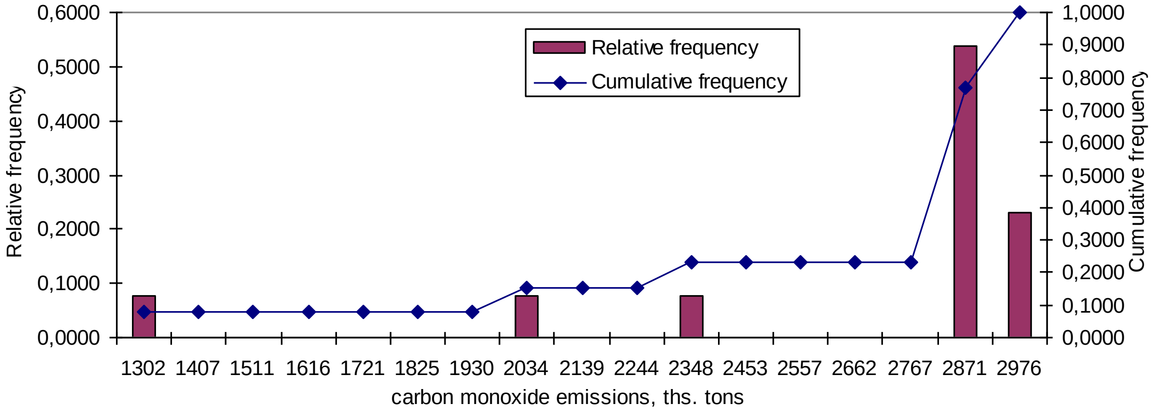

Fig. 2.5 Relative and cumulative frequencies of carbon monoxide emissions during 1996 – 2016

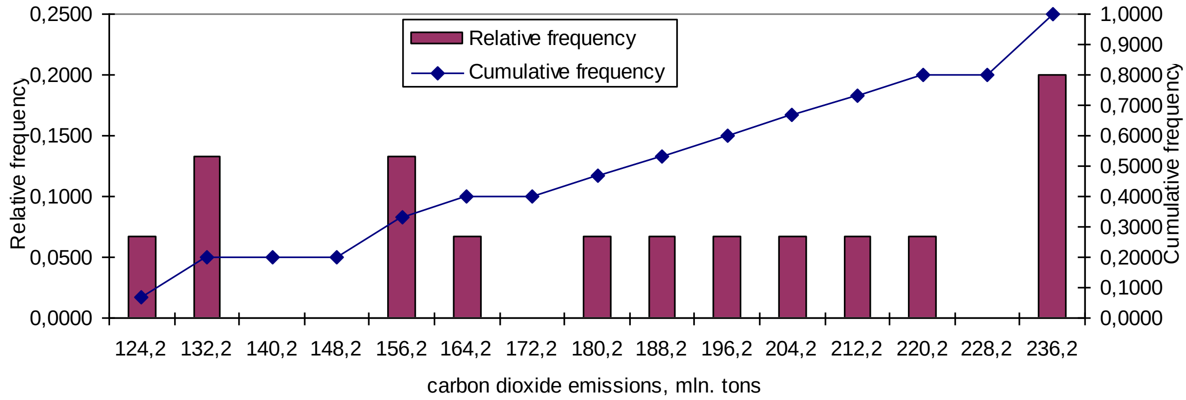

Fig. 2.6 Relative and cumulative frequencies of carbon dioxide emissions during 2000 – 2018

The second method application gives the following results: 0,95 confidence level provide the limits of allowable emissions of 181,68 ± 21,9718 million tons.

Similarly, the optimal values of growth rates of pollutant emissions have been estimated. It has been found out that for sulfur dioxide they are 96,0%. This means that emissions` growth rate`s slowdown at 4% per year will strengthen ecosystem`s sustainability. When applying the second method, we obtain the following results: 0,95 confidence level provide the limits of allowable changes of sulfur dioxide emissions growth rates of 97,28 ± 5,0356%.

It has been proved that the following growth rates of nitrogen dioxide emissions can be considered as optimal values: 85,6 and 101,6%. That is, these values that do not pose a threat to the ecosystem`s sustainability. Slowing down the emissions` growth rate at 14,4% per year will strengthen the ecosystem`s sustainability. When applying the second method, we obtain the following results: 0,95 confidence level provide the limits of allowable changes of nitrogen dioxide emissions` growth rate at 99,28 ± 4,9249%.

The growth rate of carbon monoxide emissions was an average 101,24%. This indicates that, unfortunately, there was no steady emission growth slowdown during 1991 – 2017, i.e. there was lack of conditions for strengthening the ecosystem`s sustainability.

The second method application provide the following results: 0,95 confidence level provide the limits of allowable changes in the growth rate of carbon monoxide emissions at 96,05 ± 5,8762%. To strengthen the ecosystem`s sustainability, the decline of emissions` growth rate at 5% or more should be achieved.

The following growth rates of carbon dioxide emissions can be considered as allowable values: 85,97 and 120,97%. That is, these values that do not threaten the ecosystem`s sustainability. The ecosystem`s sustainability strengthening could be achieved due to the emissions` growth rate at 14,1% or more downtrend.

The second method allows to obtain the following results: 0,95 confidence level provide the limits of allowable changes in the growth rate of carbon dioxide emissions at 100,82 ± 8,9367%.

Economic and statistical modelling of totals and rates of growth of pollutant emissions during 1991 – 2017 has been made applying statistical indicators of relative and cumulative frequency for the following substances: sulfur dioxide, nitrogen dioxide, carbon monoxide and carbon dioxide. In the process of modelling, statistical indicators of relative and cumulative frequency have been used. The model takes into account that the values of emissions that were most often achieved during the studied period are the limits of emissions that do not pose a threat to the environmental situation in the country. It has been proposed to consider the reliability degree equal to half of the confidence interval for the general arithmetic mean of pollutant emissions growth rate as the limits of Ukrainian environmental system`s sustainable development. The obtained results have been verified by the estimation using the confidence interval for the mean values. It has been found out that the optimal values of sulfur dioxide emissions are from 1077,5 and 1375,5 thousand tons. The emission limit values will be 1397,6 ± 200,0689 thousand tons. Optimal values of nitrogen dioxide emissions are 453 and 642 thousand tons. The emission limit values will be 568,6852 ± 53,2569 thousand tons. Optimal values of carbon monoxide emissions are 2871,2 and 2975,8 thousand tons. The emission limit values will be 2763,62 ± 257,0544 thousand tons. Optimal values of carbon dioxide emissions are132,2; 156,2 and 236,2 million tons. The emission limit values will be 181,68 ± 21,9718 million tons. The optimal growth rates of sulfur dioxide emissions are 96,0%. The limits of the allowable change in sulfur dioxide emissions growth rate will be 97,28 ± 5,0356%. Optimal values of nitrogen dioxide emissions growth rates are 85,6 and 101,6%. That is, these values do not pose a threat to the ecosystem`s sustainability. Emissions` growth rate slow down at 14,4% per year will help to strengthen ecosystem`s sustainability. The limits of the allowable change in nitrogen dioxide emissions growth rate will be 99,28 ± 4,9249%. It has been revealed that carbon monoxide emissions` growth rate was an average 101,24%, i.e. during 1991 – 2017 the emission growth was in steady slowdown, that is there were no prerequisites for ecosystem`s sustainability strengthening. It will be possible to strengthen the ecosystem`s sustainability, if the emissions` growth rate slows down by 5% or more. The following growth rates of carbon dioxide emissions can be considered as allowable: 85,97 and 120,97%. That is, these values do not threaten the ecosystem`s sustainability. Ecosystem`s sustainability could be strengthened as a result of emission growth rate retard at 14,1% or more. The limits of the allowable change in carbon dioxide emissions` growth rate will be 100,82 ± 8,9367%. The study results could be used to forecast the parameters of environmental pollution in Ukraine, which will not lead to the environmental disaster.

2.3 Environmental end mathematical model of united communities' (UTC) sustainable development

It has been illustrated that among the most important environmental and economic instruments of environmental protection in Ukraine are the environmental tax and the environmental pollution fee. It has been proved that emissions have recently decreased according to the analysis results of the environmental pollution fee dynamics. It has been confirmed that the growth of the environmental tax revenue in local budgets in 2015 was due to the beginning of decentralization and changes in the Tax Code. It has been defined that the slowdown in the environmental tax revenue growth rate in local budgets is caused by inefficient system of united territorial communities` (UTG) budgets and local budgets` pumping up. It has been proved that the rate of national budget funding is ebbed accompanied by the growing funding from local budgets. This gave grounds to claim that the established UTCs have a more balanced approach to environmental policy and the need to fund environmental activities. The analysis of the dynamics and sources of environmental capital and current investment funding have revealed that capital investment in environmental activities is unstable, without radical changes in national environmental situation. It has been proved that the growth rate of current investment is insignificant and equals the inflation rate, which reduces its economic efficiency.

In the context of decentralizing governance, the issue of territorial sustainable development needs to be solved. It is based on both economic and social components, and environmental one too. Thus, there is a need for the formation of organizational and economic mechanisms and regulators of the process of environmental policy in Ukraine amid decentralizing governance. That is why the issue of territorial environmental funding and management is worth serious consideration. National sustainable development cannot be ensured without creating conditions for sustainable development of its administrative-territorial units. That is why there is a need to build the model of united territorial communities` (UTCs) sustainable development. The main components of this model, in our opinion, are economic, social and environmental components. One of the most important environmental and economic instruments of environmental protection in Ukraine is the environmental tax and pollution fee (the system of relevant funds formed on its basis).

Tables 2.1, 2.2 show the data on the dynamics of environmental tax and pollution fee revenues in the budgets of different levels in Ukraine during 2012 – 2018.

Table 2.1 Environmental tax and pollution fee revenues in the budgets of different levels in Ukraine, million UAH

Year	Environmental tax (consolidated budget), million UAH	Environmental tax (national budget), million UAH	Environmental tax (local budget), million UAH	Pollution fee (consolidated budget), million UAH
2012	2816,00827795	1263,5661176	1552,4431603	42,6976062
2013	3899,48699600	2364,9265094	1534,5604870	16,9346611
2014	4830,90870700	3614,4809841	1216,4277230	6,21910521
2015	2691,04014100	1105,4138181	1585,6263232	8,72057723
2016	4987,43524500	1619,1701273	3368,2651186	11,98535279
2017	4698,43846100	1720,7890443	2977,6494168	1,94888500
2018	4921,50361100	2779,6176683	2141,8859433	1,02112770

The analysis of the above data shows that during 2015 – 2016, there was acceleration of the growth rate of the pollution fee; during 2017 – 2018, it significantly ebbed. This may be the evidence of the emissions` recent contraction. Environmental tax rates increased during 2014 – 2018; they grew on 01.04.2014, 01.01.2016, 01.01.2017 and 01.01.2018. The revenue growth rates of consolidated and national budget fully correspond to these periods. The boost of total environmental tax revenue in local budgets in 2015 can be explained by the beginning of decentralization and the Tax Code adjustments. Table 2.2 demonstrates that there is slowdown in growth rate of environmental tax revenue in local budgets during 2017 – 2018. We consider, this can be explained by the current inefficiency of UTCs and local budgets` pumping up, as most communities do not have big polluting enterprises on their territory and, accordingly, they lack the revenue source.

Table 2.3 analyzes the dynamics of environmental protection expenditure changes.

Table 2.2 Dynamics of environmental tax and pollution fee revenues in the budgets of different levels in Ukraine, %

Year	Growth rate of environmental tax (consolidated budget), %	Growth rate of environmental tax (national budget), %	Growth rate of environmental tax (local budget), %	Growth rate of a pollution fee (consolidated budget), %
2013	138,48	187,16	98,85	39,66
2014	123,89	152,84	79,27	36,72
2015	55,70	30,58	130,35	140,22
2016	185,33	146,48	212,42	137,44
2017	94,21	106,28	88,40	16,26
2018	104,75	161,53	71,93	52,40

Table 2.3 Dynamics of budget environmental protection expenditure changes, million UAH

Year	Budget environmental protection expenditure	Growth rate, %	National budget environmental protection expenditure	Growth rate of national budget expenditure, %	Local budgets environmental protection expenditure	Growth rate of local budgets expenditure ,%
2007	2241,3		1809,1		432,2	
2008	2764,7	123,35	2230,2	123,28	534,5	123,67
2009	2538,8	91,83	1824,3	81,80	714,5	133,68
2010	2872,4	113,14	2292,7	125,68	579,7	81,13
2011	3890,7	135,45	3008,40	131,22	882,30	152,20
2012	5297,9	136,17	4135,40	137,46	1162,50	131,76
2013	5594,2	105,59	4595,00	111,11	999,20	85,95
2014	3481,7	62,24	2597,00	56,52	884,70	88,54
2015	5529,7	158,82	4053,00	156,06	1476,70	166,92
2016	6255,4	113,12	4771,60	117,73	1483,80	100,48
2017	7349,3	117,49	4739,90	99,34	2609,30	175,85

Table 2.4, 2.5 analyze the dynamics and sources of funding of capital and current investment in environmental protection.

Table 2.4 Dynamics and sources of funding of capital investment in environmental protection, million UAH

Year	Capital investment in environmental protection, million UAH	Growth rate, %	i.a. from national budget, million UAH	i.a. from local budgets, million UAH	i.a. equity capital, million UAH	i.a. form another source, million UAH	Local budgets and another source together, million UAH
2007	3080,7		297,90	357,50	2291,50	133,80	491,30
2008	3731,4	121,12	707,80	569,90	2269,00	184,70	754,60
2009	3040,7	81,49	276,40	346,70	2325,90	91,70	438,40
2010	2761,500	90,82	240,50	261,30	2145,10	114,60	375,90
2011	6451,035	233,61	285,00	347,60	4297,60	1520,80	1868,40
2012	6589,300	102,14	89,50	371,50	3893,00	2235,30	2606,80
2013	6038,800	91,65	78,70	350,20	3593,50	2016,40	2366,60
2014	7959,900	131,81	39,80	no data	3924,54	no data	3995,56
2015	7675,600	96,43	314,70	no data	2692,25	no data	4668,65
2016	13390,500	174,46	374,93	no data	3896,90	no data	9118,67
2017	11025,600	82,34	385,90	no data	5132,10	no data	5507,60

As one can see, the growth rates of capital investment are volatile; the slowdown was observed in 2013, 2015 and 2017, acceleration, on the contrary, in 2014 and 2016. This approach cannot be considered systemic and, as a result, there is lack of radical changes in the environmental situation in the country.

The rates of current investment have been increasing since 2015, but not significantly, at the official inflation rate, which reduces their economic efficiency.

Tables 2.6 and 2.7 analyze the structure of the funding sources of environmental capital and current investment.

As one can see, the largest share of total investment belongs to the equity capital. Growth of another source share during 2011 – 2013 can be explained by the international financial assistance for environmental activities. The share of national budget expenditures has been declining in recent years and amounted to only 3,5% in 2017. Nevertheless, in the past few years, the share of local budgets` funding has slightly increased. The obtained results confirmed the conclusions made about the ineffectiveness of environmental national policy of capital investment.

Table 2.5 Dynamics and sources of funding of current investment in environmental protection, million UAH

Year	Current investment in environmental protection, million UAH	Growth rate, %	i.a. from national budget, million UAH	i.a. from local budgets, million UAH	i.a. equity capital, million UAH	i.a. form another source, million UAH	Local budgets and another source together, million UAH
2007	6610,30		146,00	58,10	6386,60	19,60	77,70
2008	8444,60	127,75	200,90	68,60	8144,50	30,60	99,20
2009	8032,70	95,12	250,40	75,50	7699,30	7,50	83,00
2010	10366,60	129,05	280,20	93,20	9983,10	10,10	103,30
2011	12039,40	116,14	314,30	113,00	11598,80	13,30	126,30
2012	13924,70	115,66	343,20	116,10	13452,40	13,00	129,10
2013	14339,10	102,98	375,80	137,50	13815,50	10,30	147,80
2014	13965,70	97,40	279,31	no data	13509,18	no data	177,21
2015	16915,50	121,12	304,48	no data	16382,10	no data	228,92
2016	19098,20	112,90	553,85	no data	18172,70	no data	371,65
2017	20466,40	107,16	470,73	no data	19114,50	no data	881,17

Table 2.7 demonstrates that both capital investment and current investment originate from equity capital. If capital investment is funded by businesses at 40 – 50%, then current investment is covered at 95% on average. Another source is not significant for current investment (less than 1%). Public funding of current investment is also not significant – at about 2%. Current financing from local budgets is even of less importance.

Thus, among the issues which could be solved due to the managerial impact are the following: improvement of environmental tax mechanisms (it is received only by UTCs with big industrial enterprises located in their territories); introduction of the mechanism of united territorial communities` environmental tax revenue accumulation aimed at their efficient application for purpose; ensuring the receipt of the part of national standing subsurface use tax into UTCs budgets, as subsurface use causes environmental damage at the local level; introduction of a number of taxes, namely a) for businesses managing waste disposal or storage facilities located within a community, b) for the use of natural resources of local standing - forest and water resources, mining of local importance (sand, gravel) , fertile soil, peat, etc.), if they are exported outside the district.

Table 2.6 Structure of funding sources of environmental capital investment, %

Year	Share of national budget in total capital investment, %	Share of local budgets in total capital investment, %	Share of equity capital in total capital investment, %	Share of another source in total capital investment, %	Share of local budgets and another source together in total capital investment, %
2007	9,67	11,60	74,38	4,34	15,95
2008	18,97	15,27	60,81	4,95	20,22
2009	9,09	11,40	76,49	3,02	14,42
2010	8,71	9,46	77,68	4,15	13,61
2011	4,42	5,39	66,62	23,57	28,96
2012	1,36	5,64	59,08	33,92	39,56
2013	1,30	5,80	59,51	33,39	39,19
2014	0,50	no data	49,30	no data	50,20
2015	4,10	no data	35,08	no data	60,82
2016	2,80	no data	29,10	no data	68,10
2017	3,50	no data	46,55	no data	49,95

The study proves that country`s sustainable development cannot be ensured without building conditions for sustainable development of its administrative-territorial units on the basis of the model, having economic, social and environmental components. It has been illustrated that among the most important environmental and economic instruments of environmental protection in Ukraine are the environmental tax and the environmental pollution fee.

The necessity of developing methodological approaches to substantiate effective trends of national policy implementation in the field of environmental protection for UTCs, recommendations on their scientific support, elaboration of the set of instruments for their implementation and improvement of current organizational and legal framework have been proved.

Table 2.7 Structure of funding sources of environmental current investment, %

Year	Share of national budget in total current investment, %	Share of local budgets in total current investment, %	Share of equity capital in total current investment, %	Share of another source in total current investment, %	Share of local budgets and another source together in total current investment, %
2007	2,21	0,88	96,62	0,30	1,18
2008	2,38	0,81	96,45	0,36	1,17
2009	3,12	0,94	95,85	0,09	1,03
2010	2,70	0,90	96,30	0,10	1,00
2011	2,61	0,94	96,34	0,11	1,05
2012	2,46	0,83	96,61	0,09	0,93
2013	2,62	0,96	96,35	0,07	1,03
2014	2,00	no data	96,73	no data	1,27
2015	1,80	no data	96,85	no data	1,35
2016	2,90	no data	95,15	no data	1,95
2017	2,30	no data	93,39	no data	4,31

Analysis of dynamics and funding sources of capital and current investment in environmental protection has been carried out. It has been demonstrated that the dynamics of capital investment growth is unstable, as a result there are no radical changes in the national environmental situation. It has been found out that the growth rates of current investment are insignificant, almost coincide with the official inflation rate, which reduces its economic efficiency. The analysis of the structure of funding sources of capital and current investment in environmental activities has been made. The obtained results have confirmed the conclusions made about national policy ineffectiveness referring to capital investment in environmental protection activities.

2.4 Economic and mathematical modelling of ecological expenditure for sustainable development of united territorial communities

The subchapter has been proved that the current system of local finances` management does not support their sustainable development and does not introduce viable mechanisms for the effective expenditure ecological distribution inside united territorial communities (UTCs) between the settlements within them.

The following arguments have been shown: UTCs include settlements characterized by different population size and income per capita.

Due to migration processes the population size is unstable. To overcome current disparities in expenditures` distribution of a UTC budget, it has been proposed to use economic and mathematical model, which allows to take into account both the dynamics of UTC per capita income and changes in its population.

It has been proposed to calculate the population size at certain time intervals, taking into account the expected (planned) changes of each settlement` s population within UTCs and to distribute programmed ecological expenditure within territories` sustainable development capacity.

Current decentralization and reform of the administrative and territorial structure of Ukraine, the issue of overcoming inequalities and disparities of territorial development in Ukraine by building effective system of financial management for united territorial communities as basis for territories` and country`s sustainable development becomes especially relevant. Therefore, there is a need for a balanced environmental policy in a decentralized environment.

The main task of the study is to simulate the system for united territorial communities (UTCs) expenditure ecological distribution management among settlements that comprise them, in order to create the mechanism that will ensure social justice and harmonious development.

Our analysis prove that the current mechanism of redistributive environmental taxation for the territories allows only certain united territorial communities (UTCs) to have the opportunity to receive their share of the tax (those having polluting companies registered on their territories).

Next problem – in Ukraine there is no effective guarantee of optimal, balanced distribution of resources among UTCs settlements - members in the legal framework for united territorial communities. This problem in ecological sphere arises first of all in UTCs, which consist of settlements with different population and different per capita income.

Thus there are some shortcoming.

The first corresponds to the fact that as single council of UTCs is formed by equal and direct elections, the number of representatives of each settlement depends on its population. That is, settlements as UTC centres may have decision-making advantages by the population size.

The second problem is that per capita income does not always depend on population size for settlements; more priority is given to natural resources abundancy, valuable land or big profitable business. It can also lead to uneven use of funds, in particular for the development of settlements.

The third problem is that fair, balanced expenditures and subsidies distribution is possible if there is reliable information on the number of UTCs residents. The information is practically absent, since UTC formation is preceded by the elections announced by the Central Election Commission (CEC), based on information about residents official registration in settlements. Practice shows that the place of registration does not always coincide with the place of residence. In addition, there is a pendulum labour migration in the process of job search between settlements. That is, there are those who work in another locality leaving every morning and returning in the evening. There are those who work during a week, there are also seasonal jobs. Thus, the number of settlements` residents fluctuates. In this case, the traditional approach is that the distribution of necessary expenditures between settlements will not be effective, since it is based on the place of registration without taking into account temporary migration. In turn, settlements accumulating additional workforce every morning must be provided with appropriate infrastructure (transport, catering, etc.). Transport infrastructure between settlements of such communities also needs additional attention.

We believe that there is another problem with the inefficiency of allocating both of UTCs budget expenditures to individual settlements and consideration of external migration, above all - labour migration outside the country. In regions with significant external labour migration (emigration), the actual population size does not always coincide with that taken into account by CEC when setting a UTC. It is also not possible to take into account the current system of expenditures and other financial resources (subsidies) distribution from the State budget.

Therefore, there is a need to build a mechanism for fair distribution of UTCs budget.

That is why, it is advisable to use differential approach for the formation of budget expenditures, taking into account their peculiarities based on social justice principles for UTCs` population.

We consider that it is possible to take into account the mentioned shortcomings in distribution of resources between UTCs settlements applying equation of the economic growth model by R. Solow.It looks as follows:

$$\dot{k} = lf(k) - (\alpha + \beta)k \tag{2.21}$$

Let us find the solution of R. Solow model equation for Cobb-Douglas macroeconomic production function: $F(K, L) = K^a L^{1-a}, 0 < a < 1$

Let *Y* = *F(K,L)* be total UTC income, that is own income, infrastructure subsidy and basic / reverse subsidy.

F – homogeneous first-order production function described by equation: *F(tK, tL) = tF (K, L)*, where *K* – UTC income; *L* – UTC population.

Let us introduce index *k* = *K/L*, which is equal to UTC own income to UTC total population ratio, so we have index of own income per capita for UTC.

Then capital productivity is:

$$f(k) = \frac{F(K,L)}{L} = F(k,l) \cdot \tag{2.22}$$

We assume that we have a natural increase in UTC population over period of time:

$$\dot{L} = \alpha L, \tag{2.23}$$

where α - coefficient (growth rate)of UTC population.

UTC investments (capital expenditures) are used to increase own resources (income) and depreciation of fixed capital, i.e $I = \dot{K} + \beta K$,

where β - depreciation rate (share of capital expenditures).

Then if *l* – rate of investment, then

$$I = lY = \dot{K} + \beta K \text{ or } \dot{K} = lF(K,L) - \beta K \qquad (2.24)$$

According to the own income per capita definition *k*, we have *lnk* = *lnK* – *lnL*.

We differentiate this equation by t, and obtain: $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$.

We substitute into the last ratio equations (2.23) and (2.24) and obtain the equation of unknown function k having the form (2.22), where function *f(k)* is defined by formula (2.21). This first-order nonlinear differential equation to own income per capita has simple economic interpretation: net own income increment is the difference between gross own income and steady-state own income.

Equations of R. Solow model for Cobb-Douglas production function take into account $F(K,L) = \frac{K^a L^{1-a}}{L} = (\frac{K}{L})^a = k$, are as follows:

$$\dot{k} = l\,k^a - (\alpha + \beta)k, 0 < a < 1. \qquad (2.25)$$

We integrate the Bernoulli equation by the substitution method.

Let *k* = *uv*. Then $\dot{k} = \dot{u}\,v + \dot{v}\,u$ equation (2.25) we set as:

$$\dot{u}\,v + \dot{v}\,u = l\,(uv)^a - (\alpha + \beta)uv \text{ or } \dot{u}\,v + u(\dot{v} + (\alpha + \beta)v) = l\,u^a\,v^a. \qquad (2.26)$$

Taking into account that one of the unknown functions, such as *v*, can be arbitrarily chosen (because only derivative *uv* must meet original equation), we take any partial equation solutions for *v*

$$\dot{v} + (\alpha + \beta)v) = 0,$$

which turns to zero coefficient of u in equation (2.26).

Obtain: $\frac{dv}{dt} = -(\alpha + \beta)v$.

After integration, we get: $\ln|v| = -(\alpha + \beta)v$ or $v = e^{-(\alpha+\beta)t}$ (we do not introduce continuous integration because only a partial solution of the auxiliary equation is required). To calculate u we have equations $\dot{u}\,v = l\,u^a\,v^a$ or $\dot{u}\,e^{-(\alpha+\beta)t} = l\,u^a\,e^{-a(\alpha+\beta)t}$.

We divide variables and obtain: $\frac{du}{u^a} = l\,e^{(\alpha+\beta)(1-a)t}\,dt$, then

$$\frac{u^{-a+1}}{1-a} = l\frac{1}{(1+a)(\alpha+\beta)}e^{(\alpha+\beta)(1-a)t} + \frac{C}{1-a} \text{ or } u = (\frac{l}{(\alpha+\beta)}e^{(\alpha+\beta)(1-a)t} + C)^{\frac{1}{1-a}}.$$

Then $k = (\frac{l}{(\alpha+\beta)}e^{(\alpha+\beta)(1-a)t} + C)^{\frac{1}{1-a}} e^{-(\alpha+\beta)t} = = (\frac{l}{(\alpha+\beta)} + C\,e^{-(1-a)(\alpha+\beta)t})^{\frac{1}{1-a}}.$

When t→∝, own income per capita is: $k \to (\frac{l}{(\alpha+\beta)})^{(1-a)^{-1}}$.

Thus, we have proposed the model that simultaneously take into account the capital expenditures growth rate and changes in population, if UTC per capita income and gross income rise. If necessary, it is advisable to build this function for average UTC indices, and then to determine the expenditures for each UTC`s settlement taking into account its change in population in case its per capita income meets UTC average.

Similar calculations can be further made within the region to assess the level of financial autonomy of both UTC and region in general. Comparison of regional indicators (in terms of districts) will allow to estimate the level of sustainable development of a territory (region). It will also make it possible to compare these parameters between different regions.

To plan UTCs and their settlements development (provision of public goods, capital expenditure, level of income, provision of resources and goods, etc.), it is necessary to take into account the population size and potential changes over a period of time.

To simulate the described situation we apply the system of linear homogeneous difference equations. Let us assume that UTC comprises $n \geq 2$ settlements $D_1, D_2, \ldots\ldots, D_n$ and there is the following migration between them: for all $i \neq j$ is the same part a_{ij} of residents of a settlement D_j goes to settlement D_i, and part a_{ji} of residents of a settlement D_j migrates to D_i, but part a_{ji} stays in it. Let $x_i(t)$ be residents of settlement D_i in t – period. Then,

$$x_1(t+1) = a_{i1}x_1(t) + a_{i2}x_2(t) + \ldots. + a_{in}x_n(t),$$

since for vector $x(t) = (x_1(t); x_2(t); \ldots.; x_n(t))$ we obtain the system of discretized equations:

$$x(t+1)=Ax(t) \qquad (2.27)$$

is an integral matrix A which elements obey these conditions:

$$0 \le a_{ij} \le 1,\ a_{1j}+a_{2j}+.....+a_{nj}=1,\ \ j=\overline{1,n}.$$

Let us study n equation solutions (2.27) $x^1(t), x^2(t),...., x^n(t),$ determined by the next initial conditions:

$$\begin{aligned} x^1(t_0) &= x_0^1 = (x_{11}^0; x_{21}^0;...; x_{n1}^0), \\ x^2(t_0) &= x_0^2 = (x_{12}^0; x_{22}^0;...; x_{n2}^0), \\ x^n(t_0) &= x_0^n = (x_{1n}^0; x_{2n}^0;...; x_{nn}^0). \end{aligned} \qquad (2.28)$$

The sum of solutions $x^1(t), x^2(t),...., x^n(t)$ of equation (2.21), which obey conditions (2.28), are called fundamental system of solutions if the determinant does not equal zero:

$$|X(t_0)| = \begin{vmatrix} x_{11}^0 & x_{12}^0 & \cdots & x_{1n}^0 \\ x_{21}^0 & x_{22}^0 & \cdots & x_{2n}^0 \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1}^0 & x_{n2}^0 & \cdots & x_{nn}^0 \end{vmatrix} \ne 0.$$

If $x^1(t), x^2(t),...., x^n(t)$ is a fundamental system of solutions of equation (1), then any solution $\overline{x}(t)$ of rhis equation can be presented as:

$$\overline{x}(t) = C_1 x^1(t) + C_2 x^2(t) + ... + C_n x^n(t),$$

where $C_1, C_2,...., C_n$ – constants.

Let us apply the system of linear homogeneous difference equations with fixed factors:

$$\begin{cases} x_1(t+1) = a_{11}x_1(t) + a_{12}x_2(t) + ... + a_{1n}x_n(t), \\ x_2(t+1) = a_{21}x_1(t) + a_{22}x_2(t) + ... + a_{2n}x_n(t), \\ .... \\ x_n(t+1) = a_{n1}x_1(t) + a_{n2}x_2(t) + ... + a_{nn}x_n(t), \end{cases} \qquad (2.29)$$

where $a_{ji}, i, j = 1, n,$ – real constants.

System solution (2.29) will be obtained in the form:

$$x_1 = \gamma_1 \lambda^t, x_2 = \gamma_2 \lambda^t, \ldots.., x_n = \gamma_n \lambda^t, \lambda \neq 0, \tag{2.30}$$

where $\gamma_1, \gamma_2, \ldots, \gamma_n$ and γ – numbers, which have to be determined.

Complementary system solution is:

$$x_1 = C_1(\beta_1\beta_2 + \beta_1^2 + \beta_1\beta_3) + C_2(\alpha_1 + \alpha_2)(\beta_3 + \beta_2 - \alpha_1 - \alpha_2)(1 - \alpha_1 - \alpha_2 - \beta_1)^t,$$

$$x_2 = C_1(\alpha_2\beta_3 + \alpha_1\beta_3 + \alpha_1\beta_1) + C_2(\alpha_1^2 - \alpha_1\beta_3 + \alpha_1\alpha_2 - \alpha_2\beta_3)^t +$$
$$+ C_3(\alpha_2\beta_3 - \alpha_1\beta_2)(1 - \beta_1 - \beta_2 - \beta_3)^t.$$

$$x_3 = C_1(\alpha_2\beta_1 + \alpha_2\beta_2 + \alpha_1\beta_2) + C_2(\alpha_1\alpha_2 - \alpha_1\beta_2 + \alpha_2^2 - \beta_2\alpha_2)(1 - \alpha_1 - \alpha_2 - \beta_1)^t +$$
$$+ C_3(\alpha_1\beta_2 - \beta_3\alpha_2)(1 - \beta_1 - \beta_2 - \beta_3)^t$$

Therefore, based on these equations, it is possible to calculate population size at certain time intervals (the beginning $t = 0$).

Taking into account the expected (planned) changes in each settlement`s population size in UTC territory, it is possible to distribute planned expenditure for providing public goods.

This allows to form the financial revenue distribution models that make up UTCs budgets and to take into account in territorial development strategies and projects such components as: budget revenue dynamics, territorial peculiarities and development determinants, employment rates and migration capacity.

Distribution issues have been identified, namely: UTCs include settlements with different population size, different per capita income; there is volatility of population caused by migration processes of different duration. Therefore, the need to build mechanism for efficient UTCs budget distribution based on the real needs and capabilities of communities has been proved. Foreign experience does not have effective mechanisms for budgets expenditures distribution, especially when we take into account Ukrainian specifics. Different UTCs can be dramatically different in terms of their capability and potential, even in one administrative region. Therefore, distribution of communities` financial resources should be as efficient as possible and independent of shortsighted decision-making.

To overcome existing ecological disparities in UTCs budget expenditures distribution, it has been proposed to apply economic and mathematical model based on the first-order differential equation, which allows to take into account simultaneously the dynamics of UTC per capita income and its population change.

2.5 Environmental investments for waste management to identify the EKC "turning point"

The study objective was to determine sustainable development conditions according to the criteria of emissions of harmful substances and waste generation when modelling impact factors of the parameters and general environmental situation in Ukraine.

It has been proved that the EKC model should be used not only to model parameters of emissions of harmful substances, but also for waste generation. Besides, it has been proved that it is necessary to take into account not only national level indicators, but also the contribution of the leading sectors driving national economy. Modelling has been carried out for the following industries: processing; mining and quarry development; agriculture, forestry and fisheries; supply of electricity, gas, steam and conditioned air; transport, warehousing, post and courier services. The models are based on correlation between GDP, average nominal income per capita, environmental costs, waste generation and emissions of harmful substances at the national level and by its leading industries. It has been determined that reaching the "turning point" on sectoral EKCs correlates waste generation and emissions with industry`s rate of remuneration, value added (sectoral GDP) and sectoral investment in environmental protection in the context of industry`s specifics.

Taking into consideration that higher waste generation poses threat to the country`s sustainable growth, this problem needs special attention. Waste is becoming perhaps the most acute environmental problem of the humanity as a whole and Ukrainian society in particular. Significant resource consumption, energy and raw material specialization of Ukraine's economy together with outdated technological base cause serious annual quantities of waste generation and accumulation. Man-caused load on the environment in Ukraine is 4 – 5 times higher than similar indicators of the developed countries. The difference between Ukrainian and developed countries` case of waste lies in the larger quantities of waste generation and lack of waste management infrastructure, which is the integral part of these economies.

Most scholars believe that correlation between income (economic growth) and pollution is non-linear and has the form of the inverted parabolic curve. Simon S. Kuznets is the author of the model.

The environmental Kuznets curve (EKC) is usually built to illustrate correlation between income per capita (GDP per capita) and pollutant emissions (or by their specific types). In our opinion, there is similar relationship between waste generation and household income or GDP, because as in the case of pollutants, if incomes surge, one can expect not only emission contraction but also waste generation shortening.

Fig. 2.7 – 2.9 illustrate the dynamics of correlation between average nominal income per employee in Ukraine, GDP, environmental expenditures and pollutant emissions, and waste generation during 2010 – 2017, respectively.

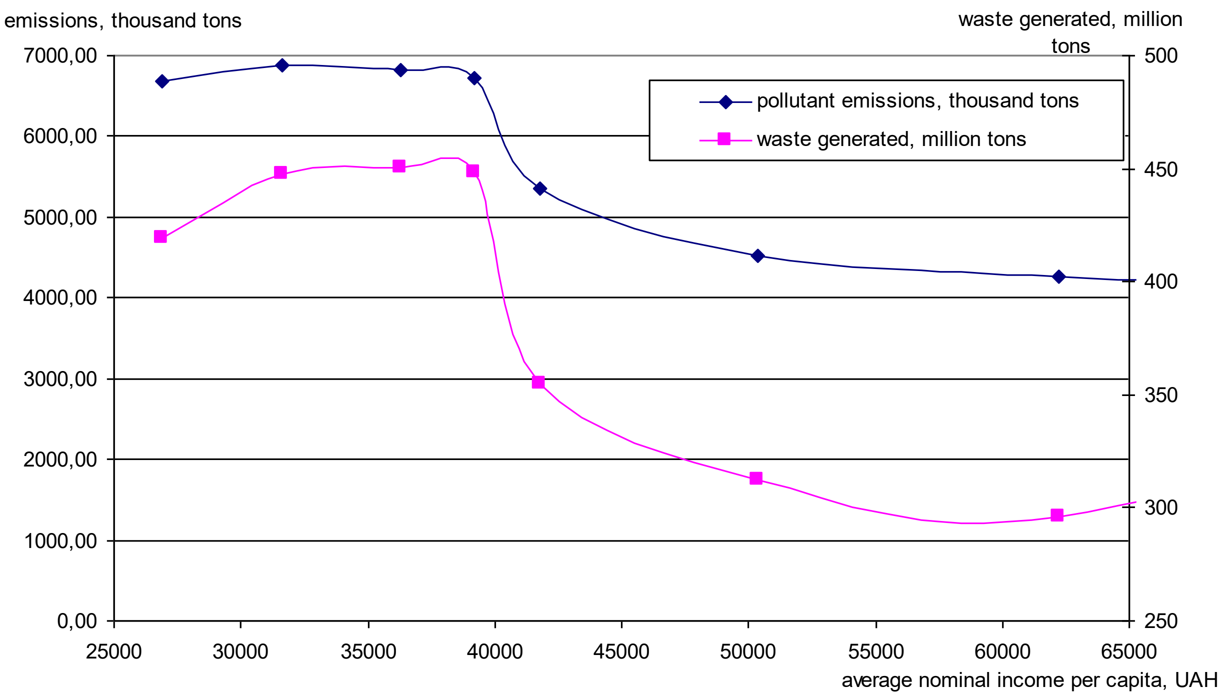

Fig. 2.7 Dynamics of correlation between average nominal income per capita in Ukraine, masses of emissions and waste generation during 2010 – 2017

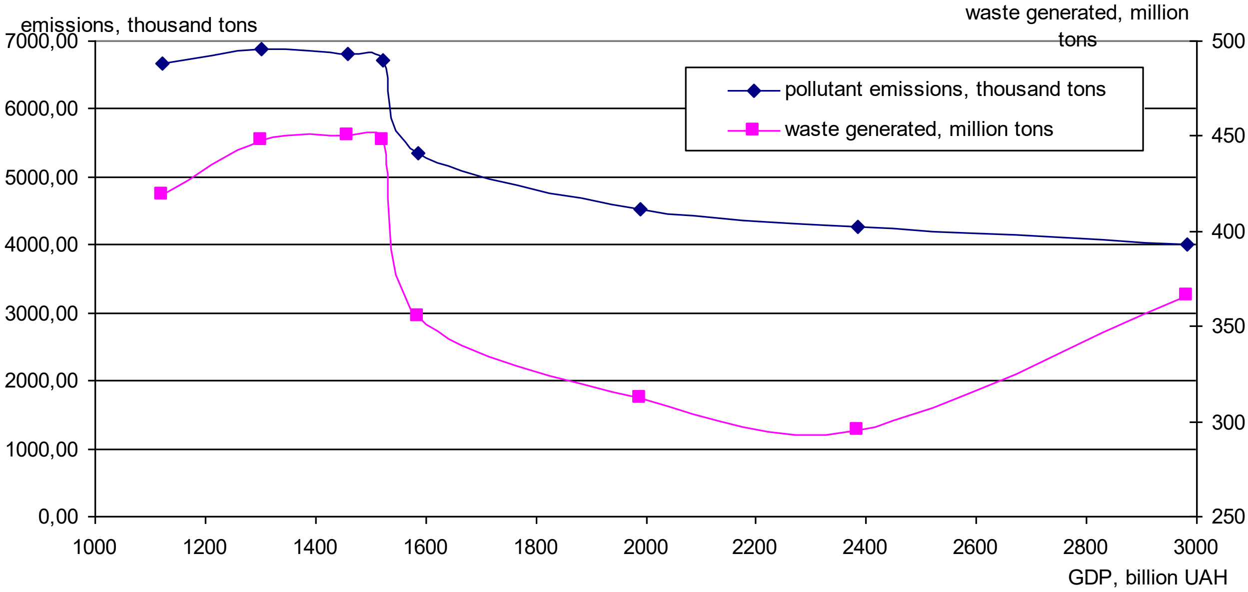

Fig. 2.8 Dynamics of correlation between GDP in Ukraine, masses of emissions and waste generation during 2010 – 2017

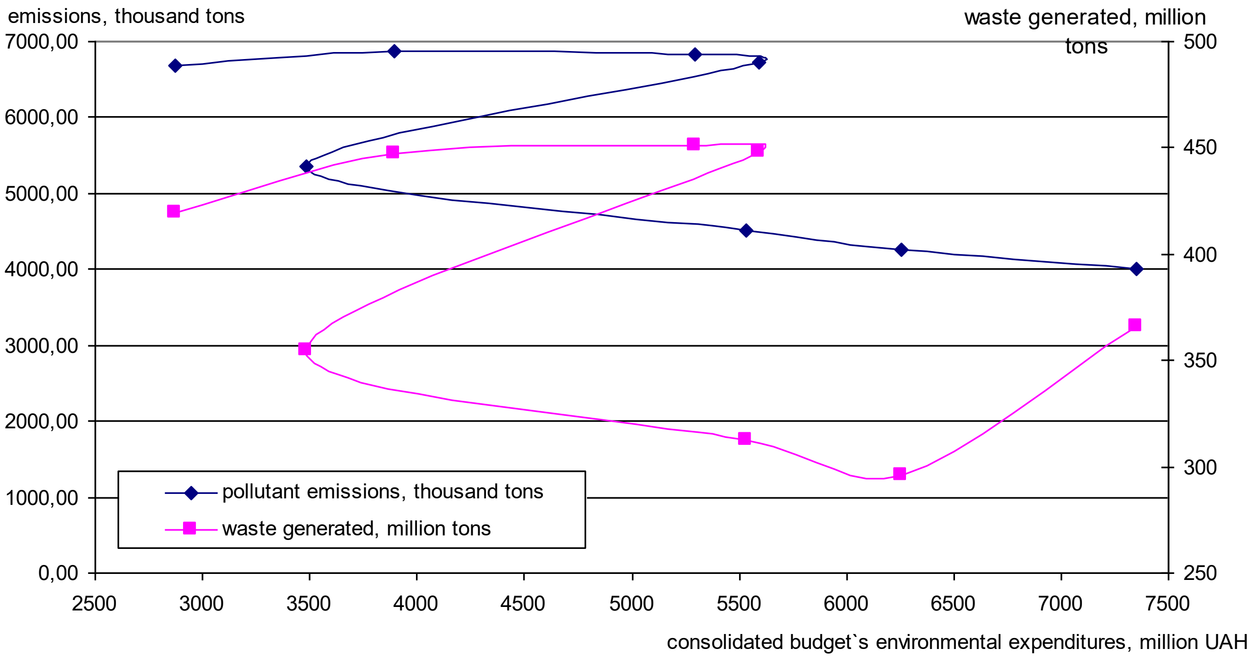

Fig. 2.9 Dynamics of correlation between consolidated budget`s environmental expenditures in Ukraine, masses of emissions and waste generation during 2010 – 2017

As one can see, the maximum growth of emissions and waste generation coincide and correspond to the level of nominal income of UAH 39180 (average monthly nominal income per employee UAH 3265), GDP of 1,522.7 billion UAH in 2013. After that, emissions and waste generation began to decline. Comparison with the EKC allows us to conclude that the average nominal income per employee and GDP is the "turning point" that ensures environmental change in the country in terms of emissions and waste. Regarding the corresponding correlation for consolidated budget`s environmental expenditures, only in 2016 – 2017 there were low quantities of emissions and waste generation alongside increasing expenditures.

Taking into account that these dependencies for Ukraine have been formed by the leading sectors of its national economy, the "turning points" analysis in the following sectors have been proved: mining and quarry development; processing; electricity, gas, steam and conditioned air supply; transport, warehousing, post and courier service; agriculture, forestry and fisheries.

Fig. 2.10 – 2.11 demonstrate interdependencies between pollutant emissions, waste generation and average monthly nominal income per capita in the mining, sectoral GDP and environmental expenditures.

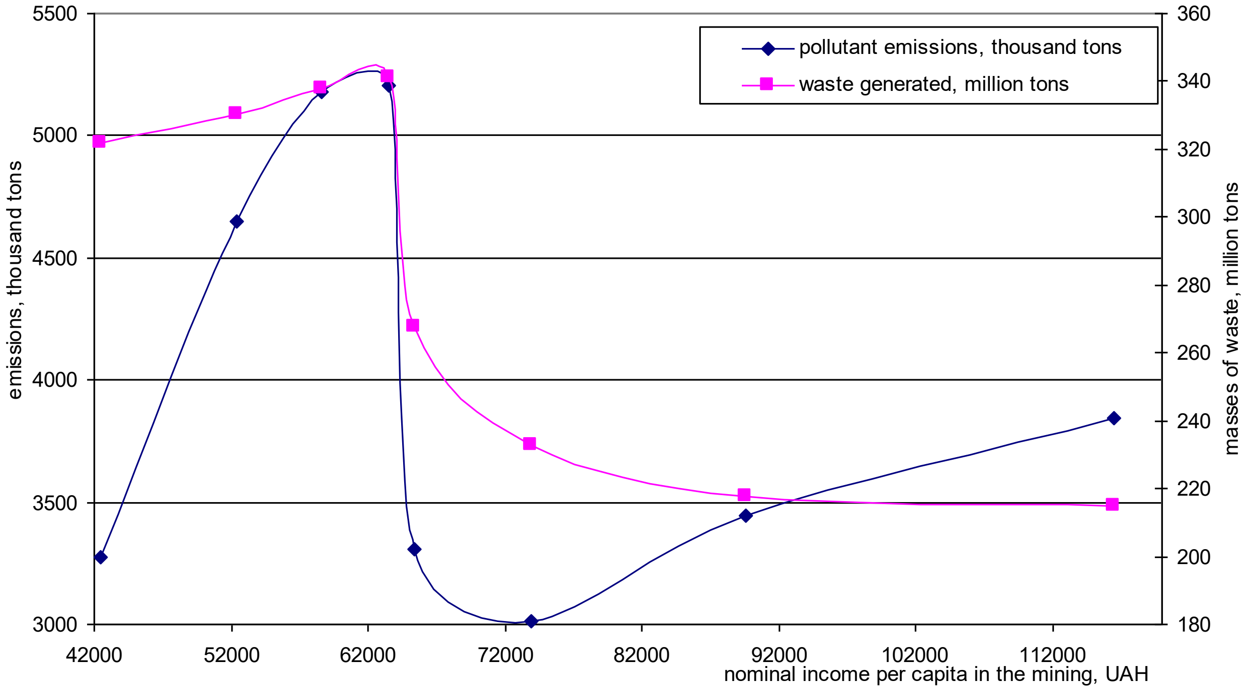

Fig. 2.10 Dynamics of correlation between average nominal income in the mining in Ukraine, masses of emissions and waste generation during 2010 – 2017

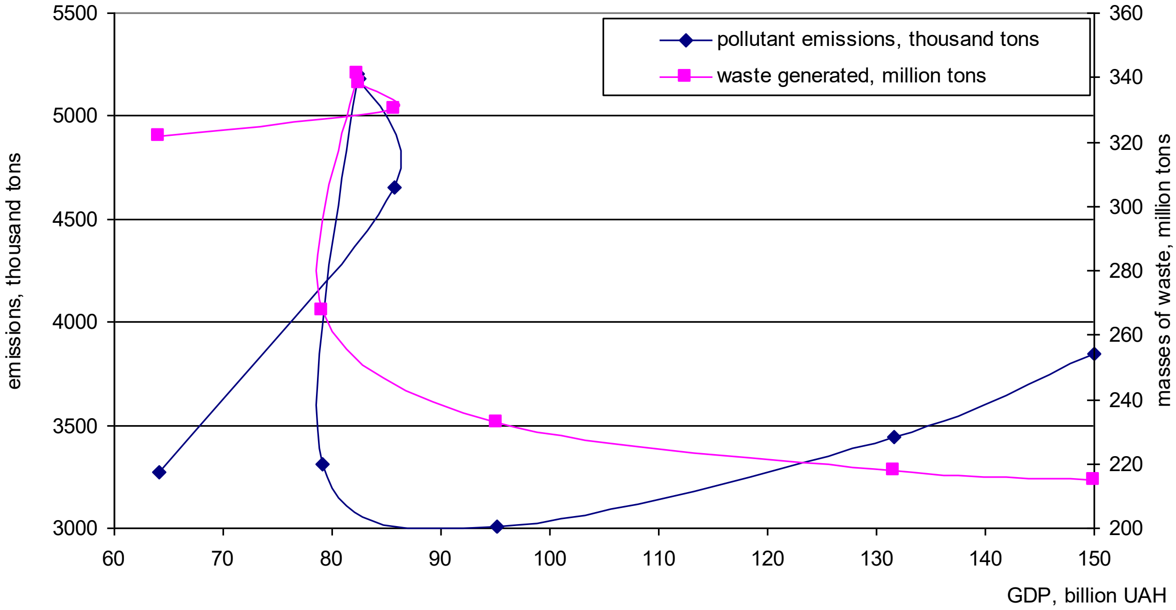

Fig. 2.11 Dynamics of correlation between GDP in the mining in Ukraine, masses of emissions and waste generation during 2010 – 2017

It has been found out that the maximums of emission growth and waste generation in the mining coincide and correspond to the level of nominal income of UAH 63468 (UAH 5289 / month) in 2013. After that emissions and waste generation began to decline.

GDP during 2012 – 2014 fluctuated between UAH 79,12 – 82,52 billion; shortening of pollutant emissions and waste generation began with GDP of UAH 82,52 billion. The following years there was growth in pollutant emissions alongside with waste generation contraction. This can be explained by the fact that the obtained level of GDP does not ensure the EKC consistent trend in terms of mining emissions. At the same time, one can associate sustainable reduction of waste generation with technical innovations that have led to lower inefficiency in the mining industry. The conclusion confirms the relationship between emissions and waste generation and sectoral environmental expenditures. The turning point was the expenditures at UAH 4394,67 million, further boost in environmental expenditures in the industry ensuring the contraction of waste generation. Nevertheless, it did not fully lead to the pollutant emissions shortening.

Similar dependencies were obtained for other leading sectors of the national economy. It has been found out for the processing industry that there are "turning points", but not as clear as in the previous case: the first point was reached with the average nominal income in the field at UAH 33252 (UAH 2771 / month) in 2011, the next – at UAH 39732 (UAH 3311 / month) in 2013. The income level in 2011 was sufficient to reach the "turning point" for waste generation.

GDP in the processing industry during 2012 – 2013 at the level of UAH 169,73 – 178,44 billion was sufficient to set the consistent trends on the EKC curve. In the same period (2013), sectoral environmental expenditures amounted to UAH 6402,47 million, their growth in the following years included positive trends, and the level of UAH 774670 million additionally stimulated contraction of both emissions and waste generation. This can be explained by the notable growth in environmental expenditures by the main pollutant – metallurgy during the period – from UAH 3575,00 million in 2015 to UAH 501,05 million in 2016.

The obtained correlations for the electricity, gas, steam and air conditioning supply in Ukraine of the average nominal income per capita and sectoral GDP can be considered as the EKC for the electricity, gas, steam and air conditioning supply. They have two "turning points" for pollutant emissions. Initially, the decline in emissions began at the average nominal income in the field at UAH 58620 (UAH 4885 / month) and sectoral GDP of UAH 44836 billion (2014). The next decline was observed at 83016 UAH (UAH 6,918 / month) and GDP of UAH 73809 billion in 2016. Thus, in 2016 the average nominal income and GDP in the electricity, gas, steam and air conditioning supply were sufficient to reach the firm "turning point". This result, in our opinion, was also achieved due to substantial growth of sectoral environmental expenditures. Thus, if in 2013 their value was UAH 1388,53 million, then in 2014 it surged by UAH 5312,20 million, in 2016 – by UAH 9605,10 million. Therefore, industry expenditure is considered to be additional positive determinant to reach the EKC "turning point".

The obtained EKC for transport, warehousing, postal and courier services has two "turning points": the first for the average nominal income in the field at UAH

32508 (UAH 2709 / month) and GDP at UAH 103,179 billion in 2011; the second for the average nominal income in the field at UAH 39180 and GDP at of UAH 104,483 million in 2013. Unfortunately, since 2016 there has been the EKC deviation on emissions due to the considerable environmental expenditure contraction in the field from UAH 1367,30 million in 2013 to UAH 576,30 million in 2017.

The EKCs for agriculture, forestry and fisheries have been obtained only since 2014 because of low income level in the industry. The "turning point" was reached for the average nominal income in the field at UAH 37680 (UAH 3140 / month). The EKC on pollutant emissions by GDP in agriculture was achieved earlier in 2012 at GDP of UAH 113,245 billion, on waste generation – in 2011 at GDP of UAH 109,961 billion. We believe that the results are caused by the growth of sectoral environmental expenditures in the period from UAH 96,167 million in 2010 to UAH 147,15 million in 2011; UAH 200,11 million in 2012 and UAH 544,17 million in 2013. Increasing pollution in the following period was due to the shortening of environmental expenditures to UAH 172,12 million in 2014 and UAH 192,38 million in 2015 despite the GDP growth.

It has been proved that the EKC model should be used not only for the pollutant emissions parameters, but also for the masses of waste generation. Moreover, it is necessary to take into account not only national indicators, but the contribution of the leading sectors of national economy. The EKC modelling for waste has been made for the following industries: processing; mining and quarrying; agriculture, forestry and fisheries; electricity, gas, steam and air conditioning supply; transport, warehousing, post office and courier service. The models are based on correlation between GDP, average nominal income per employee, environmental expenditures and waste generation at the national level and for the leading industries of national economy.

It has been found out that to reach the "turning point" in sectoral EKCs for waste generation, one has to consider the remuneration rate in the industry, the value added (sectoral GDP) and total sectoral environmental investment by the field specifics.

2.6 Innovations, environmental investments and incomes of the Environmental Kuznets curve: national economy of sustainable development

The study objective was to determine sustainable development conditions according to the criteria of emissions of harmful substances and waste generation when modelling impact factors of the parameters and general environmental situation in Ukraine. Besides, it has been proved that it is necessary to take into account not only national level indicators, but also the contribution of the leading sectors driving national economy. Modelling has been carried out for the following industries: processing; mining and quarry development; agriculture, forestry and fisheries; supply of electricity, gas, steam and conditioned air; transport, warehousing, post and courier services. The models are based on correlation between GDP, average nominal income per capita, environmental costs, waste generation and emissions of harmful substances at the national level and by its leading industries. It has been determined that reaching the "turning point" on sectoral EKCs correlates waste generation and emissions with industry`s rate of remuneration, value added (sectoral GDP) and sectoral investment in environmental protection in the context of industry`s specifics. It has been demonstrated that in Ukraine the "turning point" on the EKC has been provided by 20% of economically active population in industries that generate 46% of emissions amid country's average nominal income per employee and steady growth of environmental expenses for at least two years. It has been proved that the EKC for Ukraine should be analyzed by the sectors of national economy. To form effective national environmental policy, sectoral EKCs should be applied to determine emissions of harmful substances. It has been determined that the key factor to ensure country`s sustainable development is environmental investment both at the national level and by its driving economic sectors. Thus, the sectoral EKC reflects the progress towards industries` sustainable development that form main revenue receipts of the government and determine the rate of remuneration in the real sector. Modelling of the EKC parameters for emissions of harmful substances fully corresponds to the trends of sustainable economic growth and its transition to the innovative type of development.

Nowadays globalization causes on the one hand new development conditions, on the other hand – new threats. That is why the problem of parameters`, conditions` and mechanisms` formation of sustainable development in Ukraine in the context of deepening ecological crisis and taking into account national economy specifics needs to be solved. Accordingly, the degree of environmental risks and threats now is largely determined by the political efficiency in the field of both emissions contraction, waste generation and its management. This requires environmental policy adjustment, taking into account the need to develop and implement comprehensive strategies aimed at lower pollutant emissions while identifying impact factors of the level of environmentally friendly manufacturing. Therefore, the study objective is to determine sustainable development conditions by the criteria of pollutant emissions including impact factors modelling of the parameters` and environmental situation in Ukraine in general.

Most researchers believe that the correlation between income (economic growth) and environmental pollution is nonlinear and has the form of inverse parabolic curve. Simon S. Kuznets is the author of the – environmental Kuznets curve (EKC). Fig. 2.12 shows the dynamics of the relationship between the per capita income in Ukraine and sulfur dioxide, nitrogen dioxide, carbon oxide and dioxide emissions volumes (EKC) model.

We have proved that these dependencies for Ukraine have been formed by the leading branches of its national economy: mining and quarrying; processing industry; supply of electricity, gas, steam and air conditioning; transport, warehousing, postal and courier services; agriculture, forestry and fisheries.

Share and dynamics of pollutant emissions and carbon dioxide emissions in these industries during 2010 – 2017 are defined in Table 2.8. As one can see, the lowest degree of emissions is in agriculture, the highest – in energy. Metallurgy occupies significant share of processing industry emissions, so special attention will be paid to its analysis.

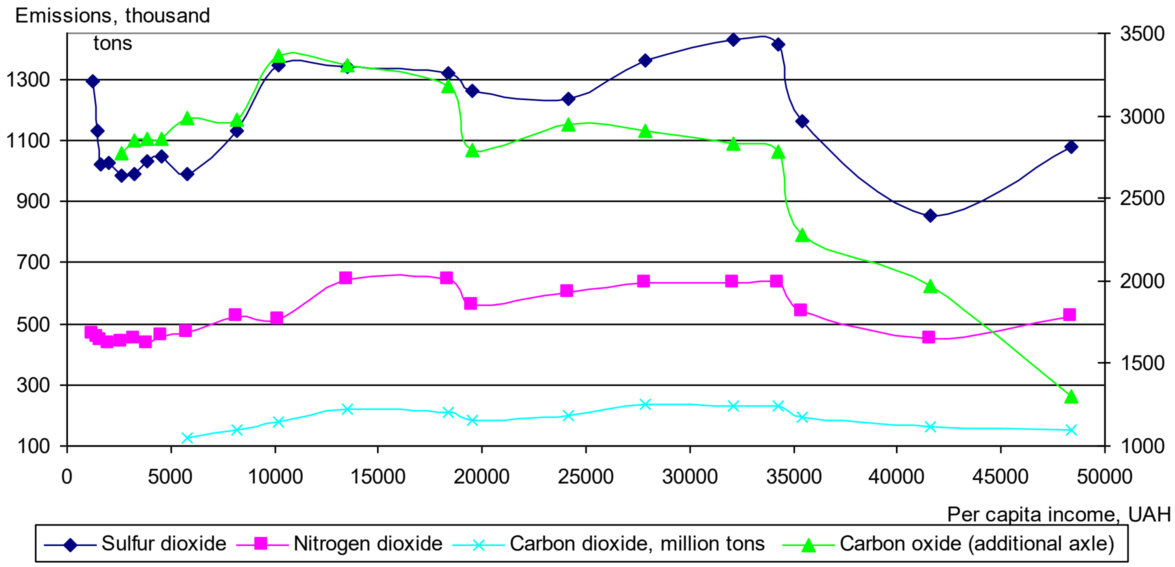

Fig. 2.12 Dynamics of the relationship between the per capita income in Ukraine and sulfur dioxide (from 1990), nitrogen dioxide (from 1990), carbon oxide (from 2000), carbon dioxide (from 2004) emissions volumes till 2017

Table 2.8 Shares of pollutant emissions, total (pollutant emissions / carbon dioxide emissions) of point sources by fields of economic activity in total emissions in Ukraine, %

Years	Agriculture, forestry and fisheries	Transport, warehousing, postal and courier services	Mining and quarrying	Processing industry	Supply of electricity, gas, steam and air conditioning
2010	0,47 (1,7/0,4)	3,42 (4,7/3,4)	1,93 (20,6/1,5)	35,75 (32,6/35,8)	57,56 (38,8/58,0)
2011	0,42 (1,7/0,4)	2,86 (4,5/2,8)	2,25 (19,6/1,9)	43,99 (31,7/44,3)	49,54 (41,3/49,7)
2012	0,98 (1,8/0,4)	2,00 (3,8/1,2)	2,56 (20,4/2,2)	41,07 (29,4/41,3)	53,15 (43,4/53,4)
2013	0,53 (2,1/0,5)	2,23 (3,9/2,2)	2,58 (21,4/2,2)	41,26 (28,7/41,5)	52,62 (42,8/52,8)
2014	0,55 (2,4/0,5)	2,15 (3,9/2,1)	2,12 (17,5/1,8)	39,11 (30,4/39,3)	54,62 (43,8/54,7)
2015	0,84 (2,7/0,8)	1,76 (2,7/1,7)	2,12 (17,2/1,8)	41,17 (32,9/41,3)	51,78 (41,1/52,0)
2016	0,62 (2,7/0,6)	2,44 (2,0/2,4)	2,24 (15,1/2,0)	40,39 (31,7/40,6)	52,38 (46,0/52,5)
2017	0,93 (3,1/0,9)	3,51 (2,4/3,6)	3,03 (18,5/2,7)	39,40 (33,8/39,5)	51,16 (39,1/51,4)

The ratio of the average monthly nominal wage in the industries with the corresponding mean values for Ukraine have been determined the same way (Table 2.9). Thus, the highest wages are in mining, the lowest – in agriculture.

In 2017 the employment rate in the analyzed industries was about 40% of all employed persons in Ukraine (Table 2.10). The largest rate was in agriculture, the smallest – in mining.

Table 2.9 Ratio of the average monthly nominal wage in the industries with the corresponding mean values for Ukraine, %

Years	Agriculture, forestry and fisheries	Mining and quarrying	Transport, warehousing, postal and courier services	Processing industry	Supply of electricity, gas, steam and air conditioning
2010	63,87	158,06	118,71	102,19	135,24
2011	70,38	165,97	102,89	105,24	127,35
2012	68,27	161,30	96,63	102,35	126,27
2013	69,59	161,99	109,92	101,41	137,83
2014	71,15	156,47	108,28	102,59	140,37
2015	74,85	146,94	110,92	106,72	130,20
2016	75,55	143,97	112,10	106,95	133,47
2017	81,10	136,60	108,22	102,74	119,55

Table 2.10 Employment rate in the field to all employed persons in Ukraine, %

Years	Agriculture, forestry and fisheries	Mining and quarrying	Transport, warehousing, postal and courier services	Processing industry	Supply of electricity, gas, steam and air conditioning
2010	15,26	2,21	5,97	9,47	2,81
2011	16,72	2,21	6,01	9,26	2,84
2012	17,18	2,16	5,94	11,41	2,87
2013	17,53	2,10	5,99	11,15	2,79
2014	17,10	1,99	6,16	11,19	2,86
2015	17,46	1,59	6,07	11,19	2,88
2016	17,61	1,47	6,13	11,01	2,85
2017	17,71	1,36	6,14	10,99	2,76

We have also compared the share of gross value added (GDP by sector) of the analyzed industries in 2017 with the total GDP of the country (Table 2.11).

The largest share of GDP was generated in the processing industry, the smallest – in the field of electricity, gas, steam and air conditioning supply. In 2017 agriculture`s, forestry and fisheries`, and the processing industry`s shares in GDP were equal.

Thus, agriculture having the lowest degree of emissions and waste generation, generates the same value added (GDP by sector) as the processing industry, which is the leader in pollution (metallurgy gives the biggest part of pollution). At the same time, the industry that accounts for the largest share of pollution – energy, generates the smallest share of GDP in the country.

Table 2.11 Share of gross value added by types of economic activity to GDP of Ukraine, %

Years	Agriculture, forestry and fisheries	Mining and quarrying	Transport, warehousing, postal and courier services	Processing industry	Supply of electricity, gas, steam and air conditioning
2010	7,4	5,7	7,8	13,0	2,8
2011	8,1	6,3	8,0	11,8	3,1
2012	7,9	5,7	7,1	12,2	3,1
2013	8,7	5,4	7,2	11,2	2,9
2014	10,2	5,0	6,4	12,2	2,8
2015	12,1	4,8	6,8	11,9	2,7
2016	11,7	5,5	6,6	12,2	3,1
2017	12,1	5,6	6,7	12,1	2,9

The obtained results reveal that the "turning point" for Ukraine on the EKC was reached in 2013 (income –UAH 34264, average nominal income per employee – UAH 39180). Industries that reached the final "turning point" in 2013 accumulated 46,07% of pollutant emissions and 20% of the employed population of Ukraine. Among 20% of employed persons, 2,1% worked in the mining characterized by 1,61 times higher wages than the average; 11,5% – in the processing industry having the average income; 5,9% worked in the transport industry with slightly higher income than the average in Ukraine.

However, energy and agriculture, reaching the turning point in 2014 – 2016, employed the same 20% and formed almost the same amount of 53% of pollutant emissions. Among 20% of employed persons, 2,8% worked in the energy sector characterized by 1,5 times higher wages than the average; 17,1% – in agriculture, having the average income.

Thus, comparison of the obtained results shows that 20% of the working population in Ukraine being employed by industries that generate 46% of pollutant emissions ensure the "turning point" on the EKC, if national average nominal income per worker and steady growth of environmental costs for at least two years are reached.

The analysis demonstrated that the industries like mining and quarrying, agriculture, fisheries and forestry have one "turning point", others – two. We believe that these sectoral features may be related to the environmental costs. Table 2.12 illustrates the results of pollutant emissions` and sectoral environmental costs` growth rates analysis. Their analysis allows us to conclude that in Ukraine, not only average nominal income per employee, but sectoral environmental costs matter.

Table 2.12 Dynamics of chain weighted growth rate of pollutant emissions and sectoral environmental costs

Years	Growth rates, %									
	Mining and quarrying		Processing industry (metallurgy)		Supply of electricity, gas, steam and air conditioning		Transport, warehousing, postal and courier services		Agriculture, forestry and fisheries	
	Emissions	Environmental costs	Emissions	Environmental costs	Emissions	Environmental costs	Emissions	Environmental costs	Emissions	Environmental costs
2011	*142,11*	*157,11*	***150,28 (161,68)***	***127,04 (126,61)***	105,10	97,95	***102,12***	***346,67***	*109,39*	*153,02*
2012	*111,34*	*106,65*	*91,52 (92,37)*	*106,65 (119,95)*	105,17	114,88	*68,58*	*122,96*	*230,93*	*135,99*
2013	***100,47***	***89,62***	***100,16 (102,19)***	***97,04 (106,85)***	98,72	38,48	***111,07***	***41,07***	*53,37*	*271,94*
2014	63,62	127,82	73,38 (70,33)	95,78 (102,92)	***80,36***	***382,58***	74,57	59,63	***80,17***	***31,63***
2015	90,94	103,73	95,47 (95,54)	100,39 (97,25)	*86,00*	*102,10*	74,37	90,64	139,30	111,77
2016	114,39	99,44	106,33 (109,93)	125,84 (140,29)	***109,62***	***177,10***	149,98	264,21	80,81	190,87
2017	111,64	114,93	80,50 (74,19)	103,33 (94,13)	80,60	64,01	118,99	70,31	122,91	116,83

As one can see, to achieve the "turning point" in the mining, it was necessary to increase environmental costs` growth rate during two years. Having high level of wages in the industry, the result was achieved in 2013. The processing industry has similar pattern: as the required income per employee was achieved in 2013, it has two "turning points". Transport, warehousing, postal and courier services industries had two "turning points" as well; the upsurge of environmental costs also lasted for two years. In addition, as one can see, after a two-year growth of environmental costs and "turning point" reached by all industries, pollutant emissions contracted. If environmental costs dropped off, emissions rose.

In the energy sector despite high average nominal income per employee (UAH 40236 in 2011), the dynamics of sectoral environmental costs constantly increased or decreased, then during 2014 – 2015 they were rising and therefore, in 2016 the "turning point" was reached.

Agriculture, forestry and fisheries had positive environmental costs growth; the "turning point" was reached only in 2014 due to low wages in the industry.

Parameters of national economy`s sustainable development by pollutant emissions have been modelled in the paper. It has been proposed to apply sectoral approach and the model of the environmental Kuznets curve (EKC). Modelling has been made for the following industries: processing; mining and quarrying; agriculture, forestry and fisheries; supply of electricity, gas, steam and air conditioning; transport, warehousing, post office and courier service.

It has been proved that the sectoral EKC reflects the progress towards industries` sustainable development that form the main budget incomes and determine wages in the real sector of the economy. The EKC parameters` modelling for waste and emissions fully corresponds to the trends of sustainable economic growth and its transition to the innovative development pattern.

The study of sectoral EKC revealed close correlation between environmental investment, investment activity and skilled labour force. Sectoral environmentally friendly investments can induce significant effect without drastic changes in the production structure. It has been confirmed that stable sectoral investments in environmental protection together with sufficient income of employees, form the conditions for national economy`s sustainable development. Environmental investments allow to modernize production, surge R&D intensity and profitability of the applied technologies. This will reduce emissions and increase wages. Indeed, higher R&D intensity of production will induce the need for highly qualified personnel with wages bigger than average. The second important outcome of environmental investments will be more qualitative and competitive products, their effective market promotion.

3 QUANTITATIVE MODELLING APPROACH TO PREDICT ABUNDANCE OF CO-INVADERS IN THE WILD ENVIRONMENT

3.1 Study area, fish sampling and parasite collection

Biological invasions are gaining increasing attention over the past decades as a major threat to biodiversity and an important element of global environmental changes. Invasive species may affect native populations and communities through different mechanisms, including competition, predation, altering habitat, changing disease dynamics, etc., while parasites play an important role in mediating such effects. However, despite considerable research effort, many aspects of the host-parasite relationships and parasite dynamics in invasive hosts remain poorly understood. Escape from the effects of parasites is a frequent explanation given for the success of introduced species that is also known as the enemy release hypothesis (ERH). This escape may occur through a reduction of the number of parasite species and a declining their infection parameters. The empirical evidence suggests that a loss of the parasite species richness and abundance, far from being mutually exclusive, are complementary.

Free-living species usually harbours a number of parasites, it is no surprise, therefore, that multiple co-introduced parasites commonly make the trip with their invasive host. The pacific so-iuy mullet, *Planiliza haematocheila* (Temminck & Schlegel), introduced in the Azov-Black Seas provides one example of parasite co-introduction pattern. Ectoparasitic species with direct life cycles were carried to the new distribution range of *P. haematocheila* and partially preserved. While our considerable effort has been devoted previously to comparative analysis of parasite species richness, abundance, prevalence and aggregation of helminth communities across the native and invasive distribution ranges of *P. haematocheila*, in the current study we focus on the abundance pattern of oioxenic monogenean parasite *Ligophorus llewellyni* Dmitrieva, Gerasev and Pron'kina. *L. llewellyni* is one out of six carried helminth species co-introduced with their host in the Azov-Black Seas region.

The variation in infection parameters of parasites among the host depends on factors acting at two levels: the host and the host's surrounding environment. The host level includes factors linked to the host age and size, the diet range, the behaviour, physiological, immunological conditions and genetic diversity. At the environment level, the host density, water temperature, seasons, months and geographic localities are the most commonly considered factors influencing the parasite infection. Deciphering the impact of the factors of both levels on parasite abundance of the wild populations is still a real challenge. Modelling the empirical data on the parasite abundance is hampered by a number of factors: excess zeros and overdispersion, non-linear relationships between independent and dependent variables, temporal or spatial correlations (pseudoreplications). Additionally, the special feature of the parasite populations is that they have an asymmetric distribution which tends to be skewed to the right. This skewness is associated with the nature of the distribution of parasites among hosts, which is known to be aggregated, with many hosts harbouring low numbers of parasites and a few fish individuals with numerous parasites. In most cases, the negative binomial distribution (NBD) proves to be a good model of the aggregated distribution of parasites. All these complexities could lead to inadequate assessment of links between the parasite abundance and host/environment-related parameters using routine methods of statistical analysis. More flexible models, such as generalized additive models (GAMs), provide a powerful tool to analyse the relative contribution of different factors in explaining the parasite population dynamics. The advantage of GAMs in respect to other models is that the shape of the response curves reflecting the relationships between dependent and continuous independent variables are data driven, instead of being predefined by parametric forms.

Here, we evaluate parasite release of invasive populations of *P. haematocheila* based on comparative analysis of the abundance of the co-introduced gill parasite, *L. llewellyni*, across localities in the Sea of Japan and the Azov Sea. The new environment of the Azov-Black Seas is favourable for the introduced host, *P. haematocheila*, whose growing rate, metric parameters and fecundity in the new region exceed those of the native range. We suppose that the effect of a favourable environment for the host may play a reverse role for its co-introduced parasite that results in decreasing the infection level. The geographic regions, host size, water temperature and months as predictor factors were evaluated to affect the abundance of the monogenean parasite using GAMs. We expect that the co-introduced parasite species will show the same tendency of dependence between the abundance and tested variables as in its native region. This research seeks to identify a key factor that may explain changes in the parasite abundance linked with the host translocation. Finally, to evaluate whether parasite release translates into a measurable advantage for the introduced host, we estimate how parasite load or infrapopulation size influences a host body condition.

This study is based on 759 individuals of *P. haematocheila* sampled from 9 marine or brackish localities in the Sea of Japan and the Azov Sea during 1998 – 2014 (Table 3.1, 3.2).

Fish were measured and surveyed for parasites within a day of their capture or after freezing. Gills were removed and the surface of each gill arch was individually examined, scraped and rinsed in water. Specimens of *Ligophorus* were collected from the surface or through rinsing of gills under a stereomicroscope. Following the preliminary identification by the stereomicroscope parasites were stored in 70% alcohol or fixed on slide in glycerin jelly for further processing. Taxonomic identification was based on morphology and was performed using key to species proposed by Sarabeev et al.

The size range for the total length of fish was 17,1 – 50 cm and 11,6 –71,5 cm in the native and invasive distribution range, respectively. In total, specimens of *L. llewellyni* from 535 fish and 6 localities were accounted for the Azov Sea (Table 3.1) and from 224 and 3, respectively, for the Sea of Japan (Table 3.2). Totally 27 samples from two host populations and across all localities, years and seasons were studied.

Data will be made available from the Mendeley Data (https://data.mendeley.com/) if published.

Table 3.1 Descriptive table of mean abundance of *Ligophorus llewellyni* from *Planiliza haematocheila* according to sampling sites, regions and dates with information on the host total length and sample size used in the study

Date	Sea	Locality	Host total length (mean (range), cm)	Sample size	Abundance		
					Mean	SD	95% CI
05-11/1998	Azov	Molochny Estuary	56.9 (28-71.5)	28	51.3	107.3	23.7-109.5
06-08/1999		Molochny Estuary	45.0 (24-67)	19	21.1	28.6	11.6-40.5
6 and 10/2001		Kerch Strait	32.1 (11.6-58)	13	0.9	1.8	0.2-2.5
05-06/2004		Kerch Strait	43.5 (30-64)	32	11.1	19.1	6.5-21.0
09/2004		Kerch Strait	42.5 (35-48.8)	30	4.5	7.8	2.6-9.0
06/2005		Kerch Strait	41.1 (31.1-44.5)	30	8.6	15.5	4.6-16.9
09/2005		Kerch Strait	39.9 (26.7-51)	38	1.9	2.8	1.1-2.9
05-07/2004		Obitochny Bay	41.2 (25-52.3)	49	5.9	15.0	3.4-14.9
10/2004		Taganrog Bay, Mariupol	40.3 (37.6-42.8)	28	2.9	5.2	1.6-6.0
09/2004		Sivash Lake	42 (32.5-52.2)	30	17.1	15.0	12.8-23.5
07/2005		Sivash Lake	30.9 (20-44.4)	62	3.2	4.4	2.3-4.8
10/2005		Sivash Lake	33.9 (23-41)	42	6.4	12.1	3.9-12.0
11-12/2007		Utluksky Estuary	36.1 (27.8-63)	31	14.5	41.6	4.0-38.3
10-12/2008		Utluksky Estuary	36.9 (29.7-55)	10	52.5	70.6	18.7-108.3
03-05/2009		Utluksky Estuary	34.2 (32-37.5)	15	5.5	7.7	2.8-11.9
06-07/2009		Utluksky Estuary	39.8 (29-68)	41	2.6	4.6	1.5-4.7
10-11/2011		Utluksky Estuary	40.2 (35-49.5)	14	11.6	21.0	4.6-33.6
05/2013		Sivash Lake	35.8 (20-42)	12	1.5	1.9	0.5-2.6
05/2014		Sivash Lake	37.6 (13-44)	11	8.5	8.7	4.6-15.9
All samples in the Azov Sea			39.0 (11.6-71.5)	535	10.2	32.4	8-13.9

Table 3.2 Descriptive table of mean abundance of *Ligophorus llewellyni* from *Planiliza haematocheila* according to sampling sites, regions and dates with information on the host total length and sample size used in the study

Date	Sea	Locality	Host total length (mean (range), cm)	Sample size	Abundance		
					Mean	SD	95% CI
06/2004	Japan	Razdol'naya Delta	34.9 (25-47)	30	31.7	62.3	16.4-71.5
10/2004		Razdol'naya Delta	35.7 (28.5-45.5)	30	57.3	117.5	31.2-135.2
05-06/2005		Razdol'naya Delta	34 (29-41)	30	47.6	103.5	25.1-118.4
10/2005		Razdol'naya Delta	40.2 (36-43)	30	50.4	66.5	32.9-88.6
05-06/2004		Posiet Bay	29.2 (17.1-50)	15	4.5	6.7	1.7-9.1
11/2004		Posiet Bay	34.6 (30-46)	29	53.2	76.3	30.9-90.5
10-11/2005		Posiet Bay	38.1 (34.5-43)	30	50.2	51.9	35.7-73.8
04-06/2005		Artemovka Delta	32.9 (28-41)	30	29	53.3	16.5-68.2
All samples in the Sea of Japan			35.3 (17.1-50)	224	42.8	77.0	34.5-56.5

3.2 Data analysis

Abundance was defined as the number of parasites in a host regardless of whether or not the fish is infected. Values of the mean abundance by samples across localities months and regions were complemented by bootstrapped confidence interval (CI) using the adjusted percentile method (BCa) as followed the recommendations by Rózsa et al. and Shvydka et al.

In order to analyse the dynamics of parasite abundance in the host, as well as the factors affecting it, we considered the infrapopulation size of *L. llewellyni* as the dependent variable. The selection of predictors was based on acknowledged biological, environmental, geographical and time factors potentially influencing parasite abundance. The host characteristics at the individual level (total body length, weight and fish body condition), the environmental abiotic factors (temperature and salinity of the sea), the geographical areas (sampling regions and localities) and the time (months, seasons and years) were used here as explanatory covariates. Values of the average monthly water temperature and salinity per locality were drawn from open sources (http://www.esimo.ru/atlas/Jap). The relative condition factor was counted, which is the ratio between an observed weight of fish and that predicted by the weight-versus-length regression across all fish specimens in the region (Le Cren, 1951).

Before statistical modelling a data exploration was performed following the protocol described by Ieno and Zuur. The dataset were inspected for identifying outliers in the data, homogeneity and zero inflation in the dependent variable, collinearity between explanatory covariates and the nature of relationships between response and independent variables. To get an initial impression of the data, a Cleveland dotplot of the abundance (Fig. 3.1) and pairplot (Fig. 3.2, 3.3) was produced.

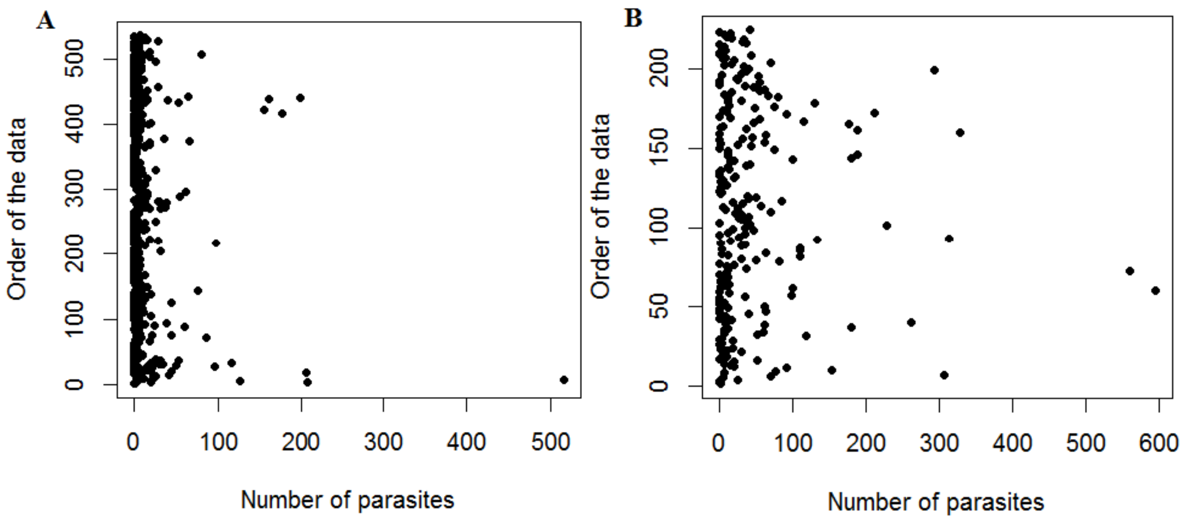

Fig. 3.1 Cleveland dotplot of parasite abundance *L. llewellyni* in the Sea of Azov (A) and Sea of Japan (B)

R code to make Fig. 3.1 follows:

```
>plot(x= myfishA$P1,
+y=1:nrow(myfishA), pch = 16, cex=0.8,
+xlab = "Number of parasites", ylab = "Order of the data")
```

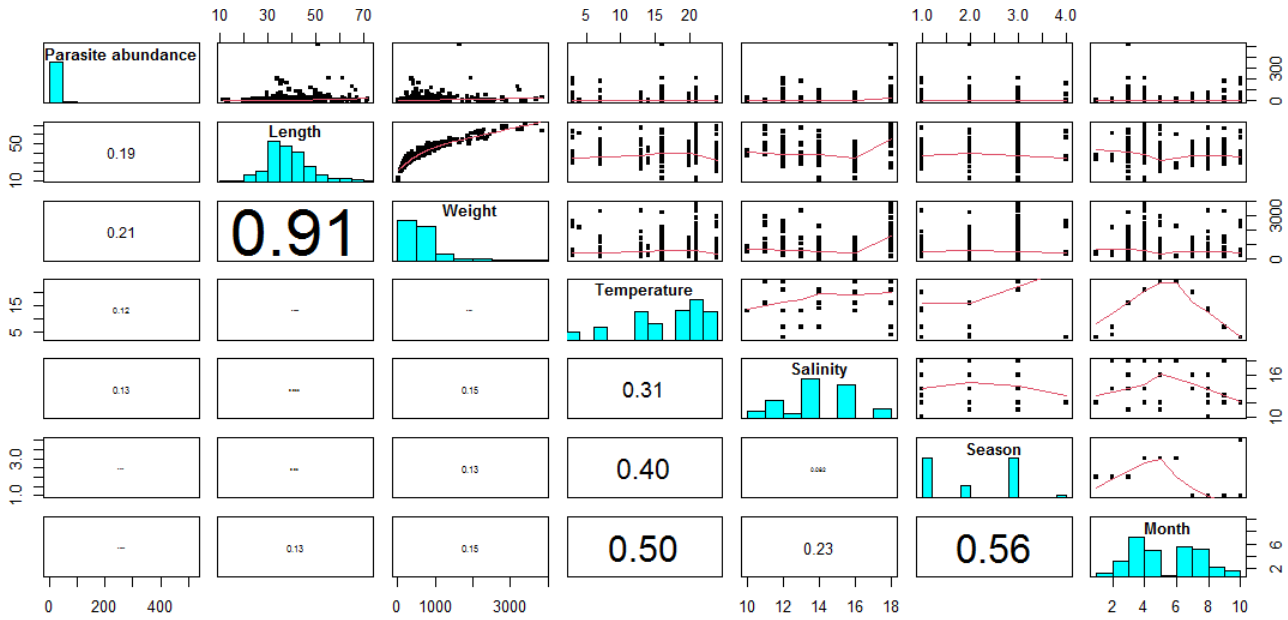

Fig. 3.2 Pairplot of parasite abundance and all explanatory variables for data from Sea of Azov

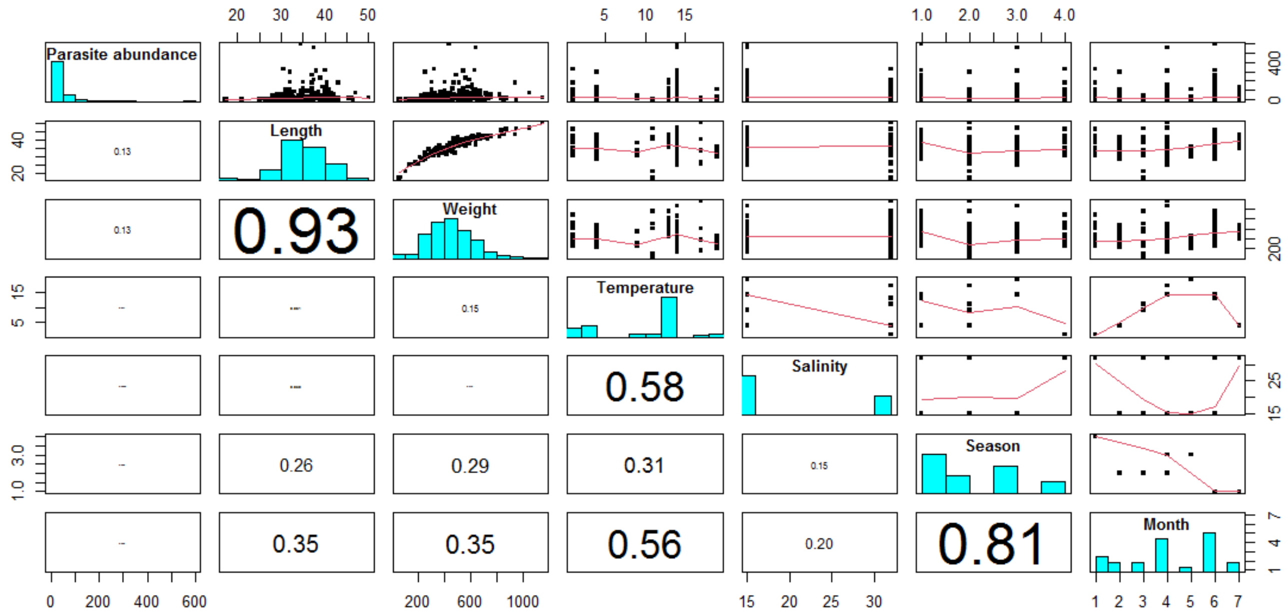

Fig. 3.3 Pairplot of parasite abundance and all explanatory variables for data from Sea of Japan

The lower diagonal part of the pairplot shows the (absolute) correlation coefficient in which the font size is proportional to the value of the correlation. The upper diagonal shows the scatterplots with smoothing curves. The diagonal part shows the bar graph of the data.

R code to make Fig. 3.2 follows:

```
>pr1<-cbind(myfishA$P1,myfishA$TL,myfishA$W,myfishA$Temperature,
+myfishA$Salinity,myfishA$season,myfishA$Month)
>pr1<-as.data.frame(pr1)
>names(pr1)<-c("Parasite abundance","Length","Weight",
+"Temperature","Salinity","Season","Month")
>pairs(pr1)
>panel.hist <- function(x, ...)
+{
  +usr <- par("usr"); on.exit(par(usr))
   +par(usr = c(usr[1:2], 0, 1.5) )
   +h <- hist(x, plot = FALSE)
   +breaks <- h$breaks; nB <- length(breaks)
   +y <- h$counts; y <- y/max(y)
   +rect(breaks[-nB], 0, breaks[-1], y, col = "cyan", ...)
+}
>panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...)
+{
    +usr <- par("usr"); on.exit(par(usr))
    +par(usr = c(0, 1, 0, 1))
    +r <- abs(cor(x, y))
    +txt <- format(c(r, 0.123456789), digits = digits)[1]
    +txt <- paste0(prefix, txt)
    +if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)
    +text(0.5, 0.5, txt, cex = cex.cor * r)
```

```
+}
>pairs(pr1, upper.panel = panel.smooth,lower.panel= panel.cor,
    +cex = 4.5, pch =".", bg = "light blue",
    +diag.panel = panel.hist, cex.labels =1, font.labels = 2)
```

For correct model processing, the parasite abundance data as the response variable was tested to fit the NBD using Pearson’s chi-square statistic. This test has been done by applying Quantitative Parasitology 3.0 software. Since the original count data does not fit the NBD ($p<0,001$), it was square root transformed and rounded to the nearest integer. The newly obtained data shows a good fit to the NBD ($p=0,057$) and was used for further analysis. Fig. 3.4 shows the parasites–total length profiles per month.

R code to make Fig.3.4 follows:

```
>library(lattice)
>myfishA$Month<-as.factor(myfishA$Month)
>myfishA$year<-as.factor(myfishA$year)
>myfishA$Locality<-as.factor(myfishA$Locality)
>MyLines <- function(xi, yi, ...){
+I <- order(xi)
+panel.lines(xi[I], yi[I], col = 1)}
>xyplot(P11~TL | Month, data = myfishA,
+groups = Locality, xlab = "Total length (cm)",
+ylab = "Number of parasites",
+panel = panel.superpose,
+panel.groups = MyLines)
```

The resulting Table 3.1, 3.2 shows that the data were not collected systematically (at the same months during several years). This fact makes it impossible to test for a month-year interaction, and we only use month as an explanatory variable.

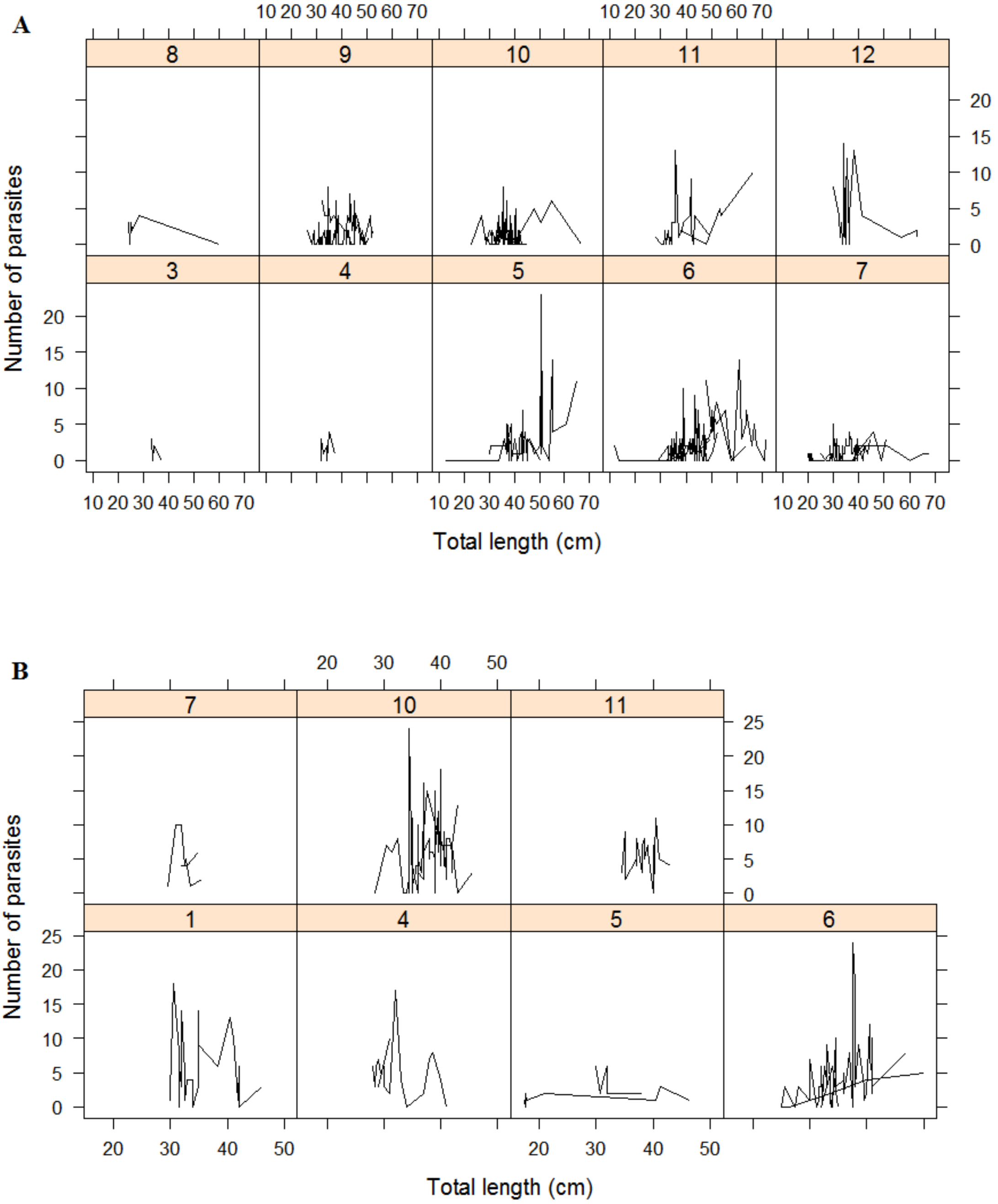

Fig. 3.4 Number of parasites–total length profiles per month for Sea of Azov (A) and Sea of Japan (B). Each line represents a Locality. There are no similar profiles

To understand the influence of different predictors on parasite abundance, a GAM with the NBD and logarithmic link function were applied.

A forward stepwise approach was used to select the continuous and nominal explanatory variables. A set of models reflecting all the various combinations of potential covariates were tested until only significant predictors were left in the final model. Co-linear predictors, such as fish body length and weight or temperature and months, were not used within the same model. Other tested host and environmental parameters (localities, years, seasons, water salinity and fish body condition), which showed no or weakly significant effect and did not improve or provided a minimal improvement to the model, were not included in the analysis.

The models were fitted using regression P-splines for the smooth terms. We model the smooth effects of continuous covariates in the different levels of factor (studied regions) using the "by" command in the «mgcv» package in R statistical data analysis software.

The extent to which each model fits the observed data was assessed using the Akaike information criterion (AIC) value and the percent of total deviance explained.

Model residuals were inspected using graphical methods by plotting randomized quantile residuals *vs*. linear predictor, quantile-quantile plot, histogram of residuals, residuals versus fitted values, versus each covariate in the model, and versus each covariate not in the model.

Since the fish condition was normally distributed for the data set from the Sea of Japan (Kolmogorov-Smirnov test; Z= 1,28, p=0,073) and close to the normal distribution from the Azov Sea (Z=1,51, p= 0,02), we used a parametric analysis of covariance (ANCOVA) to test for the effects of parasite abundance (transformed) on fish condition, controlling for region. ANCOVA were carried out with analytical software PAST v3. A significance level of 0,05 was used in all test procedures.

3.3 Results and discussion

The mean abundance with 95% CI of *L. llewellyni* per each sample and region is presented in Table 3.1. Although values of abundance varied considerably among samples within each region, overall abundance per region was around four-fold lower with no overlapping 95% CI in the Azov Sea than in the Sea of Japan.

Our search of an optimal GAM model for abundance of *L. llewellyni* in *P. haematocheila* across its native and introduced distribution ranges had resulted in two equal models:

$(Abundance_i)^{1/2} \sim NB\ (\mu_i,k)$;

$\mathrm{E}((Abundance_i)^{1/2}) = \mu_i$;

$\mathrm{Var}((Abundance_i)^{1/2}) = \mu_i + \mu_i^2/k$;

model 1: $\log(\mu_i) = s$(total length) : factor(Sea) + s(temperature) : factor(Sea);

model 2: $\log(\mu_i) = s$(total length) : factor(Sea) + s(months) : factor(Sea),

where μ_i is the mean parameter; k is the dispersion parameter; s is the thin plate smoothing spline function. These models included host length and water temperature (model 1) or months (model 2) as predictor variables after controlling for regional effect and had deviance explained of 23,3% (AIC=3138,7) and 24% (AIC=3136,4), respectively.

R code to execute the model 1 is

```
>fit0<-gam(P11~s(TL,by=Sea,bs="ps")
+ +s(Temperature,by=Sea,bs="ps",k=8)+
+ +Sea,family =nb(),data = myMay,method = "REML")
```

The summary of model 1 shows the following (starting from the parametric coefficients):

```
Parametric coefficients:
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)        0.58583    0.05126  11.428  < 2e-16
SeaSea of Japan    0.89424    0.12975   6.892 5.51e-12

(Intercept)       ***
SeaSea of Japan ***
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                                   edf Ref.df Chi.sq
s(TL):SeaSea of Azov             2.183  2.715 57.934
s(TL):SeaSea of Japan            2.158  2.617 18.990
s(Temperature):SeaSea of Azov    2.432  2.916 17.278
s(Temperature):SeaSea of Japan   1.001  1.002  2.133
                                  p-value
s(TL):SeaSea of Azov             3.34e-12 ***
s(TL):SeaSea of Japan            0.000335 ***
s(Temperature):SeaSea of Azov    0.001075 **
s(Temperature):SeaSea of Japan   0.145261
```

```
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.211   Deviance explained = 23.3%

Parametric coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)       0.55542    0.04954  11.212   <2e-16 ***
SeaSea of Japan   0.98290    0.11552   8.509   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                             edf Ref.df Chi.sq  p-value
s(TL):SeaSea of Azov       2.143  2.664  52.69 3.93e-11 ***
s(TL):SeaSea of Japan      2.077  2.522  14.10 0.004548 **
s(Month):SeaSea of Azov    3.896  8.000  19.57 0.000151 ***
s(Month):SeaSea of Japan   1.105  6.000   2.77 0.072120 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.224   Deviance explained =   24%
-REML = 1567.7  Scale est. = 1         n = 759
```

The GAM models indicate the significant regional variability of the abundance of *L. llewellyni* and that the fish length, temperature of the sea and months are highly significant explanatory continuous variables (Figs. 3.5 and 3.6). The response curves in the GAM showed that relationships between the abundance of *L. llewellyni* and the three variables selected were not linear. The probability of having parasites increases with longer fish length (Figs. 3.5 A, B and 3.6 A, B), low temperature (Figs. 3.5 C, D) and cold season (Figs. 3.6 C, D) in both regions.

The significance level (*p*-values) of the effect of predictors is displayed in each plot. The ranges of the variables are represented on the *x-axis* and the contribution of the smoothing functions to the fitted values is represented on the *y-axis*. The solid and dashed lines represent the smoothers (main effects) and the 95% confidence intervals respectively. The marks along the *x*-axis indicate a single observation. The edf value reflects of the amount of non-linearity of the smooth.

The smoother for the partial effect of fish length on parasite abundance shows initial growth that is followed by a plateau. The abundance curve reaches an asymptote when fish size reaching 39 – 40 cm and 63 – 65 cm for the native and introduced distribution range, respectively (Figs. 3.5 A, B and 3.6 A, B).

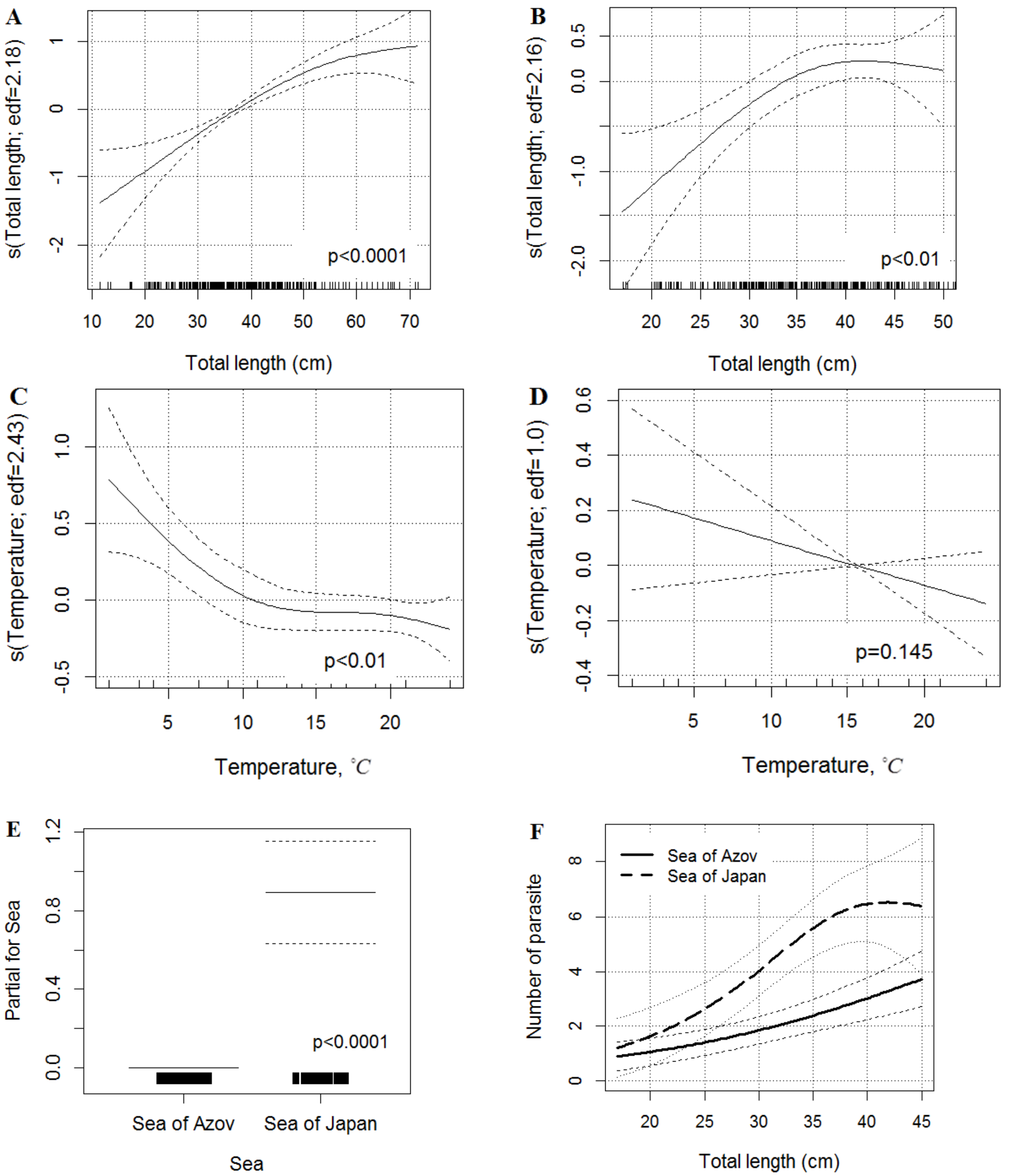

Fig. 3.5 The effect of host total length (A and B), temperature (C and D) and geographic region (E) on the abundance of *Ligophorus llewellyni* in *Planiliza haematocheila* from the Sea of Azov (A and C) and the Sea of Japan (B and D) are shown as estimated in the first GAM model. Plot F represents results of modelling the mean parasite abundance (in square root transformed values) for Sea of Azov (solid lines) and Sea of Japan (dashed lines) with fixed water temperature (5°C) and variable fish length (17 – 45 cm). The dotted lines represent the 95% confidence intervals for the response curve.

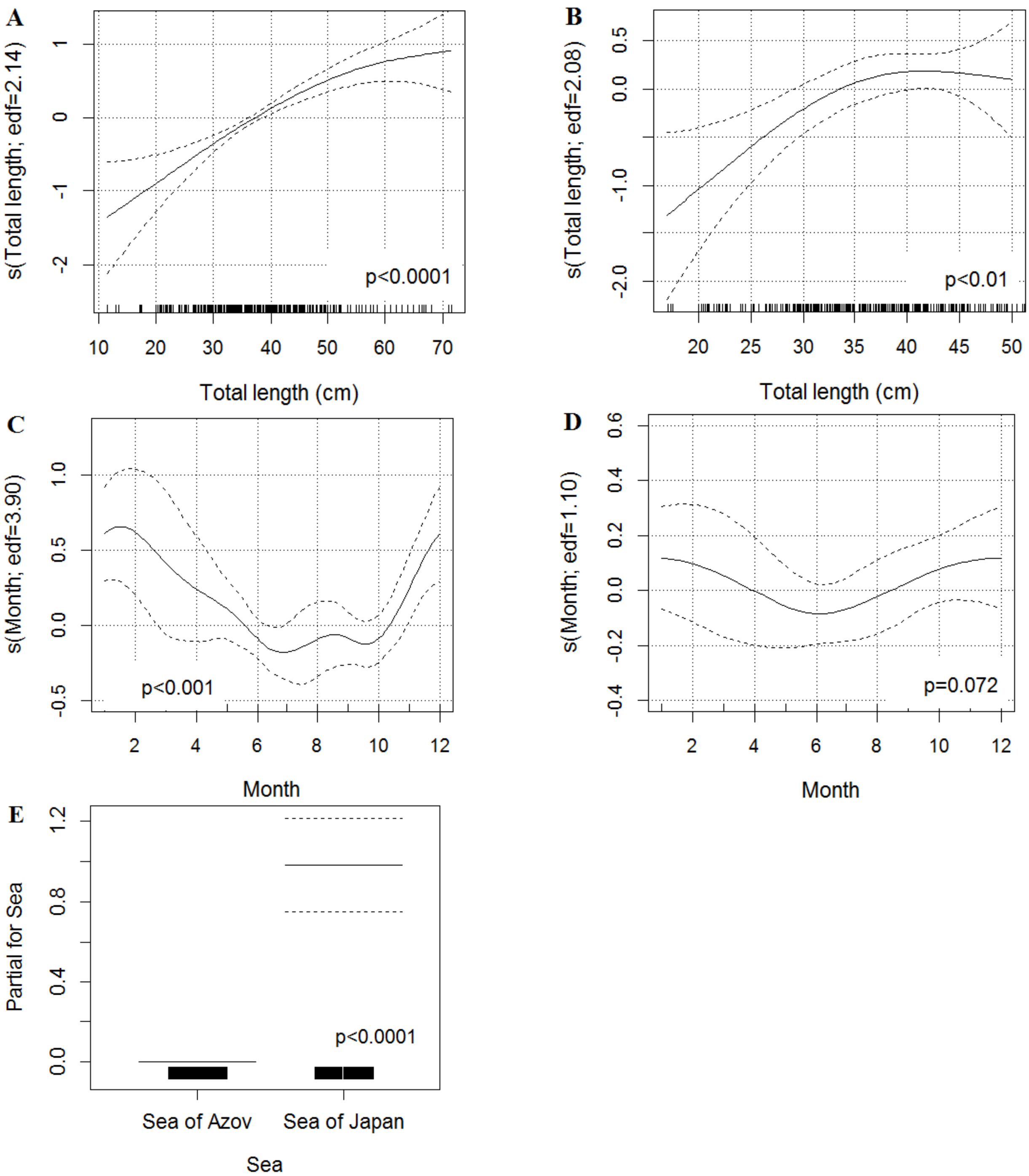

Fig. 3.6 The effect of host total length (A and B), month (C and D) and geographic region (E) on the abundance of *Ligophorus llewellyni* in *Planiliza haematocheila* from the Sea of Azov (A and C) and the Sea of Japan (B and D) are shown as estimated in the second GAM model. Labels as in Fig. 3.5.

In response to a change of the water temperature the abundance demonstrates initial decrease up to 13 – 15°C and reaches an asymptote (Fig. 3.5 C). The modelled mean parasite abundance with fixed water temperature (5°C) and variable fish length shows more rapid rise in the native region comparing with introduced one (Fig. 3.5 F).

Parasite abundance had no significant effect on the host condition (F= 2,6, P= 0,1) after controlling for regional effect. On the other hand, GAMs also did not identify the fish condition as a significant predictor of the parasite abundance (p>0,05).

At the same time, we should mention that the model suffered from a lack of data in the smallest and largest fish size categories for both regions. Taking the time scale, the dataset represents irregular samples that make impossible to test for a month-year interaction, so only month was used as an explanatory variable.

Residual checks did not show any unwanted trends that would suggest inadequacy of the fitted model to the data (Fig. 3.7, 3.8).

The main hypothesis tested here was that the host translocation led to decreasing abundance of co-introduced parasite. Our results suggest that this pattern occurred. *L. llewellyni* to be a core parasite species of *P. haematocheila* across its native and invasive distribution areas was much less abundant in the new geographic range. The multiple GAM approach applied here revealed that there is a non-linear negative relation between parasite abundance and water temperature suggesting that the latter may be responsible for such a phenomenon. The average cumulative water temperature across all samples was 11°C in the Sea of Japan and 18°C in the Azov Sea reflecting the difference in climatic conditions in these regions, so that explains observed higher abundance of *L. llewellyni* in the native area. On the other hand, the modelled abundance dynamics showed that *L. llewellyni* increases population size in fish more rapidly in the Sea of Japan than in the Azov Sea under the fixed temperature condition. This means that there is another factor(s) affecting the population growth of the parasite in the new distribution area. Although our study found no evidence for the link between parasite abundance and host body condition, we suppose that the so-iuy mullet escaping the most number of parasites of its native range, and thus benefiting from reduced infection pathology, may build a stronger immune defence to the carried parasites.

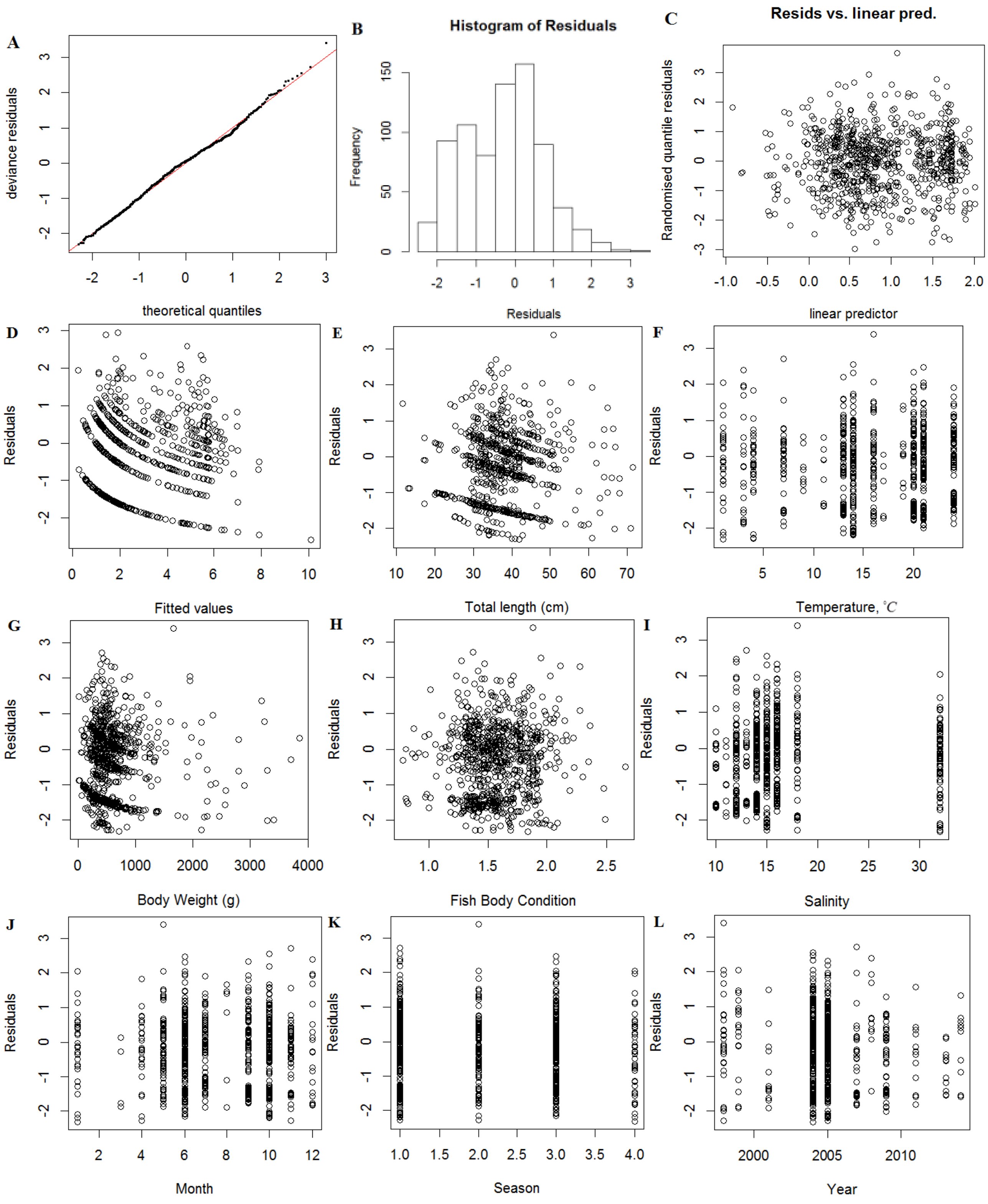

Fig. 3.7 Basic diagnostic residual plots for checking the generalized additive model 1. The normal (Q-Q) plot (A) is very close to a straight line, suggesting reasonable distributional assumption. Although the histogram of residuals (B) shows a slight right-skewed asymmetry, it reveals an approximately normal distribution. The randomized quantile residual plot (C), plot residuals versus fitted values (D), plot residuals versus each covariate in the model (E, F), and versus each covariate not in the model (G – L) did not show any unwanted trends that would suggest inadequacy of the fitted model to the data

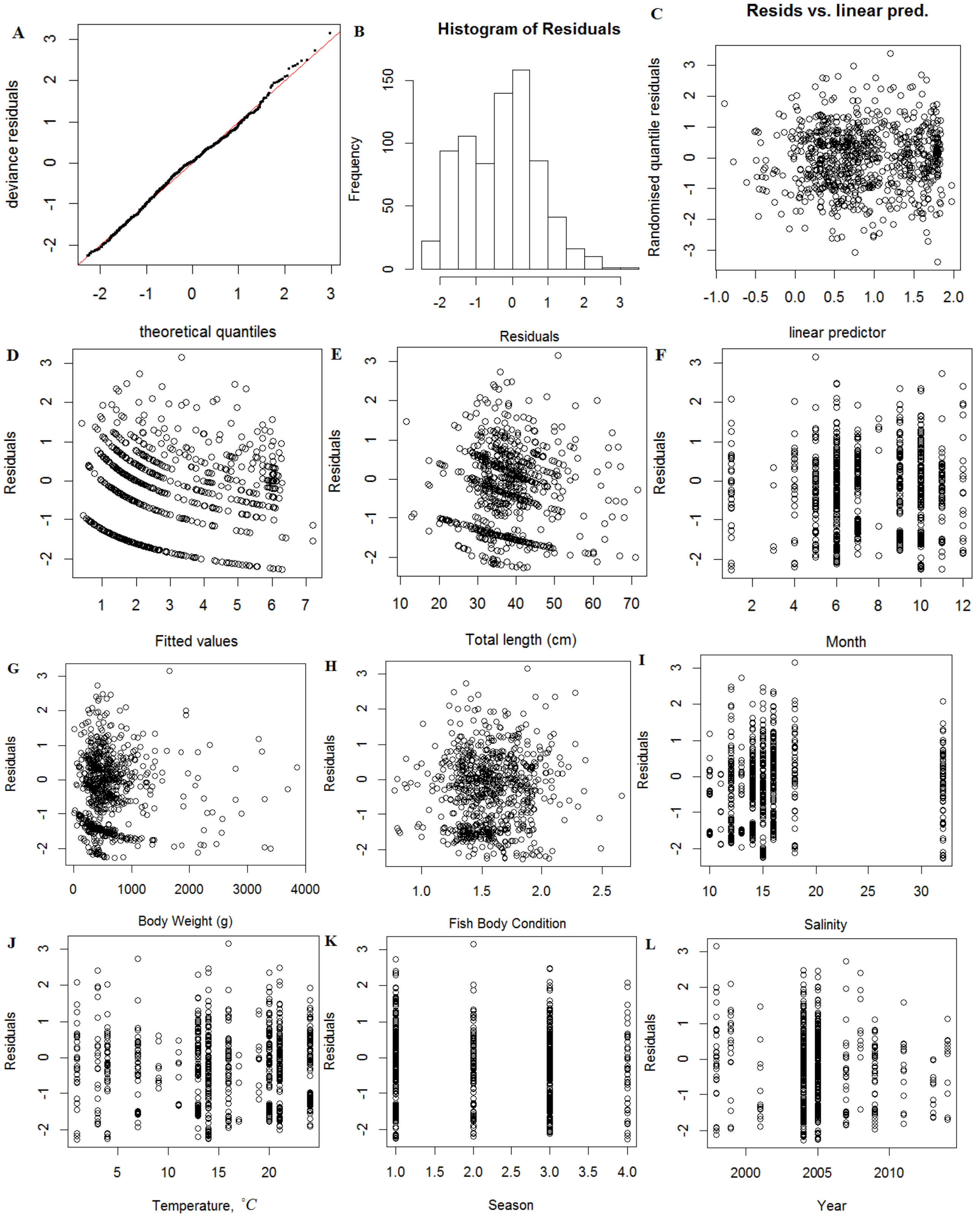

Fig. 3.8 Basic diagnostic residual plots for checking the generalized additive model 2. Labels as in Fig. 3.7

An introduced species may free up internal resources no longer needed in the deployment of the immune system against pathogens and reallocate them in fitness parameters, reproduction, etc.

Thus the effect of a warm climate that is mediated by stronger host immunity due to a reprieve from pathogens, may negatively affect the abundance of a co-introduced evolutionary related parasite species. The new environmental conditions, especially higher temperature rate, also may affect oncomiracidial survival and moving. A negative effect of increased temperature on survival of oncomiracidia *Sparicotyle chrysophrii* from gilthead sea bream in the Mediterranean was demonstrated experimentally by Villar-Torres et al.

Temperature is commonly accepted as one of the most important factors determining the abundance of monogeneans parasites. Some of them tend to increase their numbers under cold conditions, while others demonstrate the peak of their abundance in warm waters. Experiments performed by Kuperman and Shul’man on dactylogyrid parasites of bream have shown that the water temperature can affect monogeneans in two ways. A warm temperature stimulates the development and reproduction of the parasites that results in the increase of their abundance. Simultaneously, the rise in the temperature increases the resistance of the fish organism to infections through the activation of immune response decreasing the parasite abundance. The host encounter by directly transmitted monogeneans may be also largely depending on host behaviour, which varies between species of grey mullets, for instance, in migration strategy and seasonal activity. In contrast to native grey mullets, which migrate to warm waters in the Black Sea to escape cold temperature, *P. haematocheila* prefers to settle down into pits for wintering in the Azov Sea, rivers and estuaries of the region. Similarly, in the Sea of Japan with the approach of a cold season it makes migration into rivers. In the autumn, as water temperature reaches the threshold of 8°C and lower *P. haematocheila* does not feed and becomes generally inactive. Thus, the abundance of *L. llewellyni* rapidly rises in autumn (Fig. 3. 6 C and D), as fish activity and immune response decrease, reaches the maximum to the winter and decreases in spring as the fish activity and immune response is activated by a warm temperature.

The second obvious parameter driving the abundance of the *L. llewellyni* was fish length. *Ligophorus* spp. infect the so-iuy mullet beginning from the second month of its life and last to the death with increasing abundance. Changes in parasite abundance and intensity related with fish size are well known and broadly discussed in the literature. A positive correlation between the host body length and abundance of ectoparasites is based on the increased resource availability, since larger fish provide more attachment space and can thus sustain more parasite individuals. The advantage of using the GAMs is revealing more precise insights about host-parasite interactions. The result of the analysis indicates that there are non-linear relationships between the host length and the mean number of parasites, where the abundance *L. llewellyni* showed an apparent initial increase until it reaches an asymptote. Our study produced results which in general coincide the findings found for endoparasitic worms, *Pseudoterranova decipiens* (Krabbe), from cod and shorthorn sculpin. The response curves built for native and introduced regions reach an asymptote at different fish body length. This distinction is linked to the fish growth rate, which is higher in the introduced range of *P. haematocheila*. Stopping abundance from rising obviously related with loss of older age groups of fish through natural death or fishery.

This study demonstrates the utility of GAMs to analyse the relationships between dynamics of parasite abundance in the host and the factors affecting it. GAMs as a flexible class of regression models provided us a powerful tool to analyse complex and non-linear relationships in host-parasite system of the invasive host and its co-introduced parasite species across native and introduced areas. GAMs help us to predict an increase in abundance of the parasite with increasing host body size to compare modelled abundance curves between regions, but there is no way of predicting how many more parasites should be expected when we go out from empirical data, as the errors of prediction sharply increase in such cases. Moreover, the use of GAMs are constrained by the fitting of the response variable to a certain pattern of distribution that required the data transformation and limited the current research to only one species, while our initial idea was to study the abundance patterns of two congenic species, *L. llewellyni* and *Ligophorus pilengas* Sarabeev and Balbuena.

In conclusions, our results suggest that the introduced host lost a large amount of parasite abundance due to the effect of warm climate that is mediated by the host defence system. Returning to the question posed at the beginning of this study, it is now possible to state that the new favourable environment for the introduced host is much less favourable for its co-introduced parasites obviously due to stronger host response. Finally, the analysis revealed that the carried parasite species holds the same trend of relationships as compared to the native area between the mean number of monogeneans per host and independent variables increasing the abundance toward the fish length, low temperature and cold months. Our results therefore open new perspectives for future researches on statistical modelling of abundance of co-introduced parasites across native and introduced distribution ranges in order to provide deeper insight into host-parasite interactions of invasive populations. The current study was unable to analyse effects of the host population size on the parasite infection due to lack of information for the Sea of Japan. The annual monitoring for the Azov stock of the so-iuy mullet performs Institute of Fisheries and Marine Ecology, Berdyansk, Ukraine, while this information was not found for the Sea of Japan. Moreover, it would be interesting to assess the effects of fish genetic diversity on the abundance of co-introduced parasites. The study of the genetic structure of the so-iuy mullet performed by Salmenkova et al. demonstrates that the Azov population did pass through a bottleneck, which caused a considerable reduction in the haplotype diversity of the introduced population. Relationship between host genetic diversity and parasites was found to affect infection intensity.

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