FRACTAL DIMENSION OF THE BOUNDARY OF CANTORVALS

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Let's consider a convergent positive series $\sum_{n=1}^{\infty} a_n$. By $E(a_n)$ we denote the set of all subsums for the series $\sum a_n$, i.e.

$$E(a_n) = \left\{ \sum_{n=1}^{\infty} \varepsilon_n a_n : \ (\varepsilon_n) \in \{0,1\}^N \right\}.$$

We study metric and fractal properties of the set

$$X(m) = \left\{ \sum_{n=1}^{\infty} \frac{\varepsilon_n}{2m+2} : (\varepsilon_n) \in \{0,2,3,\dots,2m+1,2m+3\}^N \right\},$$

which is the set of subsums for the series

$$3q + \underbrace{2q + \cdots + 2q}_{m} + 3q^2 + \underbrace{2q^2 + \cdots + 2q^2}_{m} + \cdots + 3q^n + \underbrace{2q^n + \cdots + 2q^n}_{m} + \dots$$

where $q = 1/(2m+2), m \in N$.

Connectivity components of the closed bounded set $X(m) \subset R$ are either closed intervals or singletons. From now on, we denote *the interior* of the above set by $X_I(m)$, while $X_C(m) = X(m) \backslash X_I(m)$ will be called *the boundary* of $X(m)$. The following theorem demonstrates the structure of $X(m)$.

Theorem. *The set $X(m)$ can be obtained as the union $X = X_I(m) \bigcup X_C(m)$, where $X_I(m)$ is an infinity union of open intervals having the Lebesgue measure equal to 1, $X_C(m)$ is a Cantor set with zero Lebesgue measure and fractional Hausdorff dimension* $\dim_H X_C = \log_{2m+2} 3$.

We say that a Cantorval X is *achievable* if there exists a convergent positive series $\sum u_n$ such that $X = E(u_n)$. Taking into account the above theorem, there naturally arises the question: Is the boundary X_C for an arbitrary Cantorval X a fractal set?

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