

Effect of eigenmode destabilisation by energetic ions on plasma rotation

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Momentum transport caused by spatial channelling – spatial transport of the energy of energetic ions by eigenmodes destabilised by these ions – is considered. The torques that result from the resonant interaction of trapped and passing energetic ions with an eigenmode are calculated and compared. The shearing rate of the toroidal flow that arises when an eigenmode transports toroidal angular momentum from the excitation region to the damping region is evaluated. The possibility that this flow can contribute to turbulence suppression is discussed.

Key words: fusion plasma, plasma flows, plasma instabilities

1. Introduction

Destabilised Alfvén eigenmodes (AEs) can transport the energy and momentum of the destabilising fast ions via spatial channelling (SC) (Kolesnichenko *et al.* 2010*a*, *b*, see also the overview by Kolesnichenko, Tykhyy & White 2020); it happens when the spatial regions of the mode drive and damping are separated or when the mode shape and amplitude evolve in time. Spatial channelling can be caused not only by AEs but also by fast magnetoacoustic modes (Belova *et al.* 2015). It could be responsible (Kolesnichenko *et al.* 2010*a*, *b*; Belova *et al.* 2015) for the deterioration of confinement in NSTX (Stutman *et al.* 2009). Spatial channelling was also proposed (Kolesnichenko *et al.* 2018) as an explanation for the improved confinement and anomalous ion heating that was possibly observed in JET DTE1 experiments with deuterium–tritium plasmas (Thomas *et al.* 1998; Weisen *et al.* 2014).

Another phenomenon indicating that SC indeed takes place is twisting of AEs – radial variation of the mode phase (Tobias *et al.* 2011). Twisting is arguably associated with radial energy transport (Ma, Zonca & Chen 2015; Kramer *et al.* 2019; Meng *et al.* 2022; Kolesnichenko *et al.* 2024). An overview of publications on this topic can be found in Heidbrink *et al.* (2022), where it was proposed to use this phenomenon for diagnostics of drive and damping of AEs.

Fast ions are known to affect plasma turbulence (Di Siena *et al.* 2018), see an overview in Citrin & Mantica (2023). Several mechanisms are proposed to explain

this effect; zonal flows generated by fast-ion-excited AEs – TAEs (toroidicity-induced Alfvén eigenmodes), RSAEs (reversed-shear-induced Alfvén eigenmodes) or BAEs (beta-induced Alfvén eigenmodes) – are among these mechanisms. Generation of zonal flows (ZFs) by AEs and EPs (energetic particle modes) has been observed in numerical experiments (Todo, Berk & Breizman 2010; Zhang & Lin 2013; Spong *et al.* 2021; Ishizawa, Poli & ans Taik Soo Hahm 2025). In several works involving experimental observations and numerical simulations, the suppression of turbulence by AEs via shear flow formation was demonstrated (Mazzi *et al.* 2022; Varela *et al.* 2024; Du *et al.* 2025). In particular, it was found in recent DIII-D experiments (Du *et al.* 2025) that the appearance of multiple TAEs with increased amplitude results in the formation of shear flow accompanied by mitigation and suppression of turbulence, which reappears after the TAE activity returns to the initial state. Analytical theories for the ZF generation by AEs have been developed (Chen & Zonca 2012; Qiu, Chen & Zonca 2017; Barberis *et al.* 2025; Yan & Diamond 2025); they treat this phenomenon in terms of nonlinear wave–wave interaction between an AE and a ZF.

As any wave possesses non-zero momentum, SC of energy is accompanied by SC of momentum (Kolesnichenko *et al.* 2010b). Momentum transport by AEs may be of interest because of the effect of rotation on turbulence and confinement. It seems that the SC of momentum is an additional factor able to affect plasma rotation and generation of plasma flows. The effect of SC on rotation was considered in Kolesnichenko *et al.* (2022); in addition, the so-called MIR – mode-induced redistribution of momentum – was found in that work.

In Kolesnichenko *et al.* (2022), equations of the particle motion during their resonant interaction with eigenmodes were written in the cylindrical approximation; they are valid only for well passing energetic ions. One of the aims of the present paper is to generalise some results of Kolesnichenko *et al.* (2022) to the case when the mode is destabilised by trapped energetic ions. Another aim is to evaluate a possible contribution of the momentum channelling by AEs to the formation of shear flows.

The structure of this paper is as follows. In § 2, equations of motion caused by resonance interaction with a wave are derived for trapped and passing particles. In § 3, relationships between the displacement of energetic particles and the plasma acceleration are established. In § 4, the effects produced by SC of momentum of trapped and passing energetic ions are compared. The excitation of shear flow due to SC is studied in § 5. Finally, conclusions are drawn in § 6.

2. Motion of resonant particles

We proceed from the particle Hamiltonian in action-angle coordinates (Kaufman 1972)

$$H(J_\xi, J_\phi, J_\theta, \xi, \phi, \theta, t) = H_0(J_\xi, J_\phi, J_\theta) + \tilde{H}(J_\xi, J_\phi, J_\theta, \xi, \phi, \theta, t), \quad (2.1)$$

where ξ , ϕ and θ are the canonical gyro-, toroidal and poloidal angles, respectively, J_ξ , J_ϕ and J_θ are the respective actions, $H_0 = \mathcal{E}$ represents the unperturbed particle motion and $\tilde{H}$ describes the motion affected by the wave. The toroidal action (the canonical angular momentum) is given by

$$J_\phi = -\frac{e}{c}\psi_p + Mv_\parallel R, \quad (2.2)$$

where e and M are the particle charge and mass, respectively, c is the speed of light, $\mathbf{v}$ is the particle velocity, the subscript ‘||’ refers to the longitudinal component, R is the distance to the axis of symmetry and ψ_p is the poloidal magnetic flux. For a passing particle with the radial orbit width much smaller than the radial scale of interest, we can take

$$J_\theta \approx \frac{e}{c} \psi_T, \quad (2.3)$$

where ψ_T is the toroidal magnetic flux (the velocity contribution to the poloidal action is small when the tokamak aspect ratio is large).

Let us consider a single resonance between the mode with a frequency ω and a particle given by

$$\omega = s\omega_\theta - n\omega_\phi + l\omega_\xi, \quad (2.4)$$

where ω_ϕ and ω_θ are the frequencies of the toroidal and poloidal particle motions, respectively; ω_ξ is the bounce-averaged cyclotron frequency; n is the toroidal mode number; and s and l are integers. In particular, it is known that the strongest resonances of low-frequency waves ($\omega \ll \omega_\xi$) with passing energetic ions are for $s = m \pm 1$, where m is the poloidal mode number of a certain mode harmonic.

It is known that motion of a resonant particle satisfies

$$\frac{\dot{\mathcal{E}}}{\omega} = \frac{\dot{J}_\theta}{s} = \frac{\dot{J}_\phi}{-n} = \frac{\dot{J}_\xi}{l}, \quad (2.5)$$

where the ‘dots’ mean d/dt (this can be easily seen if one disregards non-resonant harmonics and takes $\tilde{H} \propto \exp(-i\omega t + is\theta - in\phi + il\xi)$).

For passing particles we combine (2.2), (2.3) and (2.5) to obtain

$$\dot{\psi}_p = \frac{cs}{eq\omega} \dot{\mathcal{E}}, \quad (2.6)$$

$$\dot{v}_\parallel = \frac{s/q - n}{MR_0q\omega} \dot{\mathcal{E}}, \quad (2.7)$$

where q is the magnetic winding number (safety factor). When writing these equations, we have assumed that the variations of R and $v_\parallel$ are small, $R \approx R_0$. Using (2.6), we find the following relationship between the radial current of resonant ions and the power density transferred from the fast ions to the wave due to a single resonance with given numbers s and l :

$$\frac{1}{c} \langle \mathbf{j}_{\alpha,sl} \cdot \nabla \psi_p \rangle = \frac{e}{c} \frac{d\psi_p}{dr} \left\langle \int d^3\mathbf{v} f_\alpha(r, \vartheta, \mathbf{v}) \dot{\psi}_p \right\rangle = -\frac{s}{q\omega} P_{sl}. \quad (2.8)$$

Here, f_α is the distribution function of fast particles, $\mathbf{j}_{\alpha,sl}$ is the energetic ion current density induced by this resonance and

$$P_{sl} = - \left\langle \int d^3\mathbf{v} f_\alpha(r, \vartheta, \mathbf{v}) \dot{\mathcal{E}} \right\rangle, \quad (2.9)$$

is the contribution of this resonance to the power density; the integral is taken over the velocity space part associated with the resonance under consideration and the angular brackets denote flux-surface averaging given by

$$\langle (\dots) \rangle = \oint \frac{d\vartheta}{\mathbf{B} \cdot \nabla \vartheta} (\dots) \bigg/ \oint \frac{d\vartheta}{\mathbf{B} \cdot \nabla \vartheta}, \quad (2.10)$$

where $\mathbf{B}$ is the magnetic field and ϑ is the poloidal angle of the magnetic coordinates $(\psi_p, \vartheta, \varphi)$ in which φ is the geometrical toroidal angle.

For trapped particles, we can neglect the velocity term in (2.2), $J_\phi \approx -(e/c)\psi_p$, when the radial orbit width is small in comparison with the mode width. Then we obtain

$$\dot{\psi}_p = \frac{cn}{e\omega} \dot{\mathcal{E}}. \quad (2.11)$$

Because this equation does not depend on s and l , we can write

$$\frac{1}{c} \langle \mathbf{j}_\alpha \cdot \nabla \psi_p \rangle = -\frac{n}{\omega} P, \quad (2.12)$$

where $P = \sum_{s,l} P_{sl}$, $\mathbf{j}_\alpha = \sum_{s,l} \mathbf{j}_{\alpha,sl}$.

3. Effect of the energetic ion flux on the plasma rotation

We proceed by deriving the relationships between the power transferred from the energetic ions to the mode and the angular acceleration of the plasma. The main difference of our approach from that of Kolesnichenko *et al.* (2022) is that we do not treat the energetic ion population in hydrodynamic terms. At the same time, the rest of particle species are treated hydrodynamically.

Following Hirshman (1978), Rosenbluth & Hinton (1996) and Helander & Sigmar (2005), we assume that the characteristic time of the angular acceleration of each thermal plasma species is much longer than the particle transit time but much shorter than the particle transport time. We also assume that this characteristic time is much longer than the mode period. Then the flow velocity of each thermal species can be considered as divergence free within the flux surface, which enables us to present it in the form

$$\mathbf{u}_a = \omega_a(\psi_p) R \hat{\boldsymbol{\phi}} + \zeta_a(\psi_p) \mathbf{B}, \quad (3.1)$$

where a is the index of the particle species, $\hat{\boldsymbol{\phi}} = R \nabla \varphi$ is the unit vector in toroidal direction and $\zeta_a(\psi_p)$ characterises poloidal rotation

$$\omega_a = -c \left(\frac{\partial \Phi}{\partial \psi_p} + \frac{1}{e_a n_a} \frac{\partial p_a}{\partial \psi_p} \right), \quad (3.2)$$

where n_a and e_a are particle density and charge, respectively. Note that $p = p(\psi_p)$, $\Phi = \Phi(\psi_p)$ and $\mathbf{u}$ in (3.1) and (3.2) are not directly associated with the wave. Considering processes on time scales much longer than the mode period, we take the equilibrium values of these quantities, i.e. the values averaged over the mode period.

We proceed from the following equations (Helander & Sigmar 2005) describing the evolution of the longitudinal and toroidal flow velocities with viscosity terms omitted:

$$\rho_a \frac{\partial \langle B u_{a\parallel} \rangle}{\partial t} = \langle B F_{a\parallel} \rangle, \quad (3.3)$$

$$\rho_a \frac{\partial \langle R u_{a\varphi} \rangle}{\partial t} = \frac{1}{c} \langle \mathbf{j}_a \cdot \nabla \psi_p \rangle + \langle R F_{a\varphi} \rangle, \quad (3.4)$$

where $\mathbf{F}_a$ is the partial force density acting on the species 'a', $\rho_a = M_a n_a$, M_a is particle mass, $\mathbf{j}_a$ is current density and the projections $X_\parallel$ and X_φ of an arbitrary vector $\mathbf{X}$ are defined as $X_\varphi = \mathbf{X} \cdot \hat{\boldsymbol{\phi}}$ and $X_\parallel = \mathbf{X} \cdot \mathbf{B}/B$.

Using (3.1), (3.3) and (3.4), one can derive the following equations describing the angular acceleration of the species (see Kolesnichenko *et al.* (2022) for details):

$$M_a n_a \frac{\partial \omega_a}{\partial t} = \frac{\langle B^2 \rangle}{\langle B_p^2 \rangle \mathcal{K} \langle R^2 \rangle} \left(\frac{1}{c} \langle \mathbf{j}_a \cdot \nabla \psi_p \rangle + \langle R F_{a\omega} \rangle \right), \quad (3.5)$$

$$M_a n_a \frac{\partial \zeta_a}{\partial t} = \frac{1}{\langle B^2 \rangle \mathcal{K}} \left(-\frac{I}{c \langle R^2 \rangle} \langle \mathbf{j}_a \cdot \nabla \psi_p \rangle + \langle B F_{a\omega} \rangle \right). \quad (3.6)$$

Here,

$$F_{a\omega} = F_{a\varphi} - \frac{IB}{R \langle B^2 \rangle} F_{a\parallel}, \quad (3.7)$$

$$F_{a\omega} = F_{a\parallel} - \frac{IR}{B \langle R^2 \rangle} F_{a\varphi}, \quad (3.8)$$

with $I = I(\psi_p) = B_\varphi R$, $\mathcal{K} = 1 + 2\hat{q}^2$ and

$$\hat{q}^2 = \frac{I^2}{2 \langle B_p^2 \rangle} \left\langle \frac{1}{R^2} - \frac{1}{\langle R^2 \rangle} \right\rangle, \quad (3.9)$$

with $\hat{q}^2 \approx q^2$ in a circular high-aspect-ratio tokamak. During the derivation, it was assumed that viscosity effects are negligible for the time scale of interest.

To find the plasma acceleration, we differentiate (3.2) with respect to time

$$\frac{\partial \omega_a}{\partial t} = -c \frac{\partial^2 \Phi}{\partial t \partial \psi_p} - \frac{c}{e_a} \frac{\partial}{\partial t} \left(\frac{1}{n_a} \frac{\partial p_a}{\partial \psi_p} \right). \quad (3.10)$$

We neglect the pressure and density changes because we assume that the characteristic time of the acceleration is short in comparison with the particle transport times. As will be shown *a posteriori*, the contribution of the radial particle transport produced by currents $\mathbf{j}_a$ excited by the acceleration itself to the pressure variation is also small. Therefore, we neglect the pressure term. Then the evolution of ω_a is synchronised for all species, being associated with a change of the radial electric field. Using Gauss's law, we express it in terms of the radial electric current

$$\frac{\partial \omega_a}{\partial t} = -\frac{4\pi c}{\langle |\nabla \psi_p|^2 \rangle} (\langle \mathbf{j}_\Sigma \cdot \nabla \psi_p \rangle + \langle \mathbf{j}_\alpha \cdot \nabla \psi_p \rangle), \quad (3.11)$$

where $\mathbf{j}_\Sigma = \sum_a \mathbf{j}_a$, the summation is done over all thermal species and $\mathbf{j}_\alpha$ is the current arising due to displacement of resonant energetic particles.

Now our aim is to exclude the thermal species current from the obtained equations. Summing up equations (3.5) for different species, we obtain

$$\frac{1}{\langle v_A^2 \rangle} \frac{\partial \omega_a}{\partial t} = \frac{4\pi}{D} \left(\frac{1}{c} \langle \mathbf{j}_\Sigma \cdot \nabla \psi_p \rangle + \langle R F_{\Sigma\omega} \rangle \right), \quad (3.12)$$

where v_A is the Alfvén velocity, $F_{\Sigma\omega} = \sum_a F_{a\omega}$ with summation over all thermal species and $D = \mathcal{K} \langle B_p^2 \rangle \langle R^2 \rangle$. Combining (3.12) with (3.11), we obtain

$$\frac{1}{c} \langle \mathbf{j}_{\text{tot}} \cdot \nabla \psi_p \rangle = \frac{\lambda}{1 + \lambda} \left(\frac{1}{c} \langle \mathbf{j}_\alpha \cdot \nabla \psi_p \rangle - \langle R F_{\Sigma\omega} \rangle \right), \quad (3.13)$$

$$\frac{1}{c} \langle \mathbf{j}_\Sigma \cdot \nabla \psi_p \rangle = \frac{1}{1 + \lambda} \left(-\frac{1}{c} \langle \mathbf{j}_\alpha \cdot \nabla \psi_p \rangle - \lambda \langle R F_{\Sigma\omega} \rangle \right), \quad (3.14)$$

where $\mathbf{j}_{\text{tot}} = \mathbf{j}_{\Sigma} + \mathbf{j}_{\alpha}$ and

$$\lambda = \frac{\langle v_A^2 \rangle \langle |\nabla \psi_p|^2 \rangle}{c^2 D}. \quad (3.15)$$

Below, we assume that $\lambda \sim v_A^2/(c^2 K) \ll 1$, which is justified for typical tokamak plasma parameters. Thus, we have recovered the fact that the polarisation current of thermal species (mostly ions) almost compensates for the fast-ion radial current (Kolesnichenko *et al.* 2010b). After excluding $\mathbf{j}_{\Sigma}$ from (3.12), we obtain

$$\rho \frac{\partial \omega_a}{\partial t} = \frac{\langle B^2 \rangle}{D} \left(-\frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle + \langle R F_{\Sigma\omega} \rangle \right), \quad (3.16)$$

with $\rho = \sum_a \rho_a$ the plasma mass density.

We proceed to considering the poloidal rotation evolution. Comparing (3.5) and (3.16), we find the radial polarisation current of each plasma species

$$\frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle = -\frac{\rho_a}{\rho} \frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle + \frac{1}{\rho} \left\langle R \left(\rho_a F_{\Sigma\omega} - \rho F_{a\omega} \right) \right\rangle. \quad (3.17)$$

Putting this formula into (3.6), we get

$$\rho_a \frac{\partial \zeta_a}{\partial t} = \frac{I}{D} \frac{\rho_a}{\rho} \left(\frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle - \langle R F_{\Sigma\omega} \rangle \right) + \frac{\langle R^2 \rangle \langle B^2 \rangle - I^2}{D \langle B^2 \rangle} \langle B F_{a\parallel} \rangle. \quad (3.18)$$

Now we can justify omitting the contribution of $\partial p_a / \partial t$ to (3.10), showing that the contribution of the radial displacements of plasma particles due to polarisation current to the pressure term in (3.2) is negligible. It follows from (3.17) that the main contribution to the polarisation current comes from ion species. For simplicity, we assume that there is only one kind of ion with the charge e . Then the pressure change due to polarisation current can be evaluated as follows:

$$\frac{\partial^2 p}{\partial t \partial \psi_p} = \frac{\partial}{\partial \psi_p} \left(\frac{T_i}{e} \nabla \cdot \mathbf{j}_i \right), \quad (3.19)$$

where T_i and $\mathbf{j}_i$ are the ion temperature and current density, respectively. Taking $\langle \mathbf{j}_i \cdot \nabla \psi_p \rangle \sim \lambda^{-1} \langle \mathbf{j}_{\Sigma} \cdot \nabla \psi_p \rangle$ due to (3.13), we express $\nabla \cdot \mathbf{j}_i$ in terms of $\partial \Phi / \partial t$. Then we find that the pressure term in (3.10) is $\rho_L^2 / \Delta_{\text{mode}}^2$ times smaller than the electrostatic term, where ρ_L is the ion Larmor radius and Δ_{mode} is the radial mode width.

Finally, we derive an equation for the evolution of $u_{a\varphi}$. Using (3.1), (3.16) and (3.18), we find

$$\frac{\partial}{\partial t} \langle R u_{a\varphi} \rangle = \frac{1}{\rho} \left(-\frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle + \langle R F_{\Sigma\omega} \rangle \right) + \frac{I}{\rho_a \langle B^2 \rangle} \langle B F_{a\parallel} \rangle. \quad (3.20)$$

We observe that both poloidal ($\dot{\zeta}_a$) and toroidal ($\dot{u}_{a\varphi}$) accelerations are the same for all species unless there is non-zero $F_{\parallel}$.

4. Effect of spatial channelling on plasma rotation

We proceed to evaluating the effect of the SC on the toroidal plasma rotation for trapped and passing energetic ions. The first term in the right-hand side of (3.20)

describes the toroidal torque density caused by the radial displacement of energetic ions due to their resonant interaction with the waves. Using (2.12), we can write it in the following form for trapped ions:

$$T_{\varphi}^{\text{trap}} = -\frac{1}{c} \langle \mathbf{j}_{\alpha} \cdot \nabla \psi_p \rangle = \left\langle \int d^3 \mathbf{v} \, j_{\varphi} f_{\alpha}(r, \vartheta, \mathbf{v}) \right\rangle = \frac{n}{\omega} P = -T_{\varphi}^{\text{mode}}, \quad (4.1)$$

where $T_{\varphi}^{\text{mode}}$ is the rate of the momentum transfer to the mode. The physical meaning of (4.1) is transparent: the torque acting on thermal plasma equals to the rate of momentum transfer to energetic ions, which is opposite to $T_{\varphi}^{\text{mode}}$.

Using (2.8), we obtain a similar relationship for passing ions

$$T_{\varphi}^{\text{pass}} = \sum_{s,l} \frac{s}{q\omega} P_{sl}. \quad (4.2)$$

Thus, the torque acting on the thermal plasma and the rate of the momentum transfer to the mode differ in this case. The difference is received by the resonant energetic ions. Indeed, as follows from (2.7), the torque acting on the energetic ions is

$$T_{\varphi}^{\alpha} = - \sum_{s,l} \frac{s/q - n}{\omega} P_{sl} = -T_{\varphi}^{\text{pass}} - T_{\varphi}^{\text{mode}} \quad (4.3)$$

The toroidal momentum fraction received by energetic ions is small for low-frequency AEs, i.e. TAEs, BAEs and RSAEs, with $n \gg 1$

$$\left| \frac{T_{\varphi}^{\alpha}}{T_{\varphi}^{\text{mode}}} \right| = \left| \sum_{s,l} \frac{s/q - n}{n} \right| \leq \frac{2}{nq}, \quad (4.4)$$

because, for each mode harmonic with the poloidal mode number m , its amplitude is significant when $|m/q - n| < 1/q$, whereas the resonances of this harmonic with passing ions are $s = m \pm 1$. In this case, trapped and passing energetic ions create an approximately equal effect on plasma rotation for equal radiated power. In other cases (high-frequency AEs or AEs with $n \sim 1$) energetic ions can contribute significantly to the momentum balance.

Note that there is another phenomenon affecting the momentum of passing energetic ions – MIR (momentum transport together with the energetic ion flux). The evolution of the longitudinal momentum is described by the following equation (Kolesnichenko *et al.* 2022):

$$\frac{\partial}{\partial t} \langle \rho_{\alpha} u_{\alpha\parallel} R \rangle = \langle F_{\parallel}^{(1)} R + F_{\parallel}^{(2)} R \rangle, \quad (4.5)$$

where $F_{\parallel}^{(1)}$ and $F_{\parallel}^{(2)}$ are the contributions to the longitudinal force density from channelling and MIR, respectively. One can see that

$$F_{\parallel}^{(1)} = T_{\varphi}^{\alpha} / R_0, \quad (4.6)$$

where R_0 is the plasma major radius. The contribution of MIR ($F_{\parallel}^{(2)}$) depends on the spatial inhomogeneity of the fast-ion distribution and can be larger than $F_{\parallel}^{(1)}$ (Kolesnichenko *et al.* 2022), but MIR is outside the scope of this work.

5. Generation of shear flow by momentum channelling

The radial profiles of the AE drive and damping rates may not coincide. For example, the continuum damping is known to be localised at the mode periphery (otherwise, the mode can hardly be destabilised) and, thus, not in the region where the mode is destabilised. Then the energy balance in the AE is impossible without a radial energy flux $\Gamma_\varepsilon \neq 0$. The mode twist (the dependence of the mode phase on the radial coordinate) is often observed in experiments; it is an indication that a non-zero radial energy flux indeed occurs, see Heidbrink *et al.* (2022). The energy flux in the mode is accompanied by the toroidal momentum flux $\Gamma_{j_\phi} = (n/\omega)\Gamma_\varepsilon$. Thus, the mode extracts energy and momentum from energetic ions at one location to release them to thermal plasma at another location. This phenomenon is referred to as SC of the energy and momentum of energetic ions.

As was shown in §§ 3 and 4, the torque acting on energetic ions when an AE is destabilised is transferred to thermal plasma (partly for passing ions and almost totally for trapped ones). Thus, the SC of momentum creates a pair of opposite torques, which act on the plasma in the regions where drive and damping dominate. This pair of torques tends to create a shear flow. Let us estimate the magnitude of this shear flow.

For simplicity, we assume that drive and damping are totally separated in different spatial regions and the distance between these regions, as well as their characteristic size, is approximately the mode width. To estimate the steady-state rotation, we write

$$\frac{\rho\omega_\phi R_0^2}{\tau} = T_\phi, \quad (5.1)$$

where $\omega_\phi \approx u_\phi/R_0$ is the plasma toroidal rotation frequency (approximately the same for all plasma species)

$$\tau = (\Delta_{\text{mode}}/a)^2 \tau_E \quad (5.2)$$

is the characteristic time of the momentum transport, a and R_0 are the plasma minor and major radii, respectively, Δ_{mode} is the characteristic width of the mode (distance between the drive and damping regions) and τ_E is the energy confinement time. When writing this estimate, we have assumed that the momentum transport is due to turbulence that results in the energy transport.

We estimate the power taken from the energetic ions by the mode as

$$P = \gamma_{\text{drive}} \frac{\tilde{B}^2}{8\pi}, \quad (5.3)$$

where γ_{drive} is the drive rate and $\tilde{B}$ is the amplitude of the magnetic field perturbation. Combining (4.1), (5.1) and (5.3), we obtain the following estimate for the steady-state plasma rotation caused by the mode destabilisation:

$$\omega_\phi \sim n \frac{\gamma_{\text{drive}}}{\omega} \frac{\tilde{B}^2}{B^2} \frac{\Delta_{\text{mode}}^2}{a^2} v_A^2 \frac{\tau_E}{R_0^2}. \quad (5.4)$$

As an example, we take the following parameters relevant to DIII-D: $B = 2.1$ T, $R_0 = 170$ cm, $a = 60$ cm, $n_i = 5 \times 10^{13}$ cm⁻³, $\tau_E = 0.5$ s. We find from (5.4) that the momentum flux produced by a mode with $n = 10$, $\gamma_{\text{drive}}/\omega = 5 \times 10^{-2}$, $\tilde{B}/B = 5 \times 10^{-4}$ and $\Delta_{\text{mode}}/a = 1/3$ can cause a flow with $\omega_\phi \sim 5 \times 10^4$ s⁻¹ and $u_\phi \sim 85$ km s⁻¹. This velocity is not much less than the rotation velocities observed in super H-mode DIII-D experiments (Ding *et al.* 2020).

It is known that a shear flow can stabilise turbulence when the flow shearing rate ($\omega_{E \times B}$) exceeds the linear growth rate (γ_{lin}) of the instabilities that produce the turbulence (Hahm & Burrell 1995). Following an overview paper by Connor *et al.* (2004), we take

$$\omega_{E \times B} = \frac{RB_\vartheta}{B} \frac{\partial \omega_\varphi}{\partial r} \sim \frac{r}{q \Delta_{\text{mode}}} \omega_\varphi, \quad (5.5)$$

$$\gamma_{\text{lin}} = k_\vartheta \rho_s \frac{c_s}{a} \left(\frac{a}{R_0} \right)^{1/2} f(\hat{s}) \left(\frac{a}{L_n} + \frac{a}{L_T} \right) \left(\frac{T_i}{T_e} \right)^{1/2}, \quad (5.6)$$

where k_ϑ is the turbulence poloidal wavenumber, $\rho_s = c_s/\omega_{Bi}$, $c_s = \sqrt{T_e/M_i}$, T_e and T_i are the electron and ion temperatures, respectively, $f(\hat{s})$ is a form factor depending on the magnetic shear $\hat{s}$ and L_n and L_T are the characteristic lengths associated with the density and temperature profiles.

Let us evaluate these quantities for the example given above. Following Connor *et al.* (2004), we take $k_\vartheta \rho_s = 0.1$ and $\hat{s} = 1$. We consider the vicinity of a flux surface at $r/a = 0.5$ and take $q(r) = 3$ and $T_i(r) = 5$ keV. When calculating L_n and L_T , we take the density and temperature profiles as follows: $n_e \propto (1 - r^2/a^2)^{1/2}$, $T_i \propto T_e \propto (1 - r^2/a^2)$. Then we find $\omega_{E \times B} = 2.5 \times 10^4 \text{ s}^{-1}$ and $\gamma_{\text{lin}} = 5.2 \times 10^4 \text{ s}^{-1}$. Thus, for this specific parameters the shearing rate is somewhat lower than that required for the total stabilisation of turbulence.

It should be emphasised that this example is purely illustrative. It is not intended to explain the experiments that we have mentioned. The only aim of this example is to attract attention to SC as a possible mechanism of the shear flow generation. Obviously, our consideration is over-simplified. On the one hand, (5.4) and (5.6) include several parameters that we do not know ($\gamma_{\text{drive}}/\omega$, $\tilde{B}/B$, $\Delta_{\text{mode}}/a = 1/3$, τ_E). It may well be that their magnitudes are actually smaller. In addition, we do not know if the spatial locations of drive and damping were strongly separated in those experiments. On the other hand, in the above estimates we assumed that the momentum transport is determined by non-stabilised turbulence. When a shear flow generated by the eigenmode suppresses turbulence, (5.2) is likely to underestimate the characteristic time of the momentum transport, and the actual shearing rate can be higher.

6. Summary and discussion

In this work, the plasma momentum transport that occurs during the SC is studied. When an eigenmode with a non-zero toroidal mode number n transports energy from one flux surface to another, it always transports toroidal angular momentum. This means that when the spatial regions where the mode is destabilised and damped are separated in space, the mode applies opposite torques to both these regions. Here, we derive relationships between the radial flux of the ions destabilising the wave, the power accepted by the mode and the torques applied to the plasma. At a fixed power channelled by the mode, the resulting torque is proportional to n ; note that TAEs with $n \sim 10$ – 15 are expected to be dominant in ITER (Todo *et al.* 2025).

When the AE is excited by trapped particles, it is mainly the thermal plasma that receives the torque resulting from the wave radiation. In the case of destabilisation by passing energetic ions, they can take a certain fraction of this torque and undergo longitudinal acceleration. This fraction is small for low-frequency AE modes (TAEs, BAEs and RSAEs) with $n \gg 1$, for which the resonance numbers s and n satisfy the condition $|s/q - n|/|n|$ in the drive region. However, this inequality may not

hold for low-frequency modes with $n \sim 1$ and for high-frequency modes excited via cyclotron resonance.

Note that momentum transport due to eigenmodes destabilised by energetic ions can also occur via the so-called MIR, where the momentum is redistributed by the flux of resonant energetic ions (Kolesnichenko *et al.* 2022). This phenomenon is not considered here.

As mentioned in §1, the formation of shear flows (ZFs) by AEs/EPs was reported in several numerical and experimental works. In the theoretical papers that we know about, the ZF formation is explained by nonlinear wave–wave coupling between the ZF and unstable AEs. Two mechanisms of nonlinear ZF excitation were found, spontaneous generation predicted theoretically by Chen & Zonca (2012) and forced generation, which was observed in numerical experiments by Todo *et al.* (2010) and analysed theoretically by Qiu, Chen & Zonca (2016). The momentum channelling (MC) by AEs seems to be an additional mechanism able to contribute to the ZF formation.

These mechanisms have different restrictions and different scalings with mode amplitude. Spontaneous ZF generation requires the AE amplitude to exceed a certain threshold and leads to a ZF growth rate proportional to the mode amplitude. Forced ZF generation has no amplitude threshold; the ZF growth rate due to forced generation is twice that of the AE. The MC requires spatial separation between drive and damping to generate ZF, whereas both nonlinear ZF excitation mechanisms are free of this restriction. The term ‘growth rate’ is not applicable to MC because the time derivative of the ZF amplitude (i.e. velocity) is not proportional to this amplitude. The toroidal acceleration due to MC is proportional to the mode amplitude squared. It seems that the MC has better chances to contribute to the ZF generation when the AE amplitude is large and the drive and damping are strongly separated in space. Further work, probably including numerical simulations, is required to compare the roles played by different mechanisms. Only then will we find out whether MC can significantly contribute to the ZF formation and when it is possible.

We have estimated the rotation shear generated by the momentum SC. We found that a shearing rate created by a mode with $n = 10$, $\gamma_{\text{drive}}/\omega = 5 \times 10^{-2}$, $\tilde{B}/B = 5 \times 10^{-4}$ can reach the order of magnitude required for turbulence suppression. It should be taken into account that our estimate does not allow for the effect of the mode on the turbulence suppression; actually, the flow shearing rate can be higher. It should be emphasised that our work is not intended to explain any experimental results cited herein. The aim of our analysis is to point out MC as a possible mechanism for the shear flow generation.

Finally, it is worth discussing the assumptions used in our analysis. We assume that the tokamak aspect ratio is large. The radial excursions of the energetic ions and the Larmor radius of thermal ions are assumed to be small in comparison with the radial width of the mode. The characteristic time of the plasma acceleration is assumed to be much less than the characteristic times of viscous and diffusive processes; we neglect the effect of viscosity on rotation and the effect of transport on the plasma pressure. At the same time, we assume that this time is much larger than both the mode period and the thermal ion transit time. It is assumed that the plasma density is sufficiently high so that $v_A/c \ll 1$, where v_A is the Alfvén velocity. Our main assumption is that the radial distributions of the mode drive and the mode damping are different, so that there are spatial regions where drive prevails over damping and *vice versa*.

All the assumptions listed above are important for the validity of our conclusions (except for the first one, which seems to play a technical role in our calculations). A shear flow is unlikely to arise when viscosity and transport are faster than the processes under consideration. When the inequality $v_A/c \ll 1$ does not hold, the thermal plasma cannot accept the total momentum produced by wave radiation; the momentum is received by the radial electric field instead. However, this inequality can hardly be broken in modern tokamaks even at the plasma periphery.

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Declaration of interests

The authors report no conflicts of interest.

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