Visualization in Maple for Learning Differential Calculus

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Abstract

Visualization plays a crucial role in understanding concepts in Differential Calculus [Arcavi, 2003].

Maple is a comprehensive general-purpose computer algebra system. It is used primarily in educational and scientific research in mathematics and engineering. Maple offers many visualization tools. Using Maple, students can explore functions, analyze their derivatives, and draw tangent lines. In this paper, we consider two problems and their visualization in Maple by providing graphical representations of a function and its tangent.

Keywords: Differential Calculus; Derivative; Tangent line equation; Cubic function; Maple.

1 Find the Function Equation

Cubic functions are useful tools in learning outcomes of Differential Calculus [Alt, 2014]. They illustrate behavior of a derivative of a function, critical points, concavity, and inflection points. Since a cubic function has both a local maximum and a local minimum, students can explore key concepts such as the first and second derivative tests. Cubic functions are widely used in physics, engineering, and economics to model real-world problems involving rates of change.

1.1 Problem 1

A cubic function has a maximum point $H(0,3)$. The equation of the tangent line at the point $P(1, y_p)$ is $3x + y = 4$. Find the equation of the function.

1.2 Solution to Problem 1

A cubic function is a polynomial function of degree 3 and is of the form $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers and $a \neq 0$. Substituting $x = 0$ into $f(x)$, we get $d = 3$.

Since $H(0, 3)$ is a maximum, the derivative $f'(x) = 3ax^2 + 2bx + c$ must be equal to zero at $x = 0$. Setting $f'(0) = 0$, we get $c = 0$.

Since the function passes through the point $P(1, y_p)$, we set $f(1) = y_p$ which must satisfy the equation $a + b + 3 = 1$.

Also, since the derivative at $x = 1$ must be equal to the slope of the tangent, we get the condition $3a + 2b = -3$. Now we write the system of equations:

$$\begin{cases} a + b + 3 = 1, \\ 3a + 2b = -3. \end{cases}$$

Solving for a and b, we get $a = 1, \quad b = -3$. Thus, the function equation is $f(x) = x^3 - 3x^2 + 3$. The final visualization of the problem is presented in Fig.1.

```
with(plots);
f := x^3 - 3*x^2 + 3;
t := 4 - 3*x;
ft := plot([f, t], x = -3 .. 4, y = -7 .. 7,
color = [blue, magenta]);
p_PH := plot([[1, 1], [0, 3]], font = [TIMES, BOLD, 20],
style = point, color = "DarkBlue",
symbolsize = 20, symbol = solidcircle);
t_PH := textplot([[1, 1, "P"], [0, 3, "H"]], color = "DarkBlue",
    'align' = {'above', 'right'}, 'font' = [TIMES, BOLD, 20]);
plots[display]([ft, p_PH, t_PH]);
```

1.3 Problem 2

The cubic function $f : \mathbb{R} \to \mathbb{R}$, given by the equation $f(x) = x^3 + ax^2 + bx + c$ passes through the points $P(-2, -1)$ and $Q(1, 2)$. The tangent to the curve f at the point P has a slope of $k = 4$. Find the function equation of f.

1.4 Solution to Problem 2

Since the point $P(-2, -1)$ lies on the curve $f(x) = x^3 + ax^2 + bx + c$ we get the equation $4a - 2b + c = 7$. Similarly, since the point $Q(1, 2)$ lies on the curve $f(x) = x^3 + ax^2 + bx + c$ we obtain the equation $a + b + c = 1$.

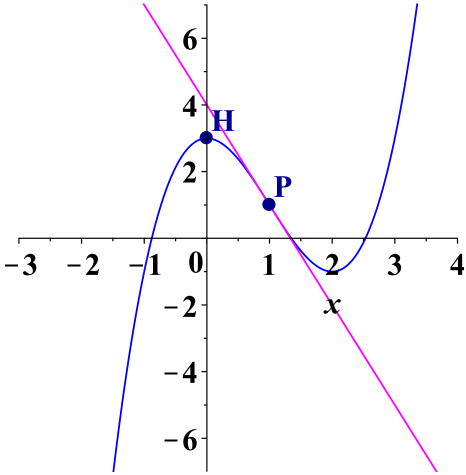

Figure 1: Visualization of Problem 1.

The condition that the derivative of $f(x)$ is $f'(x) = 3x^2 + 2ax + b$ and the slope at the point $P(-2, -1)$ is 4 leads to the equation $-4a + b = -8$. Then, solving the system:

$$\begin{cases} 4a - 2b + c = 7, \\ a + b + c = 1, \\ -4a + b = -8, \end{cases}$$

we get the values $a = 2, b = 0, c = -1$. Thus the function equation is: $f(x) = x^3 + 2x^2 - 1$.

1.5 Visualization to Problem 2

To visualize problem 2, (Fig.2), we use the following Maple commands:

```
with(Student[Calculus1]);
f := x^3 + 2*x^2 - 1;
tf := Tangent(f, x = -2);
with(plots);
ft := plot([f, tf], x = -3 .. 2,
    y = -6 .. 8, color = [blue, magenta]);
PQ := plot([[-2, -1], [1, 2]],
    color = "DarkGreen",
    font = [TIMES, BOLD, 10],
```

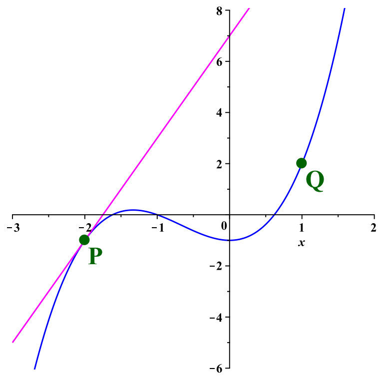

Figure 2: Visualization of Problem 2.

```
    style = point,
    symbolsize = 20,
    symbol = solidcircle);
sP := textplot([[-2, -1, "P"],
    [1, 2, "Q"]],
    'font' = [TIMES, BOLD, 20],
    style = point,
    color = "DarkGreen",
    'align' = {'below', 'right'});
plots[display]([ft, PQ, sP]);
```

2 Conclusion

Visualization is essential for grasping concepts of Differential Calculus. Maple allows students to visualize 2-D and 3-D plots, and create beautiful and informative animations. The purpose of the paper is to get students involved in learning mathematics including graphical representation using Maple tools.

References

Arcavi A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, vol. 52, pp. 215–241.

Alt A. (2014). A Laboratory on Cubic Polynomials. *Crux mathematicorum*, vol. 40, no. 5, pp. 207–210.