STRONG LAW OF LARGE NUMBERS FOR PAIRWISE INDEPENDENT IDENTICALLY DISTRIBUTED RANDOM VARIABLES

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Consider the case of a two-dimensional $\{X_{i,j}\}_{i,j\in\mathbb{N}}$ sequence of pairwise independent and identically distributed random variables and let's investigate the strong law of large numbers employing multi-indexed sums.

If $\{X_{i,j}\}_{i,j\in\mathbb{N}}$ are independent, then the law of large numbers is known for $a_{n,m} = nm$. If $\{X_{i,j}\}_{i,j\in\mathbb{N}}$ are pairwise independent, then the law of large numbers is proven [1, theorem 2]. We use another normalization in the law of large numbers [2, Corollary 9.11].

Theorem 1. *Assume $\{X_{i,j}\}_{i,j\in\mathbb{N}}$ is a two-dimensional sequence of pairwise independent identically distributed random variables, r is a number in $(1,2]$. Then one has*

$$\left(\mathbb{E}|X|^{\frac{2}{r}} < \infty\right) => \left(\lim(\max)_{n,m\to\infty} \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(X_{i,j} - \mathbb{E}X_{i,j})}{(m+n)^r} = 0, \quad a.s.\right)$$

The proof of this theorem combines methods from both [1] and [3]. Moreover, the statement of the theorem can be generalized in a similar fashion to [2, Corollary 9.11] i.e. not two-dimensional sequences, but for $d-$dimensional sequences.

References

[1] N. Etemadi, *An elementary proof of the strong law of large numbers*, Z. Wahrsch. Verw. Gebiete, 55(1) (1981), 119–122.

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