Asymptotic Methods in Optimization of Inventory Business Processes

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***Abstract*—The use of inventory management models enables the management of the company to optimize logistics business processes and to reduce fixed and variable costs of production, order and sales. The study proposes a multi-nomenclature model for optimizing inventory business processes using asymptotic methods. An easy-to-use analytical formula for determining optimal order interval, when order and inventory holding costs meet insufficient cyclical changes, is obtained. Testing of the developed model of optimization of inventory business processes was carried out on the example of the enterprise operating in the HoReCa segment of Zaporizhzhia the regional market. The proposed multi-nomenclature inventory management model enables the company management to apply situational management of these processes, modeling the changes caused by increased product storage costs and order fulfillment. Keywords: logistic model, small parameter, asymptotic methods, total logistical costs, multi- nomenclature order.**

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I. Statement of the Problem in General Way and Its Relation to Important Scientific or Practical Tasks

Intensification of competition in the markets of consumer and industrial goods forces domestic manufacturers to search constantly for ways to maintain their own competitiveness as well as competitiveness of the supplied goods.

Inventories are formed at all stages and links of production process, distribution and consumption, so inventory problems are one of the most important in cost management of domestic enterprises.

Modern inventory models allow us to solve problems of business processes` optimization of transport, production, information, financial and other systems` planning.

II. Analysis of Recent Studies and Publications That Have Initiated Problem Solution, Which the Author Refers On

R.R. Larina, O.Yu. Lukianova [1], G.V. Mel`nyk, V.I. Skitska [2], A.O. Kolomitseva, V.S. Yakovenko [3] and other scholars dedicated their research to the issue of logistics` business processes` modeling at an enterprise. S.V. Ocheretenko [4], P. Tripaty, S. Shukla [5] built inventory`s mathematical models; N.O. Markova, I.A. Kiosseva [6], O.V. Posilkina, Yu.E. Novitskoy, Yu.M. Penkin, O.Yu. Gorbunov [7], R. Iassa, S. Ikatrinasari [8] and other researchers studied peculiarities of multi-nomenclature systems of inventory. S. Jaggi, S. Goel, and M. Mittal [9] proposed the specifics of inventory management system taking into account deficit for retail trade with allowable late payment. In the context of uncertainty caused, for example, by inflation, unexpected ups and downs in the economy, scientists like A. Einan and D. Kropp [10], S. Wang [11], A. Yousefli and M. Gazanfari [12] built appropriate inventory models that take these indicators into account. The proposed optimization methods solve some of the inventory management problems; however, real management often requires building of analytical models that provide solutions in the form of easy-to-use formulas.

III. Article Objectives

Objective of the study is to develop a multi-nomenclature model of inventory business processes using asymptotic methods and to obtain easy-to-use analytical formulas to determine model parameters when ordering and inventory holding costs meet insufficient cyclical changes.

IV. The Main Material of the Research with Justification of Findings

Application of inventory models enables company management to reduce fixed and variable production costs, ordering and sales. All costs associated with a multi-nomenclature ordering of resources or goods from a supplier can be represented by two components:

$$C = C_0 + \sum_{i=1}^{k} C_i = \sum_{i=0}^{k} C_i \cdot \quad (1)$$

C_o – costs incurred during transportation,

C_i – costs that depend on the amount of transactions performed when forming a specific order. Therefore, the costs of ordering k of product lines from one supplier can be represented as (1).

If to specify total commodity consumption as i during a certain period as S_i, unit storage costs as Cx_i, cycle of supply as T, period as D, and if synchronistic supply k of product lines take place, then total costs are (2) [13].

$$C_{\sum} = \frac{D}{T}\sum_{i=0}^{k} C_i + \frac{T}{2D}\sum_{i=1}^{k} S_i\, Cx_i \rightarrow \min . \quad (2)$$

If model parameters are fixed (2), the optimal value of multi- nomenclature supply cycle T_{opt} is defined as follows [13]:

$$T_{opt} = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}}} . \quad (3)$$

Assumption about the model`s fixed parameters reduces possibility and effectiveness of its practical application. Let us apply asymptotic methods that allow, without violating this assumption, to perturb the model parameters. One of these parameters is ordering costs, which are fixed in the inventory optimization model. In practice, this parameter may increase cyclically due to inflationary processes, consumer and producer expectations, the ratchet effect, etc. Let us assume that during a fixed period ordering costs, namely their transport component, rise manyfold by l%. Then in n periods it is $C_0 \cdot \left(1 + \frac{l\%}{100\%}\right)^n$. Specifying addends` perturbation as a small parameter $\frac{l\%}{100\%}$, this correlation can be presented as: $C_0 \cdot (1+\varepsilon)^n$, when ε<<1.

In practice, ordering costs and inventory holding costs increase as a result of higher electricity and utilities prices. Assume that holding cost increases with each period by j%. Analogously, considering value $\beta = \frac{j\%}{100\%}$ (β<<1) as a small parameter, we obtain correlation of inventory holding costs as $C_{xi}(1+\beta)^m$.

To increase the proposed model`s efficiency in manufacturing enterprises` activity, it is advisable to take into account different combinations of parameter values n i m, ε i β. Multiplicity m, which characterizes frequency of change in inventory holding costs is relatively less than the multiplicity n, which characterizes changes in order fulfillment. These terms can be taken into consideration in the model, for example when m=[n/2], m=[n/3] etc., where [ ] – a quotient.

Let T^*_{opt} be an asymptotic decomposition by two small parameters ε i β:

$$T^*_{opt} = T_0 + T_1\varepsilon + T_2\beta + T_3\varepsilon^2 + T_4\varepsilon\beta + T_5\beta^2 + \ldots, \quad (4)$$

where ε i β – parameters of perturbation.

Let us substitute perturbed values of the transport component of order fulfillment costs and storing inventory costs, as well as decomposition T^*_{opt} by degrees of small parameters ε and β to formula (3), not taking into account terms of order ε^3, β^3, $\varepsilon^2\beta$, $\varepsilon\beta^2$ and higher:

$$\left(T_0 + T_1\varepsilon + T_2\beta + T_3\varepsilon^2 + T_4\varepsilon\beta + T_5\beta^2\right)^2 = D^2 \frac{\left(2C_0(1+\varepsilon)^n + 2\sum_{i=1}^{k} C_i\right)}{(1+\beta)^m \sum_{i=1}^{k} S_i C_{xi}} . \quad (5)$$

Decomposition of function $(1+\varepsilon)^n$ i $(1+\beta)^{-m}$ to Taylor's series expansion and neglecting members of higher orders, after raising square of both equality parts (5), we obtain:

$$T_0^2 + 2T_0T_1\varepsilon + 2T_0T_2\beta + \left(T_1^2 + 2T_0T_3\right)\varepsilon^2 + \\ + \left(2T_0T_4 + 2T_1T_2\right)\varepsilon\beta + \left(T_2^2 + 2T_0T_5\right)\beta^2 = \\ = D^2 \frac{\left(2C_0\left(1 + n\varepsilon + \frac{n(n-1)}{2}\varepsilon^2\right) + 2\sum_{i=1}^{k} C_i\right)\cdot\left(1 - m\beta + \frac{m(m+1)}{2}\beta^2\right)}{\sum_{i=1}^{k} S_i C_{xi}} \quad (6)$$

Equating coefficients with similar parameters` degrees ε and β, we obtain system of six equations to identify unknowns T_i ($i=0,\ldots,5$):

$$\varepsilon^0: \quad T_0^2 = \frac{D^2}{\sum_{i=1}^{k} S_i C_{xi}}\left(2C_o + 2\sum_{i=1}^{k} C_i\right) = D^2 \frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i C_{xi}}, \quad (7)$$

$$\varepsilon^1: \quad 2T_0T_1 = D^2 \frac{2C_0 n}{\sum_{i=1}^{k} S_i C_{xi}}, \quad (8)$$

$$\beta^1 \quad 2T_0T_2 = D^2 \frac{-m\left(2C_0 + 2\sum_{i=1}^{k} C_i\right)}{\sum_{i=1}^{k} S_i C_{xi}} = D^2 \frac{-2m\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i C_{xi}}, \quad (9)$$

$$\varepsilon^2: \quad T_1^2 + 2T_0T_3 = D^2 \cdot \frac{C_0 n(n-1)}{\sum_{i=1}^{k} S_i \cdot C_{xi}}, \quad (10)$$

$$\varepsilon\beta: \quad 2T_0T_4 + 2T_1T_2 = D^2 \cdot \frac{-2nmC_0}{\sum_{i=1}^{k} S_i \cdot C_{xi}}, \quad (11)$$

$$\beta^2: \quad T_2^2 + 2T_0T_5 = D^2 \frac{\frac{m(m+1)}{2}\left(2C_0 + 2\sum_{i=1}^{k} C_i\right)}{\sum_{i=1}^{k} S_i C_{xi}} = D^2 \frac{m(m+1)\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i C_{xi}} . \quad (12)$$

Solving equations (7)-(12), we obtain:

$$T_0 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}}} \qquad T_1 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}}} \cdot \frac{C_0 n}{2\sum_{i=0}^{k} C_i},$$

$$T_2 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}} \cdot \left(\frac{-m}{2}\right)},$$

$$T_3 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}} \cdot \left(\frac{C_0 n(n-1)}{4\sum_{i=0}^{k} C_i} - \frac{C_0^2 n^2}{8\left(\sum_{i=0}^{k} C_i\right)^2}\right)}, \qquad (13)$$

$$T_4 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}} \cdot \frac{C_0(-mn)}{4\sum_{i=0}^{k} C_i}},$$

$$T_5 = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}} \cdot \frac{m(m+2)}{8}}.$$

Asymptotic representation of the optimum value of multi- nomenclature supply cycle T^*_{opt} will be presented as (14) or (15):

$$T^*_{opt} = D\sqrt{\frac{2\sum_{i=0}^{k} C_i}{\sum_{i=1}^{k} S_i\, C_{xi}}} \cdot \left(1 + \frac{C_0 n}{2\sum_{i=0}^{k} C_i}\varepsilon - \frac{m}{2}\beta + \left(\frac{C_0 n(n-1)}{4\sum_{i=0}^{k} C_i} - \frac{C_0^2 n^2}{8\left(\sum_{i=0}^{k} C_i\right)^2}\right)\cdot\varepsilon^2 - \frac{C_0 nm}{2\sum_{i=0}^{k} C_i}\varepsilon\beta + \frac{m(m+2)}{8}\beta^2\right), \qquad (14)$$

$$T^*_{opt} = T_{opt} \cdot \left(1 + \frac{C_0 n}{2\sum_{i=0}^{k} C_i}\varepsilon - \frac{m}{2}\beta + \left(\frac{C_0 n(n-1)}{4\sum_{i=0}^{k} C_i} - \frac{C_0^2 n^2}{8\left(\sum_{i=0}^{k} C_i\right)^2}\right)\cdot\varepsilon^2 - \frac{C_0 nm}{4\sum_{i=0}^{k} C_i}\varepsilon\beta + \frac{m(m+2)}{8}\beta^2\right) \qquad (15)$$

One can see, that when $\varepsilon=0$ and $\beta=0$ we obtain marginal transformation to "not perturbed" solution of this kind (3).Let us test the built multi-nomenclature model of optimization of inventory business processes under «perturbed» parameters on the example of the enterprise operating in HoReCa market of Zaporizhzhia.

Output data and calculation of "not perturbed" system parameters are given in Table I.

TABLE I. OUTPUT DATA AND CALCULATION RESULTS OF MULTI-NOMENCLATURE DELIVERY OF HORECA MARKET (ZAPORIZHZHIA)

Type of commodities	**Annual demand S_i, un.**	**Holding costs, Cx_i, currency unit**	**Order fulfillment costs, currency unit**		**Optimal value of supply cycle T_{opt}, full days**	**Order quantity, $q_i = T_{opt}\frac{S_i}{D}$, units**
			C_0	**C_i**		
Tea (10 sachets)	700	1,0	50	2	90	173
Whole bean coffee (1 kg)	240	4,5	50	2		59
Sugar (800 sticks)	85	0,5	50	2		21

«Perturbed» values of a multi-nomenclature delivery T^*_{opt}, corresponding output data of Table I, as well as its relation with «not perturbed» parameter (T^*_{opt}/T_{opt}) for different values of ε, n and $\beta=0,1$ (parameter that characterizes holding cost increase) are presented in Tables II and III. There is comparative analysis of optimal cycle`s values of multi-nomenclature delivery of HoReCa (Zaporizhzhia) when $m = \left[\frac{n}{6}\right]$ in Table II.

TABLE II. COMPARATIVE ANALYSIS OF OPTIMAL CYCLE`S VALUES OF MULTI-NOMENCLATURE DELIVERY, Β=0,1, $m = \left[\frac{n}{6}\right]$*

Multiplicity		**ε= 0,01**		**ε=0,015**		**ε=0,02**	
n	**m**	T^*_{opt}/T_{opt}	T^*_{opt}	**n**	**m**	T^*_{opt}/T_{opt}	T^*_{opt}
0	0	1,0000	90,00	1,0000	90,00	1,0000	90,00
1	0	1,0045	90,40	1,0067	90,60	1,0089	90,80
2	0	1,0089	90,80	1,0134	91,21	1,0179	91,61
3	0	1,0134	91,21	1,0202	91,82	1,0270	92,43
4	0	1,0180	91,62	1,0270	92,43	1,0361	93,25
5	0	1,0225	92,03	1,0339	93,05	1,0454	94,09
6	1	0,9795	88,16	0,9926	89,34	1,0059	90,53
7	1	0,9839	88,55	0,9993	89,94	1,0149	91,34
8	1	0,9883	88,95	1,0060	90,54	1,0241	92,17
9	1	0,9927	89,34	1,0128	91,15	1,0333	93,00
10	1	0,9972	89,75	1,0196	91,77	1,0426	93,84
11	1	1,0017	90,15	1,0265	92,39	1,0521	94,68
12	2	0,9597	86,38	0,9857	88,72	1,0125	91,12

*Note: Authors own product

Based on Table II data, it can be concluded that increased multiplicity of periods (n and m) of higher ordering and holding costs causes ordering period`s fluctuations.

For example, gradual ramp up of order fulfillment costs` transport component by 1% (ε = 0.01) during the first 5 periods and unchanged holding costs, entail the raise of order interval from 90 to 92 days. Even a one-time increase in holding costs (n = 6 and m = 1) leads to decrease of the studied parameter from 92 to 88 days, or about 4%. Further changes tend to recur.

Optimal periods of multi-nomenclature delivery of HoReCa segment in Zaporizhzhia when $m = \left[\frac{n}{3}\right]$ are calculated in Table III.

TABLE III. CALCULATION OF OPTIMAL CYCLE`S VALUES OF MULTI-NOMENCLATURE DELIVERY OF HORECA (ZAPORIZHZHIA), B=0,1, $m = \left[\frac{n}{3}\right]$*

Multiplicity		*ε= 0,01*		*ε=0,015*		*ε=0,02*	
N	*m*	T^*_{opt}/T_{opt}	T^*_{opt}	T^*_{opt}/T_{opt}	T^*_{opt}	T^*_{opt}/T_{opt}	T^*_{opt}
0	0	1,0000	90,00	1,0000	90,00	1,0000	90,00
1	0	1,0045	90,40	1,0067	90,60	1,0089	90,80
2	0	1,0089	90,80	1,0134	91,21	1,0179	91,61
3	1	0,9665	86,99	0,9729	87,56	0,9794	88,14
4	1	0,9708	87,37	0,9794	88,15	0,9881	88,93
5	1	0,9752	87,76	0,9860	88,74	0,9969	89,73
6	2	0,9344	84,10	0,9469	85,22	0,9595	86,35
7	2	0,9386	84,47	0,9532	85,79	0,9680	87,12
8	2	0,9428	84,85	0,9596	86,36	0,9767	87,91
9	3	0,9037	81,33	0,9218	82,96	0,9403	84,62
10	3	0,9077	81,69	0,9279	83,52	0,9487	85,38
11	3	0,9117	82,06	0,9342	84,08	0,9572	86,15
12	4	0,8744	78,69	0,8977	80,79	0,9218	82,96

* Note: Authors own product

Calculations presented in Table III prove that growing multiplicity of ramping up ordering and holding costs (*n* and *m*) causes periods fluctuations of an optimal order. For example, with a threefold increase in holding costs (*m=3*) when *ε= 0,02, β=0,1,* and when *n=9* the period of ordering shortens from 90 to 85 full days, or 6%, which is quite significant. The following trend is repeated.

Let us analyze dependence of the order volume dynamics of one product category from HoReCa delivery (Zaporizhzhia), precisely tea (Figure 1).

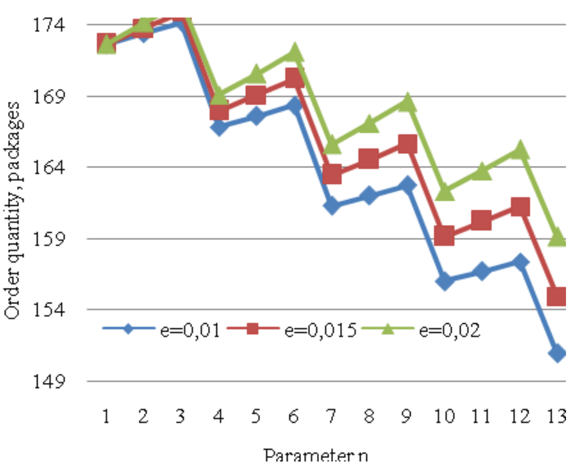

Fig. 1. Correlation of tea delivery`s order quantity change to parameters values *ε* and *n**.

* Note: Authors` calculations

As a result of calculations we found out that increase of parameter *n*, which characterizes frequency of rising transportation costs to value 6 with a simultaneous rise of parameter *m* (multiplicity of holding costs growth) to 2 leads to decrease in optimum order volume by 12 packages or almost 7% when ε = 0.01; by 10 packages or 6% when ε = 0,15; by 7 packages or almost 4.1% when ε = 0.02.

Figure 2 illustrates correlation of company`s total costs, operating in HoReCa segment in Zaporizhzhia from multiplicity of order and holding costs growth (*n* and $m = \left[\frac{n}{3}\right]$).

Holding costs growth causes a sinuous total costs` surge. A twofold increase in holding costs entails total costs growth approximately by 13% when *n* = 6.

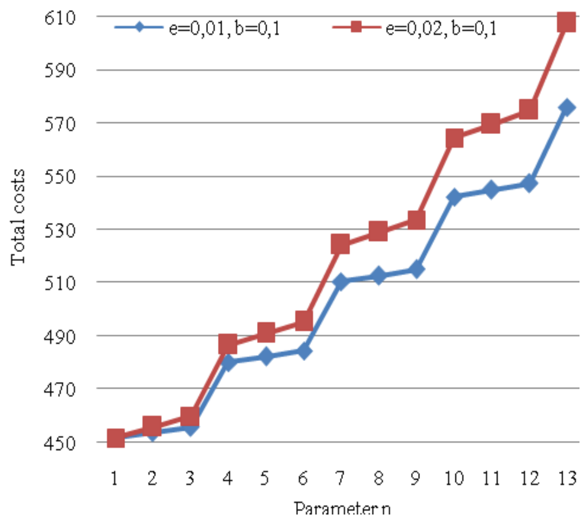

Fig. 2. Correlation of customer`s total costs` change to parameters values *ε* and *β**

* Note: Authors` calculations

V. CONCLUSIONS

In this paper, we use asymptotic approach to modify the model for determining optimal order interval. It is based on perturbation methods that allow to solve problems, varying system parameters. This eliminates constraints imposed on the basic Wilson formula for fixing input parameters.

The asymptotic formula handy for companies` management has been obtained for the situation when order fulfillment and holding costs cyclically increased. This formula allows to determine the optimum period between orders for multi- nomenclature delivery.

Multi-nomenclature model of optimization of inventory business processes was tested on the example of the enterprise operating in HoReCa segment of Zaporizhzhia regional market. The results indicate that increased multiplicity of ordering and holding costs growth leads to periods` fluctuation of an optimal order.

Thus, the developed multi-nomenclature inventory model will allow company`s management to optimize its business processes of order logistics, as well as apply situational management of these processes, modeling changes for businesses caused by holding and order fulfillment costs growth.

Prospects for further research are related to the application of asymptotic approach to inventory management models in case of system input variables. In particular, they can be used to model the situation associated with price change for resources which depends on order volume.

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