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Investment risks and their measurement

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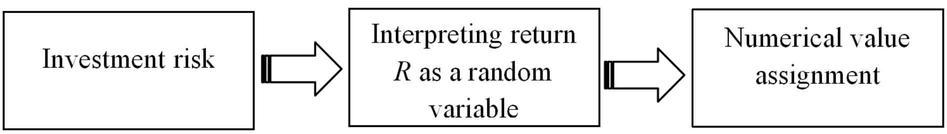
Abstract

The article is devoted to analysis of investment risks and their measurement. Three approaches for risk measurement are examined. These approaches have been applied to risk estimation of basic cryptocurrencies. Statistical assessment of basic risk measures from each approach was accomplished. The investigation shows that cryptocurrencies have completely distinctive characteristics of risk-return corresponding. It distinguishes cryptocurrencies from traditional investment assets and from new investment opportunities. The results are important for investment and risk management purposes.

1 Introduction

Risk takes a central place in the framework of investment decision making. The relationship between return and risk is in the core of modern investment thinking. As a rule, higher return should be associated with higher risk. The opposite correspondence is also true: higher risk should be covered by additional return (risk premium). Consequently, it is very logically to analyze risk-return correspondence before investment decision.

A source anchor of construction of such correspondence is risk measurement. Risk measurement is a procedure of assigning some numerical value to risk. This procedure can be formalized for investment risk by following scheme:



Procedure of risk measurement

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So, risk measurement supposes to introduce some mapping ρ which each random variable R (representing return of investment asset) assigned nonnegative number $\rho(R) \in [0; +\infty]$. Let us consider this procedure in details. The return of investment over a period of time $[t; t+1]$ will be expressed through the formula:

$$R_{t,t+1} = \frac{P_{t+1} - P_t}{P_t},$$

where P_t and P_{t+1} are prices of an investment asset at times t and $t+1$, respectively. $R_{t,t+1}$ will be a random variable, because the future price P_{t+1} is unknown. Thereafter R, which reflects return over the time, is also a random variable.

Assigning a numerical value for risk is complicated because various approaches for presentation of mapping ρ exist. Three conceptual approaches are the most significant ones:

- Risk measurement is based on reflecting the variability of return and income.
- Risk measurement is focused on losses in negative situation.
- Risk measurement associates with sensitivity of return to some factors. Measurement is focused on response level.

Each approach incorporates some important characteristics of multifaceted notion of risk and has a number of indicators. In general, there are several dozen of risk measures, which represent one or another aspect of risk (example is presented in Szego (2004)). An attempt of understanding the essence of properties which should be represented in risk measure was formulated in Artzner et al. (1999). The authors created a notion of a coherent risk measure. Risk measure ρ is coherent if satisfies the following properties (axioms):

Axiom 1 Sub-additivity. For all R_1 and R_2 we have

$$\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$$

Axiom 2 Positive homogeneity. For all R and for all $\lambda \geq 0$ we have

$$\rho(\lambda R) = \lambda \rho(R)$$

Axiom 3 Monotonicity. If $R_1 \geq R_2$ then $\rho(R_1) \leq \rho(R_2)$

Axiom 4 Translation invariance. For all R and for all $\alpha \geq 0$ we have

$$\rho(R + \alpha) = \rho(R) - \alpha$$

Each of these axioms formalizes some essential investment risk property. Thus Axiom 1 presents diversification effect. Axiom 2 describes linear increase of risk if some investment position is linearly increased. Axiom 3 presents a natural property: if returns for one investment are always higher than returns for other investment, then risk of the first investment is lower. Axiom 4 formalizes adding to investment a free-risk asset.

Examples of coherent risk measures are Conditional Value-at-Risk (considered below, see Rockafellar and Uryasev (2000)) and the Fischer measure (see Fischer (2003)).

It is necessary to note that presented approach for coherency is not unique. Approaches of coherency are considered in Kaminskyi (2006).

Below we consider applications of three approaches of risk measurement to cryptocurrencies. Cryptocurrencies are one of the alternative investment assets which demonstrated high developing since last years. The investment problems of cryptocurrencies are discussed in Lee, Guo and Wang (2018), Chan et al. (2017), Gangwal (2018) and Trimborn, Mingyang and Härdle (2017),

We have chosen for analysis cryptocurrencies with capitalization higher than 1 billion USD. They are:

Table 1. A list of cryptocurrencies chosen for analysis

Cryptocurrency	Ticker tape	Start day of trading	Capitalisation on 17.08.2018	Share of total market capitalization
Bitcoin	BTC	18.07.2010	$111.23B	52.1%
Ethereum	ETH	10.03.2016	$30.27B	14.2%
Ripple	XRP	22.01.2015	$12.28B	5.8%
Bitcoin Cash	BCH	03.08.2017	$9.31B	4.4%
EOS	EOS	02.07.2017	$4.34B	2.0%
Stellar Lumens	XLM	22.02.2017	$4.20B	2.0%
Litecoin	LTC	24.08.2016	$3.31B	1.6%
Tether	USDT	14.04.2017	$2.68B	1.3%
Cardano	ADA	31.12.2017	$2.59B	1.2%
Monero	XMR	30.01.2015	$1.55B	0.7%
Ethereum Classic	ETC	28.07.2016	$1.42B	0.7%
TRON	TRX	14.11.2017	$1.39B	0.7%
IOTA	MIOTA	14.06.2017	$1.34B	0.6%
Dash	DASH	04.03.2017	$1.29B	0.6%
NEO	NEO	08.09.2017	$1.14B	0.5%

Source: Data sources used for investigation were https://investing.com and http://thecrix.de

2 The variability approach for risk measurement

The variability approach is focused on dispersion or deviation from an expected outcome. The most simple risk measure is a range which equals to difference between maximum and minimum possible values:

$$L(R) = \max_{[0,T]} R(t) - \min_{[0,T]} R(t).$$

This risk indicator is important for the investor from the point of view of receiving a general picture about future possibilities (it is assumed that future distribution will be the same as historical distribution). The shortcoming of this risk indicator is that maximum and minimum prices were on peak and crisis times. These may be rare events and not relevant for periods of stability. Consequently, it is more efficient to use inter-quartile range:

$$Q(R) = Q_{75\%}(R(t)) - Q_{25\%}(R(t))$$

Of course, the most famous risk measure used in this approach is standard deviation which characterizes deviation from the expected value of R:

$$\sigma(R) = \sqrt{\int_{-\infty}^{+\infty} (R - E(R))^2 \,\mathrm{dF}(R)}$$

Expected value of R is defined as

$$E(R) = \int_{-\infty}^{+\infty} R dF(R)$$

where F is the distribution function of the random variable R.

If we use statistical estimations of R, then unbiased estimate of standard deviation is:

$$\hat{\sigma}(R) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T} (R(t) - E(R))^2}$$

Statistical estimation of $E(R)$ can be calculated by formula:

$$E(R) = \frac{1}{T}\sum_{t=1}^{T} R(t) \ \ (T\text{-number of periods}).$$

The other indicators which can be used for risk measurement in the frameworks of the variability approach are skewness and kurtosis. Skewness summarizes divergence from symmetry of distribution:

$$S(R) = E\left(\frac{R - E(R)}{\sigma(R)}\right)^3 = \frac{\mu_3(R)}{\sigma(R)^3}$$

where $\mu_3(R) = E(R - E(R))^3$.
The unbiased statistical estimation of skewness is:

$$\hat{S}(R)_{\text{Unbiased}} = \frac{\sqrt{(T-1)T}}{T-2} \cdot \frac{\frac{1}{T}\sum_{t=1}^{T}(R(t) - E(R))^3}{\left(\frac{1}{T}\sum_{t=1}^{T}(R(t) - E(R))^2\right)^{(3/2)}}.$$

Negative skewness indicates a long left tail of distribution, or the possibility of larger losses than profits. Positive skewness is a desirable characteristic for risk-averse investors. The motivation of that is based on the expected utility theory. Typically, the third derivative of the utility function of a risk-averse investor is positive (see e.g. Scott and Horvath (1980)) and this derivative is a multiplier for skewness in the Taylor expansion of expected utility.

The kurtosis (sometimes the term "excess kurtosis" is used) coefficient K can be considered as assessment of the size of distribution tails:

$$K(R) = E\left(\frac{R - E(R)}{\sigma(R)}\right)^4 - 3 = \frac{\mu_4(R)}{\sigma(R)^4} - 3$$

where $\mu_4(R) = E(R - E(R))^4$.

Kurtosis can be considered as measure of risk associated with heavy tails or outliers. Kurtosis greater than 0 indicates a fatter tail than the normal distribution has. Hence, this distribution may generate more extreme values which lead to potential catastrophic risks. The sample kurtosis is

$$\hat{K}(R) = \frac{\frac{1}{T}\sum_{t=1}^{T}(R(t) - E(R))^4}{\left(\frac{1}{T}\sum_{t=1}^{T}(R(t) - E(R))^2\right)^2} - 3$$

An unbiased estimator of the sample excess kurtosis is

$$\hat{K}(R)_{\text{Unbiased}} = \frac{(T-1)}{(T-2)(T-3)} \cdot \left((T+1)\hat{K}(R) + 6\right)$$

Results of statistical estimations for considered risk measures are presented at the Table 2.

Risk-return correspondence at the frameworks of classical consideration expected return and standard deviation (Markowitz (1959)) is presented on Figure 1.

Table 2. Statistical estimations of indicators from variability approach for risk measurement (daily return, time period: 01.01.2018 – 17.08.2018)

Crypto-currency	Expected return	Range	Inter-quartile	Standard deviation	Skew-ness	Kurt-osis
BTC	-0.22%	31.00%	3.20%	4.70%	-0.16	1.15
ETH	-0.23%	34.80%	3.70%	5.69%	-0.10	0.66
XRP	-0.58%	54.50%	4.60%	6.69%	0.14	2.43
BCH	-0.40%	60.60%	4.90%	7.05%	0.39	3.23
EOS	0.17%	64.10%	4.80%	8.91%	1.23	4.75
XLM	0.15%	86.80%	5.00%	8.66%	1.60	10.07
LTC	-0.41%	52.60%	3.90%	6.24%	0.94	4.82
USDT	-0.01%	4.70%	0.10%	0.44%	0.58	9.72
ADA	-0.54%	63.70%	4.60%	8.11%	1.19	4.57
XMR	-0.33%	45.80%	4.90%	6.56%	-0.07	1.18
ETC	-0.03%	53.20%	4.70%	7.16%	-0.06	1.59
TRX	0.31%	142.60%	4.80%	12.32%	3.86	31.50
MIOTA	-0.58%	52.70%	5.40%	7.36%	-0.06	0.63
DASH	-0.64%	37.20%	4.10%	5.79%	0.07	0.92
NEO	-0.16%	92.60%	5.60%	9.93%	1.70	9.11

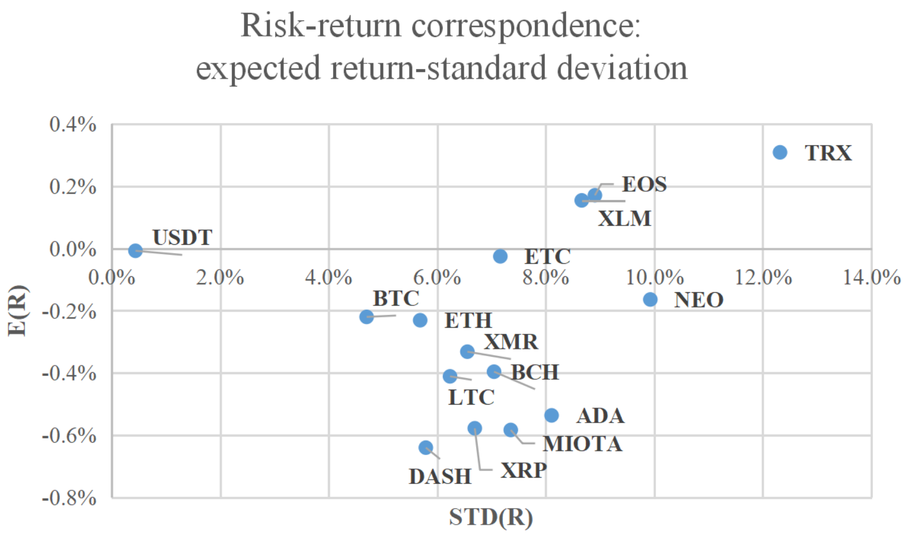

Figure 1. Correspondence "expected return – standard deviation"

So, Figure 1 illustrates an interesting property of risk-return correspondence for cryptocurrencies: a transparent dependency between risk and return is absent.

3 Risk measurement as losses in negative situation

This conceptual approach is based on considering different measures relating to the interpretation of "negative situation" for the investor. Among others, it is possible to mark out downside deviation risk measure. This measure focuses on the returns that below MAR (minimum acceptable return). MAR should be considered as a minimum threshold. Another risk measure at analysing frameworks is TUW (time under the water). This measure calculates how long does the investor wait to recover its money at the start of the drown down period. But, of course, the most popular in this group is the left-tail risk measures, such as Value-at-Risk (VaR) (Holton 2003). This risk measure presents a quantile corresponding to some level of safety (example 95%, 99% or 99.9%). The economic logic of VaR is based on risk covering. If, for example, VaR orients for 95%, then 5% biggest losses will throw off. VaR will cover maximum losses at the framework of 95% possibilities.

VaR is a very efficient measure for market risk. Moreover, it is a regulative risk measure in banking. But together with advantages this measure has shortcomings, too. First shortcoming raises from the fact that VaR is really only one point of probability distribution function (pdf). Behaviour of pdf left-side and right-side from VaR is out of consideration. Second gap of VaR is absence of coherency property. Coherency property of Value-at-Risk occurs only for elliptical class of distributions.

Risk measure Conditional Value-at-Risk (CVaR) is based on generalization of VaR. This is conditional mathematical expectation:

$$\mathrm{CVaR}(R) = E(R|R \leq \mathrm{VaR}(R))$$

The essence of VaR and CVaR is illustrated by picture at Figure 2.

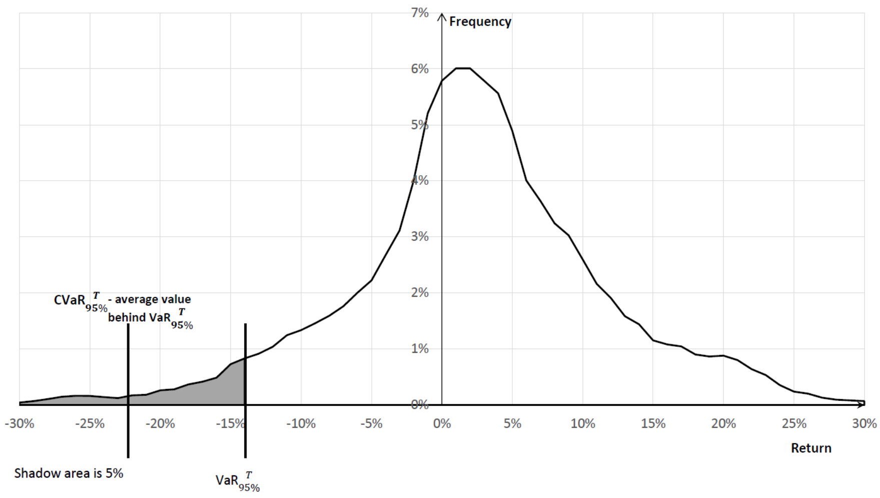

Figure 2. Essence of VaR and CVaR

Advantages of CVaR include coherency of this risk measure and more correct considering of possible losses.

Statistical estimations of VaR and CVaR for cryptocurrencies under consideration we present below in Table 3.

Table 3. Statistical estimations of VaR and CVaR (daily return, safety level – 95%; time period: 01.01.2018 – 17.08.2018)

Cryptocurrency	VaR	CVaR	Cryptocurrency	VaR	CVaR
BTC	-8.2%	-11.0%	ADA	-12.2%	-15.1%
ETH	-9.6%	-12.8%	XMR	-11.1%	-14.3%
XRP	-11.7%	-14.9%	ETC	-12.0%	-15.9%
BCH	-10.5%	-15.3%	TRX	-15.1%	-19.1%
EOS	-12.9%	-17.4%	MIOTA	-13.3%	-16.2%
XLM	-12.8%	-16.1%	DASH	-9.4%	-13.1%
LTC	-8.7%	-12.7%	NEO	-13.1%	-17.8%
USDT	-0.6%	-1.1%			

The ratio CVaR/VaR characterizes correspondence between "catastrophic" losses and maximal losses at the frameworks of 95% safety level. Our consideration shows that ratio belongs to interval [1.22; 1.81]. Such interval is relatively wide, so cryptocurrencies are quite different in behaviour of left pdf tails.

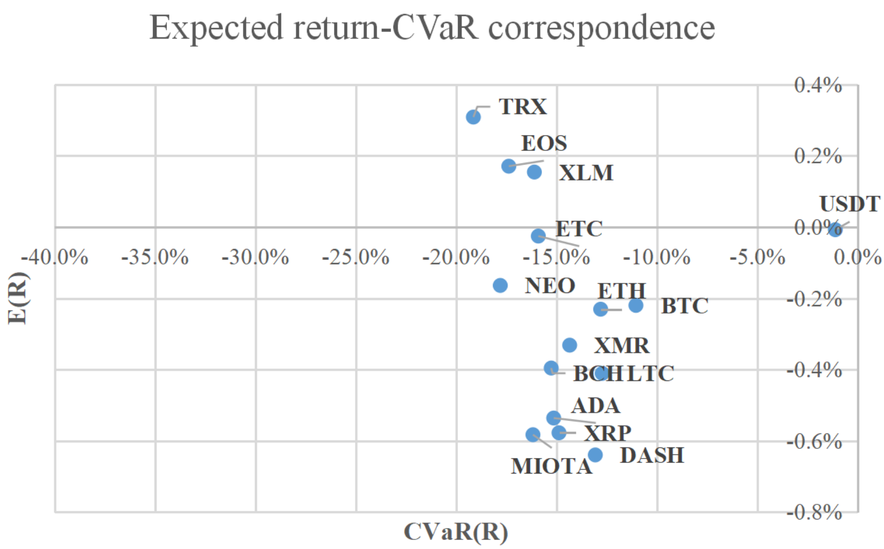

Figure 3. Correspondence "expected return – CVaR"

4 Risk measurement at the frameworks of sensitivity analysis

One of the most important approaches for investment risk measurement is based on sensitivity analysis. The importance of this approach is based on possibility to structure risk into systematic and nonsystematic risks. Systematic risk reflects impact of market changes to return of an investigated asset. Sensitivity analysis involves procedures for assessment of such impacts. Classical approach consists in using a linear regression model for return:

$$R_A = \alpha_A + \beta_A R_I + \varepsilon_A$$

where

- R_I indicates return of some market index (source of systematic risk);
- R_A is return of investment asset;
- β_A - coefficient of sensitivity (more precisely, this coefficient explains sensitivity numerically);
- α_A- coefficient of linear regression;
- ε_A is a random variable which indicate "own" – nonsystematic risk (not caused by the index).

One of the crucial suppositions in this model is independence between random variables R_I and ε_A. So, covariance between those random variables equals 0.

Risk structuring on systematic and nonsystematic risk can be obtained after applying operator of variance to formula for R_A:

$$\sigma^2(R_A) = \beta_A^2 \cdot \sigma^2(R_I) + \sigma^2(\varepsilon_A).$$

Ratios

$$\frac{\beta_A^2 \cdot \sigma^2(R_I)}{\beta_A^2 \cdot \sigma^2(R_I) + \sigma^2(\varepsilon_A)}$$

and

$$\frac{\sigma^2(\varepsilon_A)}{\beta_A^2 \cdot \sigma^2(R_I) + \sigma^2(\varepsilon_A)}$$

will be indicators of significance of systematic risk and nonsystematic risk correspondingly. Ratios are measured as percentages.

In our research we applied such approach to the index model which is based on the cryptocurrencies index CRIX (Trimborn and Härdle (2017)). The results – beta-coefficients to index CRIX are given in Table 4.

Table 4. Statistical estimation of beta-coefficient daily return, period: 01.01.2018–17.08.2018

Cryptocurrency	Beta coefficient	Cryptocurrency	Beta coefficient
BTC	0.1262	ADA	0.2382
ETH	0.1361	XMR	0.0544
XRP	0.1840	ETC	0.1214
BCH	0.1485	TRX	0.2518
EOS	0.1925	MIOTA	0.0495
XLM	0.1579	DASH	0.0753
LTC	0.0852	NEO	-0.0353
USDT	-0.0171		

Structure of the risk is presented at the Figure 4.

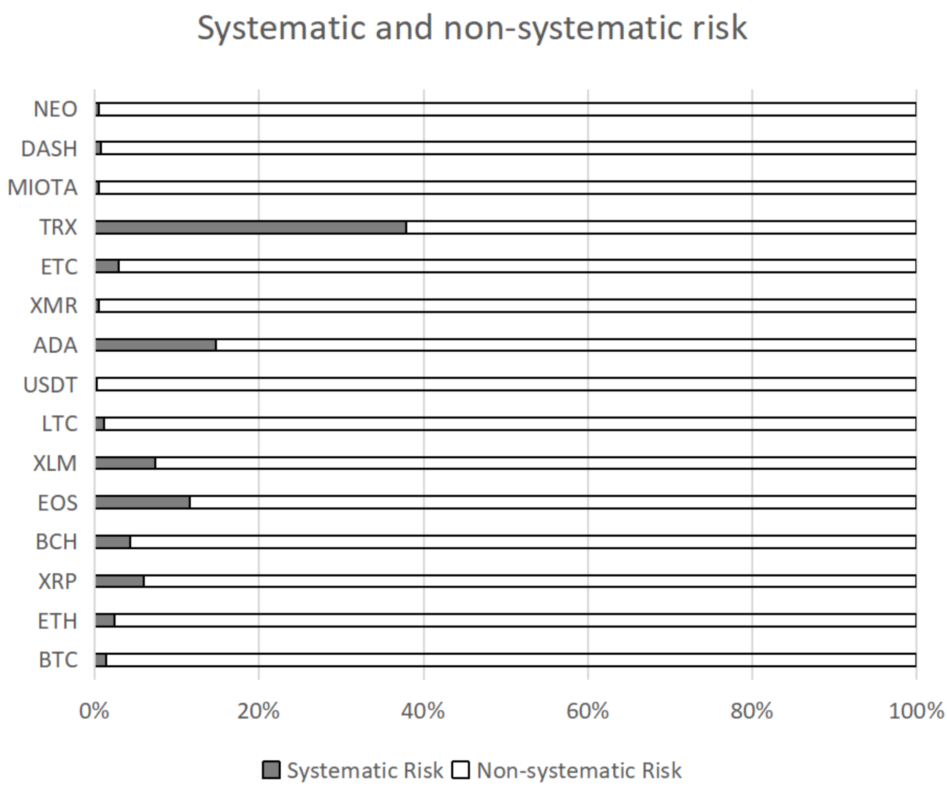

Figure 4. Correspondence between systematic and nonsystematic risks

Results show that nonsystematic risks are dominated.

5 Conclusions

The measurement of investment risk is multifaceted task which supposed to apply different approaches. Each approach points out specific features of risk.

The application of different approaches for the risk measurement of basic cryptocurrencies makes it possible to form some conclusions. First conclusion indicates a relatively high level of risk at the frameworks of volatility and significant outliers. Most cryptocurrencies demonstrate 5%–10% of standard deviation. The ratio of Range/Interquartile range is also relatively high. Kurtosis demonstrates high values. The risk measurements on the base of VaR and CVaR also indicate their values as high as ratio CVaR/VaR. All these results can be explained by significant outliers.

Second conclusion concerns exclusively high proportion of nonsystematic risk. Economically this can be explained by absence of meaningful factor which affects for all cryptocurrencies. This also revealed in low values of beta-coefficients in CRIX index model. On the other hand, such results may be raised from imperfection of index construction.

Third conclusion, maybe the most interesting, consists in fact that “classical” relationship between risk and return cannot be identified.

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