Matrix algorithms in commutative domains: the history of development in recent decades

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Abstract

We discuss the history of development of matrix algorithms in commutative domains, starting from the 1983 year.

Of course, the history of matrix algorithms over commutative domains begins in the nineteenth century. But in this short review we will only cover the last three decades. Today this theory is an independent chapter in modern computer algebra and is actively developing.

J. Dongarra at his talk at International Congress ICMS-2016 [1] put attansion on the difficult challenges. The task of managing calculations on a cluster with distributed memory for algorithms with sparse matrices is today one of the most difficult challenges.

The main achievement of modern matrix algorithms in commutative domain is the high efficiency of application in computing systems with distributed memory.

You can see four stages in the development of matrix algorithms in commutative domains. Each stage took about ten years.

I. Algorithms for solution of a system of linear equations of size n in an integral domain, which served as the basis for creating recursive algorithms

(1983) Forward and backward algorithm ($\sim n^3$) [2].
(1989) One pass algorithm ($\sim \frac{2}{3}n^3$) [3], [4].
(1995) Combined algoritm with upper left block of size r ($\sim \frac{7}{12}n^3$ for $r = \frac{n}{2}$) [5].

II. Recursive algorithms for solution of a system of linear equations and for adjoint matrix computation in an integral domain without permutations

(1997) Recursive algorithm for solution of a system of linear equations [6].
(2000) Adjoint matrix computation (with 6 levels) [7].
(2006) Adjoint matrix computation alternative algorithm (5 levels) [9].

III. Main recursive algorithms for sparse matrices

(2008) Computation of adjoint and inverse matrices and kernel [10].
(2010) Bruhat and LEU decompositions in the feilds [11].
(2012) Bruhat and LDU decompositions in the domains [12], [13].
(2015) Bruhat and LDU decompositions in the domains (alternative algorithm) [14].

IV. New achivements

(2013) It is proved that the LEU algorithm has the complexity $O(n^2r^{\beta-2})$ for matrices of rank r. [15].
(2017) It is proved that the LEU algorithm has the complexity $O(n^2s^{\beta-2})$ for quasiseparable matrix, if any it's submatrix which entirely below or above the main diagonal has small rank s [16].

The block-recursive matrix algorithms for sparse matrix require a special approachs to managing parallel programs. One approach to the cluster computations management is a scheme with one dispatcher. Another approach is a scheme with multidispatching, when each involved computing module has its own dispatch thread and several processing threads. It was developed in [17] - [20].

Acknowledgments

The work is partially supported by RFBR grant No 16-07-00420.

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