

MODERN NONLINEAR AND ASYMMETRIC ECONOMETRIC MODELING OF SOCIO-ECONOMIC PROCESSES

Today we observe the active search of new approaches that include the modern economic and mathematical tools to reflect adequately the complex nonlinear behavior and information asymmetry that characterize the socio-economic processes not only in Ukraine but also in other countries. In this regard given the great interest in studies of asymmetry in the social and labor sphere of national economy and foreign research experience, in terms of economic instability in Ukraine are important to conduct relevant analysis and modern econometric models that allow to identify the characteristics of nonlinear dynamics of basic macroeconomic socio-economic and labor market indicators and their asymmetric response to positive and negative shocks disturbing economic environment.

Analysis and study asymmetric and nonlinear behavior of labor market indicators for different countries were based on econometric studying of asymmetric nonlinear time series models. In the early work, W. Wecker (1981) first described the properties of asymmetric moving average processes and evaluated several asymmetric price indices of industrial products in the United States. Afterwards S. Elwood (1998) showed asymmetry of the innovations impact on the gross domestic product and the volume of industrial production by using of threshold asymmetric autoregressive models. G. Koutmos (1999) as well as R. Kumar and R. Dhankar (2010) received empirical evidence that the conditional volatility of income is asymmetric/ They showed that the negative disturbances and bad news had much stronger influence on volatility than positive. In the work of A. Diongue and D. Guégan (2007) were proposed a study of asymmetric seasonal time series behavior by means of seasonal APARCH hyperbolic model. L. Kilian and J. Vigfusson (2011) used a vector auto regression model for estimation of asymmetric impulse response function. The authors proved that the dynamics of output fluctuations that were occurred due the deviations from equilibrium had the different amplitudes in different directions. However, there are not enough researches of asymmetry dynamics of the labor force, the level of economic activity, employment, unemployment, labor productivity and other labor market indicators, particularly in Ukraine. We also need the modeling tools to investigate the asymmetric reaction on macroeconomic shocks and disturbances as well as the estimation of their size and direction.

Asymmetric Time Series Moving Average Models

Assuming symmetry responses on shocks of different signs the linear time series models such an autoregressive $AR(p)$ models:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + u_t, \quad (2)$$

moving average models $MA(q)$;

$$y_t = \beta_0 + u_t + \beta_1 u_{t-1} + \dots + \beta_q u_{t-q} \quad (3)$$

and mixed moving average autoregressive models $ARMA(p,q)$;

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + u_t + \beta_1 u_{t-1} + \dots + \beta_q u_{t-q}, \quad (4)$$

are known for univariate modeling of economic indicators behavior.

The scientists also used the nonlinear time series models which have the form (J. Nelson, J. Vanness, 1973)

$$y_t = \alpha_0 + \sum_{i=1}^{\infty} \alpha_i y_{t-i} + \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \alpha_{jk} y_{t-j} y_{t-k} + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} \alpha_{lms} y_{t-l} y_{t-m} y_{t-s} + u_t \quad (5)$$

and bilinear models (C. Granger, A. Anderson, 1978)

$$y_t = \alpha_0 + \sum_{i=1}^{\infty} \alpha_i y_{t-i} + \sum_{j=1}^{\infty} \beta_j u_{t-j} + \sum_{k=1}^{\infty} \gamma_k y_{t-k} u_{t-k} + u_t. \quad (6)$$

In each of these models y_t denotes the time series observations; u_t is a sequence of independent identically distributed random variables that are not directly observed; α_i , β_j and γ_k are the unknown model parameters. If the innovation sequences u_t is determined by the sequence of random variables with asymmetric density function, particularly in the case of lognormal distribution, the previous models will describe the behavior of economic time series with asymmetric marginal or conditional probability density.

However, the researchers reveal that the behavior of many economic variables has another type of asymmetry. The short-term fluctuations of economic indicators demonstrate various asymmetric responses to positive and negative disturbances that can't be described by using of models (2)-(6) with asymmetric innovations. As a result, the dynamics of asymmetric time series at different periods are characterized by different properties. They depend on the values of current and previous innovations that can be positive or negative.

For modeling of labor market indicators and aggregate output the choice of theoretical basis is usually based on different assumption about their stochastic nature and the assumption of symmetry. However, theoretical models of real business cycle argue that technological shocks primarily affect the variance of output but its level is described by a random walk model (L. Ljungqvist, T. Sargent, 2004). Many researchers emphasize that an important sources of changes in GDP variance are positive technological shocks. The theories do not accept the existence of negative technological shocks and insist that technological regression is rare. On the other hand, neo-Keynesian theory focuses on the demand shocks as a source of significant changes in the variance of output and price rigidities. They can explain the short-term deviations from the natural level of production and the impact of these shocks have symmetrical characters. Therefore, if the observed fluctuations in output and employment are the result of supply shocks and significant technological innovations, the average positive shocks will have a longer effect than negative. As a result, the persistence of influence of positive and negative disturbances on economic activity, unemployment, employment and productivity will be asymmetric.

For estimation of various shocks influence and measuring of their correlation with future values of the Ukrainian labor market indicators we use nonlinear threshold specifications which interpret the disturbances as unobservable components of time series. For taking into consideration the differences between the effects of positive and negative shocks, we include in time series model some threshold parameter.

Therefore, we examine the models that take into account several regimes of behavior. These regimes depend on the specific value of the variable-indicator that characterizes the value of past disturbances. In particular, if the disturbances are positive the fluctuations are followed by the first regime. For negative disturbances the second regime determines the alternative series behavior.

The threshold-disturbance moving average model of the first order (TDMA) has the form (W. Wecker, 1987):

$$y_t = u_t + \theta_+ u_{t-1}^+ + \theta_- u_{t-1}^-, \quad (7)$$

where u_t is a sequence of independent identically distributed random variables; $u_t^+ = \max \{u_t, 0\}$ – a sequence of positive innovations; $u_t^- = \min \{u_t, 0\}$ – a sequence of negative innovations; θ_+ and θ_- – unknown parameters of the model. If the both filters of asymmetric model coincide, the asymmetrical TDMA model (7.6) is reduced to a symmetric MA model:

$$y_t = u_t + \theta u_{t-1}. \quad (8)$$

Unlike the model (7.7) by which the expectation of sequence y_t is equal to zero, in the case of asymmetric MA(7.1) model (7.6) y_t has the expectation that is a function of the parameters θ_+ and θ_- and generally different from zero. In particular, the expectation is determined by the formula

$$\mu = \theta_+ \int_0^{\infty} u^+ \varphi(u^+) du^+ + \theta_- \int_{-\infty}^0 u^- \varphi(u^-) du^- = (\theta_+ - \theta_-) / (2\pi)^{1/2}. \quad (9)$$

The variation of asymmetric time series is

$$\gamma_0 = 1 + ((\theta_+)^2 - (\theta_-)^2) / 2 - \mu^2. \quad (10)$$

The first order auto covariation is

$$\gamma_1 = (\theta_+ - \theta_-) / 2 \quad (11)$$

and higher than one order auto covariations are zero. Since symmetric model (8) is a particular case of asymmetric patterns (7), the expectation and auto covariations of symmetric model we can determine if in (9), (10) and (11) the parameters θ_+ and θ_- accept as being equal to each other. We get the well-known results of MA (1) model: $\mu = 0$; $\gamma_0 = 1 + \theta^2$, $\gamma_1 = \theta$. Since both asymmetric TDMA(1) and symmetrical MA(1) model are characterized by zero autocorrelation coefficients of order greater than one, the common analysis of the autocorrelation function can't determine whether the model is asymmetrical or symmetrical.

In addition if the asymmetrical model parameters are equal in magnitude but opposite in sign ($\theta_+ = -\theta_-$) the asymmetrical time series which is described by dynamic TDMA(1) model we can't distinguish it from implementation of simple sequence of independent identically distributed random variable under common autocorrelation function features. In this case, we have

$$\mu = 2 \theta_+ / (2\pi)^{1/2}, \quad \gamma_0 = 1 + ((\pi-2)/\pi) (\theta_+)^2, \quad \gamma_1 = 0.$$

Therefore the estimated future value of the sequence y_t that characterizes the error forecast variance $1 + ((\pi-2)/\pi)(\theta_+)^2$ exceed the error forecast variance that would have been made with true asymmetric model.

Asymmetric TDMA(q) model

$$y_t = u^+_t + \theta_{+1} u^+_{t-1} + \dots + \theta_{+q} u^+_{t-q} + u^-_t + \theta_{-1} u^-_{t-1} + \dots + \theta_{-q} u^-_{t-q}$$

is characterized by the autocorrelation function like the first-order model, which becomes zero for lags greater than q . If $\theta_{+i} = -\theta_{-i}$, $i = 1, 2, \dots, q$, the symmetrical TDMA(q) model have zero auto covariation all lags. Given the sample autocorrelation function the time series can't be distinguished from the sequence of independent random variables.

We verify the symmetry of the response to positive and negative shocks the series *LFPR*, *UR*, *UROF*, *EMPL*, *PROD*, *RGDP*, that determine the percentage of economic activity, the unemployment rate as defined by the ILO, registered unemployment rate, nominal employment, productivity and real gross domestic product in Ukraine, respectively. We base our analysis on the evaluation of TDMA model (7). We conduct the research for the level of series and their natural logarithms, as well as for the first differences of their natural logarithms and seasonal differences. At first, we removed trend from all the series and also the series were seasonally adjusted. The seasonal method depended on statistical properties of each series.

We used regression with dummy variables that determined seasonal factors and seasonal adjustment method that was based on multiplicative moving average approach. We check the stationary of series with augmented Dickey-Fuller unit root test. In Tables 12-13 we show some of the evaluation results.

We give the parameters estimation of asymmetric TDMA models together with appropriate estimation symmetric models, in which both slope parameters are identical. For the estimation we used maximum likelihood the method.

The numerical calculations were performed by the creating of the programs that was developed by the author in MATLAB.

Table 12

Evaluation results for symmetric and asymmetric moving average models and asymmetry test of percentage of economic activity, unemployment rate (defined by the ILO), registered unemployment rate

	Percentage of economic activity		Registered unemployment rate		Unemployment rate (ILO)	
Parameter	$\Delta_4 \log LFPR$		$\Delta_4 UROF$		$\Delta_4 UR$	
	MA	TDMA	MA	TDMA	MA	TDMA
θ	0,26		0,99		0,88	
θ_+		0,25		0,99		0,99
θ_-		0,27		0,99		0,24
$\hat{\sigma}^2$	0,0044	0,0044	5,9336	5,9336	39,9504	37,5826
LR-statistic	0,0023		0,0000		2,7493	

Source: author's evaluations

To test asymmetries we use the likelihood ratio statistic

$LR = -2 (\log L_R - \log L_{UR}) = 2 \log [(\hat{\sigma}_s / \hat{\sigma}_{as})^n]$, where $\log L_R$ is logarithm of likelihood function and $\hat{\sigma}_s$ - estimate of standard deviation of residues that were found for symmetric hypothesis; $\log L_{UR}$ is logarithm of likelihood function and $\hat{\sigma}_{as}$ is estimate of standard deviation of residues that were found for asymmetric hypothesis.

Table 13

Evaluation results for symmetric and asymmetric moving average models and asymmetry test of nominal employment, productivity and real gross domestic product

	Real gross domestic product		Productivity		Nominal employment	
Parameter	$\Delta_4 \log RGDP$		$\Delta \log PROD$		$\log EMPL$	
	MA	TDMA	MA	TDMA	MA	TDMA
θ	-0,01		-0,05		0,29	
θ_+		-0,05		-0,06		0,51
θ_-		0,31		0,04		0,07
$\hat{\sigma}^2$	0,2008	0,1980	0,2035	0,2032	0,0097	0,0093
LR-statistic	0,6910		0,0718		2,0585	

Source: author's evaluations

This statistic has an asymptotic chi-square distribution. The degree of freedom is equal to the number of restrictions. The calculated values likelihood ratio statistics are given in the last column of Tables 12-13.

We analyzed the evaluation results for different transformations of seasonally and trend adjusting in registered unemployment, productivity and percent of economic activity. We obtained that sum of residual squares symmetric and asymmetric MA models do not differ statistically significantly. LR-value statistics makes it impossible to reject the hypothesis of symmetry, and therefore we can argue that these series of symmetrically respond to positive and negative shocks of the previous period. For series *UR*, *EMPL* and *RGDP* the hypothesis of symmetry MA process is rejected at the 5% level. The parameters of symmetric and asymmetric moving average processes differ. These results indicate that the positive and negative innovations have different influences on the behavior of unemployment, the number of employed and real GDP. Therefore, their predictions that were based on previous innovations have different property and depend on the sign of past disturbances.

Threshold Autoregressive Models

A promising area of current research are an approach based on the using of econometric methods and models for nonlinear time series analysis with atypical distribution functions, in particular there are threshold autoregressive model. Accordingly, the development and evaluation of this kind of models are useful to study the socio-economic variable behavior in Ukraine. Their analysis will allow to predict adequately the nonlinear path of processes and to forecast the change in their dynamics. They also will help us to estimate the value of threshold parameters that define different regimes of their behavior, including the dynamics of growth and reducing unemployment and employment. The model will allow predicting the short-term fluctuation of processes and point of transition to long-term economic growth as well as predicting the phases of their decreasing and increasing to order to improve economic policy.

We consider the time series u_t that describes the register unemployment rate among the working age population in Ukraine. Unemployment is socio-economic phenomenon in which some fraction of the workforce for some reason is not engaged in economic activity. Oscillation nature of the unemployment rate dynamics is an important factor of influence on all aspects of market. The research is impossible without effective study of economics behavior property. The registered unemployment rate is defined as the percentage of unemployed that are registered in State Employment Service, in the average annual number of working age population. The analysis of its correlation with the unemployment rate that is defined by the ILO, makes possible to estimate the degree of trust of people to the government in the issue of employment and state social unemployment insurance. We estimated several various autoregressive models:

$$\Delta u_t = c + \alpha_1 \Delta u_{t-1} + \alpha_2 \Delta u_{t-2} + \dots + \alpha_{p-1} \Delta u_{t-p+1} + \alpha_p \Delta u_{t-p} + \varepsilon_t \quad (12)$$

and obtained that most parameters were insignificant. The statistical characteristics of residual denied the adequacy of these specifications. The study revealed that the linear structure of the model is not correct. Therefore, we need some deeper investigation. Exploring the statistical properties and the plots of unemployment rate in Ukraine we could to assume that his behavior was describes by different processes for different periods. To simulate this type of nonlinearity we apply threshold autoregressive models that allow explaining the change of time series behavior that depend on the value of some function defined economic structure.

To model the nonlinear dynamics of the registered unemployment in Ukraine we use threshold autoregressive models that characterize the changing behavior of time series based on some threshold value, which is unknown, as is also the subject of study. The approaching methods of total nonlinear structure by threshold autoregression were developed in the articles of famous econometrists K. Chan, H. Tong (1996), B. Hansen (1997), D. Peel, A. Speight (2000) and others.

The most common type of this class of models is two-regime threshold autoregressive (TAR) model that for unemployment rate has such a general form:

$$\Delta u_t = (\alpha_0 + \alpha_1 \Delta u_{t-1} + \alpha_2 \Delta u_{t-2} + \dots + \alpha_p \Delta u_{t-p}) \cdot I(q_{t-1} \leq \gamma) + (\beta_0 + \beta_1 \Delta u_{t-1} + \beta_2 \Delta u_{t-2} + \dots + \beta_p \Delta u_{t-p}) \cdot I(q_{t-1} > \gamma) + \varepsilon_t, \quad (13)$$

where $I(q_{t-1} \leq \gamma)$ defines the indicator-function, which takes the value of 1 or 0, depending on the value of the argument. The function $q_{t-1} = q(u_{t-1}, \dots, u_{t-s})$ is some function of previous data. The value $p > 1$ determines the order autoregression. The parameter γ is called as threshold parameter. The parameters α_j are the slope autoregression coefficients if $q_{t-1} \leq \gamma$. The parameters β_j are the slope autoregressive coefficients by lagged variables if $q_{t-1} > \gamma$. It is assumed that the disturbance ε_t are independent on previous values y_t and that they are identically independent distributed random variables.

We define $y_t = (1 \ \Delta u_{t-1} \ \dots \ \Delta u_{t-p})'$ $y(\gamma)_t = (y_t' \cdot I(q_{t-1} \leq \gamma) y_t' \cdot I(q_{t-1} > \gamma))'$. Then we can rewrite the equation (12) in the form

$$\Delta u_t = (y(\gamma)_t)' \delta + \varepsilon_t, \quad (14)$$

Where $\delta = (\alpha' \ \beta)'$. The unknown parameters of the model that we need to estimate are the parameter vectors δ and γ . In addition, one of the main issues by

investigation TAR models is a choice of functional relationship, which is responsible for regime change in behavior of economic variable, namely the threshold function q .

Since the model (14) is not linear in the parameters so the appropriate methods of estimation method is maximum likelihood (ML). Furthermore, assuming that ε_t is iid $N[0, \sigma^2]$ and taking into account that the regression equation (14) have break in parameter estimates we used sequential conditional LS consistent estimation.

To estimate the model parameters for each threshold function q_{t-1} we define various possible values that it may acquire during the studied period. For each such point values γ we find pointes of LS estimates of model parameters δ (B. Hansen, 1997).

$$\hat{\delta}(\gamma) = \left(\sum_{t=1}^T y_t(\gamma) y_t'(\gamma) \right)^{-1} \left(\sum_{t=1}^T y_t(\gamma) \Delta u_t \right). \quad (15)$$

On the base of residues of estimated model

$$\hat{\varepsilon}_t(\gamma) = \Delta u_t - y_t'(\gamma) \hat{\delta}(\gamma)$$

we calculate the estimates of unknown error variations

$$\hat{\sigma}_T^2(\gamma) = 1/T \sum_{t=1}^T (\hat{\varepsilon}_t(\gamma))^2. \quad (16)$$

Given a set of residues variances that correspond to different threshold function, we choose the smallest $\hat{\sigma}_T^2$ and the corresponding threshold. Then LS estimation of threshold parameter γ is a value that minimizes (17):

$$\hat{\gamma} = \operatorname{argmin}_{\{\gamma \in \Gamma\}} \hat{\sigma}_T^2(\gamma), \quad (17)$$

where $\Gamma = [\gamma_1, \gamma_2]$.

The residues variance $\hat{\sigma}_T^2(\gamma)$ can take up to T different values depending on the values, which takes the parameter γ . These values correspond to $\hat{\sigma}_T^2(\gamma)$ (q_{t-1}), $t=1, \dots, T$. Therefore for estimation of the model parameters we apply LS to regressions (15) taking $\gamma = q_{t-1}$ for each $q_{t-1} \in \Gamma$. For each regression we calculate residues variance

$$\hat{\sigma}_T^2(q_{t-1}) = 1/T \sum (\hat{\varepsilon}_t(q_{t-1}))^2.$$

After obtaining a set of variances of residues corresponding to different values of the threshold function, we select the least $\hat{\sigma}_T^2(\gamma)$ and the corresponding value γ , which corresponds to the smallest value of the variance, that is

$$\hat{\gamma} = \operatorname{argmin}_{\{q \in \Gamma\}} \hat{\sigma}_T^2(q_{t-1}).$$

Then the estimate of δ is obtained as $\delta = \delta(\hat{\gamma})$.

The corresponding reduces are calculated as

$$\hat{\varepsilon}_t = \Delta u_t - y_t'(\hat{\gamma}) \delta \text{ with sample variance } \hat{\sigma}_T^2 = \hat{\sigma}_T^2(\hat{\gamma}).$$

If we define the threshold variable as $q_{t-1} = \Delta u_{t-d}$ for some integer $d \in [1, d_0]$ we obtain self-exciting threshold autoregressive (SETAR) model. The integer d is determined as delay lag. Usually it is unknown and need to be estimated. In the

case using SETAR models the method of parameter estimation require the estimation nearly $T \times d_0$ regression. In the case of SETAR model the estimation task expands by searching the parameter d , which is estimated along with other parameters of the model. To select the appropriate SETAR threshold unemployment model we also consider long differences in unemployment levels $q_{t-1} = u_{t-1} - u_{t-d}$, as well as long differences in unemployment changes $q_{t-1} = \Delta u_{t-1} - \Delta u_{t-d}$ for different orders ($3 < d < 12$) as an indicator function. These values show a change in the behavior of the indicator over a long time interval. The results of model estimations that take into account various kind of threshold functions are given in Table 14.

Table 14

**Evaluation Results of SETAR unemployment models
for different threshold function**

Threshold function	The least variance $\hat{\sigma}_T^2 \cdot 100$	Testing of adequacy SETAR model $F_T(\gamma)$	Estimated threshold parameter
$q_{t-1} = u_{t-1} - u_{t-3}$	3,6412	91,75 ($p=0,000$)	0,145
$q_{t-1} = u_{t-1} - u_{t-4}$	3,2672	120,11 ($p=0,000$)	0,034
$q_{t-1} = u_{t-1} - u_{t-5}$	3,5204	100,25 ($p=0,000$)	0,060
$q_{t-1} = u_{t-1} - u_{t-6}$	3,4766	103,48 ($p=0,000$)	-0,055
$q_{t-1} = u_{t-1} - u_{t-7}$	3,7248	86,19 ($p=0,000$)	-0,085
$q_{t-1} = u_{t-1} - u_{t-8}$	3,7923	81,88 ($p=0,000$)	-0,265
$q_{t-1} = u_{t-1} - u_{t-9}$	3,5280	99,70 ($p=0,000$)	-0,425
$q_{t-1} = u_{t-1} - u_{t-10}$	3,9513	72,31 ($p=0,000$)	-0,440
$q_{t-1} = u_{t-1} - u_{t-11}$	4,1848	59,56 ($p=0,000$)	-0,305
$q_{t-1} = u_{t-1} - u_{t-12}$	4,4938	44,74 ($p=0,022$)	-0,270
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-3}$	3,4747	103,62 ($p=0,000$)	0,095
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-4}$	3,7231	86,30 ($p=0,000$)	0,137
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-5}$	3,3494	113,33 ($p=0,000$)	0,183
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-6}$	3,4178	107,94 ($p=0,000$)	0,238
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-7}$	3,3903	110,09 ($p=0,000$)	0,235
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-8}$	3,5397	98,85 ($p=0,000$)	0,173
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-9}$	3,3582	112,63 ($p=0,000$)	0,072
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-10}$	3,4699	103,98 ($p=0,000$)	0,075
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-11}$	3,7067	87,36 ($p=0,000$)	0,165
$q_{t-1} = \Delta u_{t-1} - \Delta u_{t-12}$	4,0615	66,11 ($p=0,000$)	-0,032

Source: author's evaluations

Since SETAR model estimation is not provided in standard econometric packages, numerical calculations are performed on the basis of the program developed by authors in the GAUSS environment. As can be seen from the table among all the estimated threshold models the smallest residuals sum of squares in the case of using long differences in unemployment levels is obtained for $q_{t-1} = u_{t-1} - u_{t-4}$. In the case of using the long differences in the changes in unemployment rate we obtained the smallest residuals sum of the squares $q_{t-1} = \Delta u_{t-1} - \Delta u_{t-6}$. At the same time the results showed that the effect of change in regime on the behavior of long differences in unemployment rate is more than the effect on long differences in unemployment changes.

The values of the statistical criterion for testing the statistical significance of the nonlinear SETAR models as alternatives to the linear AR(p) model, as well as the significance level in which the null hypothesis is not rejected, is given in the third column of Table 14. The test results show that in both cases the application of long differences with an accuracy of 99.9% we can argue that the nonlinear SETAR models have an advantage over AR specification in the study of unemployment. In this case, the application with threshold function for long differences in the level of unemployment gives better results than for long differences in unemployment changes. In addition, in the case of using the long differences $q_{t-1} = u_{t-1} - u_{t-4}$ we get less variance rather than by using the lags of the change in unemployment $q_{t-1} = \Delta u_{t-6}$ (Table 14).

Estimation of confidence intervals of the threshold parameter and slope parameters for SETAR model

We can evaluate the statistically significant boundary of the threshold parameter of the SETAR model. To construct an asymptotic confidence interval for γ we use the likelihood ratio statistics $LR_n(\gamma)$. In the case of testing the hypothesis $H_0: \gamma = \gamma_0$ under certain conditions we obtain the following:

$$LR_T(\gamma_0) \rightarrow^d \xi, \text{ where } \xi = \max_{s \in R} [2W(s) - |s|], \quad (18)$$

$W(v) = \{W_1(-v), \text{ if } v < 0; 0, \text{ if } v = 0; W_2(v), \text{ if } v > 0\}$, where $W_1(v)$ and $W_2(v)$ two independent standard Brownian motions on $[0, \infty)$. The confidence interval for γ is

$$\Gamma^{\wedge} = \{\gamma: LR_T(\gamma) \leq c_{\xi}(\beta)\}.$$

This interval can be found by the graphical method, which consists in constructing a graph of $LR_T(\gamma)$ dependence on γ and also the line $c_{\xi}(\beta)$, where $c_{\xi}(\beta)$ is the critical value of the distribution ξ for the level of significance β (B. Hansen, 1997). The graph of the residual variance $\hat{\sigma}_T^2(\gamma)$ dependence on γ and the straight line $\hat{\sigma}_T^2(1 + c_{\xi}(\beta)/T)$ can be represented equivalently. However, the domain Γ can be discontinuous in practice. A more conservative procedure is to determine the convex region $\Gamma^{\wedge}_C = [\hat{\gamma}_1, \hat{\gamma}_2]$, where $\hat{\gamma}_1 = \min_{\gamma} \Gamma^{\wedge}$ and $\hat{\gamma}_2 = \max_{\gamma} \Gamma^{\wedge}$. Empirical studies that were based on the Monte Carlo method showed that the levels of rejection of the hypothesis in its testing by the likelihood ratio statistics are generally liberal, that is, the confidence region Γ contains less than the nominal percentage of parameter values. Note that the level of rejection decreases with the increase of the threshold effect, and also that the magnitude of the distortion does not decrease evenly with the increasing in the sample size. The best approximations were obtained by using the convex region Γ^{\wedge}_C , for which the rejections are generally close to the nominal ones. The point estimates for the threshold parameter γ for various SETAR unemployment models are given in the last column of Table 14. In Figure 10 is depicted a dependency graph $\hat{\sigma}_T^2(\gamma)$ on γ , as well as a straight line $\hat{\sigma}_T^2(\gamma)(1 + c_{\xi}(\beta)/T)$. The evaluation results show that for a model that uses the threshold function $q_{t-1} = u_{t-1} - u_{t-4}$ the 95% confidence interval for γ is [0.02; 0.11]. The point estimate of the threshold is the value of the threshold function q_{t-1} for which the value $\hat{\sigma}_T^2(\gamma)$ is the smallest. Such an estimate is $\gamma = 0.034$.

To find the confidence intervals of the slope parameters we use the asymptotic conclusions of the classical regression model analysis that confirm the following: if γ is known, then

$$\sqrt{T} (\hat{\delta}(\gamma) - \delta_0) \rightarrow^d N[0, \Psi(\gamma)], \quad (19)$$

where $\Psi(\gamma) = (E[x(\gamma)'x(\gamma)])^{-1}\sigma^2$. In the case if γ is known the region

$$\Omega^\wedge(\gamma) = \hat{\delta}(\gamma) \pm z_\beta s_\delta(\gamma)$$

is a β -significant confidence interval for δ . Here z_β denotes the β -significant critical value of the normal distribution, $s_\delta(\gamma) = \Psi^\wedge(\gamma)/T$ is the standard error of $\hat{\delta}(\gamma)$.

However, for TAR models the threshold parameter γ is estimated along with the slope parameters δ . Therefore, in order to obtain a confidence interval for δ for some $0 < \varphi < 1$ we construct a φ -significant confidence interval $\Gamma^\wedge(\varphi)$ for the threshold parameter γ and for each γ from this interval we calculate the confidence interval $\Omega^\wedge(\gamma)$ for δ . Next, they combine all these sets (B. Hansen, 1997) as following

$$\Omega^\wedge_\varphi = \bigcup_{\{\gamma \in \Gamma^\wedge(\varphi)\}} \Omega^\wedge(\gamma). \quad (20)$$

By construction Ω^\wedge_φ grows over φ in the sense that $\Omega^\wedge_{\varphi_1} \subseteq \Omega^\wedge_{\varphi_2}$ if $\varphi_1 < \varphi_2$. Therefore, the question arises: what choice of the significance level φ of the interval of the threshold parameter γ should be for constructing an adequate β -significant confidence interval for δ .

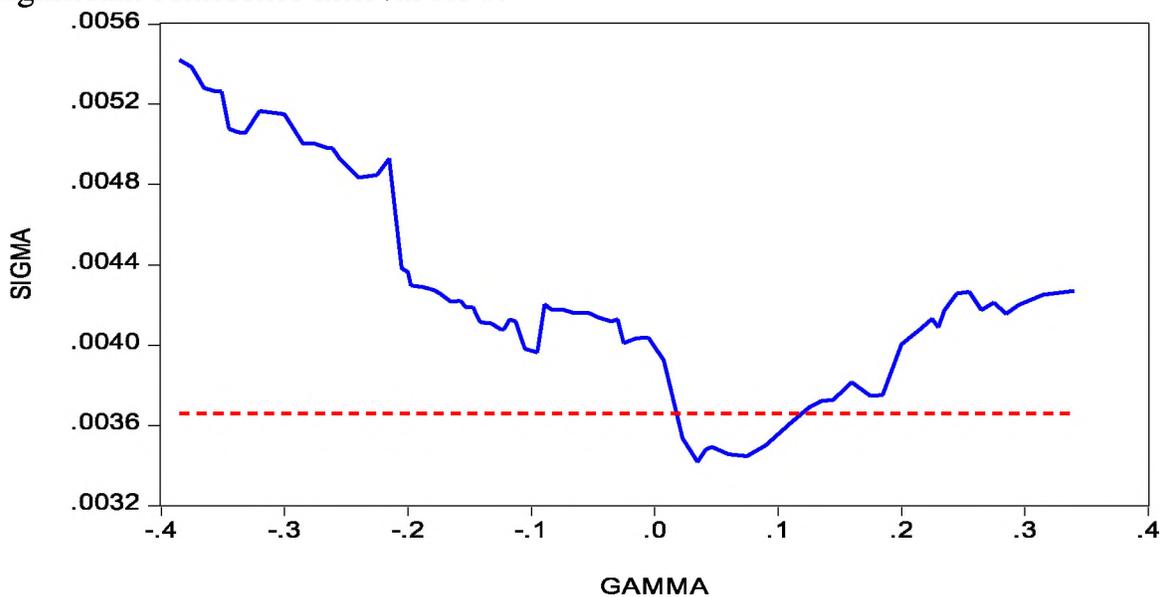


Figure 10: Confidence interval of threshold parameter for SETAR model with $q_{t-1} = u_{t-1} - u_{t-4}$

Source: author's evaluations

The study of the accuracy of confidence regions conducted with Monte Carlo experiments found that for the construction of regression parameters confidence intervals for the SETAR model it is best to use the domain $\Omega^\wedge_{0,8}$. The obtained confidence intervals for the slope parameters of the estimated threshold unemployment model are shown in Table 15. The results indicate the significance of most coefficients.

Table 15

Confidence intervals of slope parameters for SETARunemployment models

Lags order j	95% confidence intervals of α_j	95% confidence intervals of β_j
0	(-0,002 ; -0,001)	(0,056;0,103)
1	(0,442 ; 0,800)	(0,642;1,164)
2	(-0,262 ; -0,144)	(-1,142;-0,629)
3	(-0,068 ; -0,037)	(0,755;1,370)
4	(-0,188 ; -0,104)	(-2,164;-1,193)
5	(0,022 ; 0,040)	(1,728;3,133)
6	(-0,215 ; -0,118)	(-1,530;-0,844)
7	(0,026 ; 0,048)	(0,048;0,087)
8	(-0,233 ; -0,128)	(0,591;1,073)
9	(0,054 ; 0,098)	(-1,322;-0,729)
10	(-0,329 ; -0,181)	(0,551;1,000)
11	(0,117;0,213)	(-0,055;-0,030)
12	(0,236;0,428)	(0,003;0,006)

Source: author's evaluations

Using the estimated threshold model based on $q_{t-1} = u_{t-1} - u_{t-4}$ we also investigate the rate of change of the registered unemployment rate as $\Delta u_t / u_t$. Figure 11 shows the estimated values of the registered unemployment levels, which the model splits into two regimes.

Figure 12 shows the estimated values of the rate of their unemployment change. The first regime includes those observations for which the difference in levels of unemployment rate for the previous three months was less than 0.034, the rest of the observations are subject to the second regime of behavior.

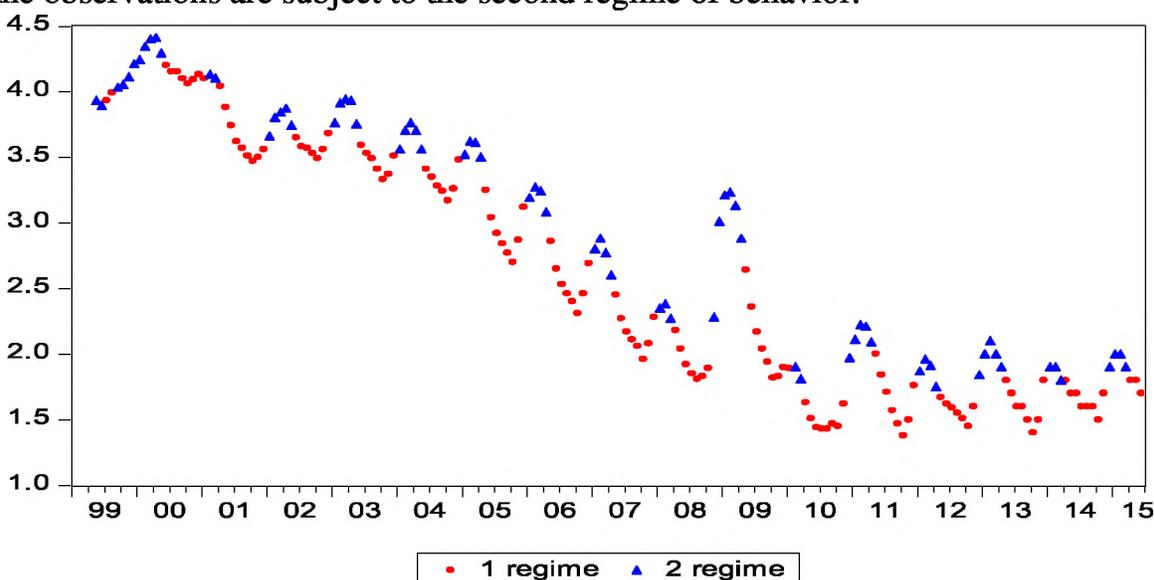


Figure 11: The estimated value of the registered unemployment rate based on the two-regime SETAR model using long differences

Source: author's evaluations

In Figure 11-12 the observations that are included in the first regime are depicted by circles, respectively, the triangles correspond to the observations that belong to the second regime.

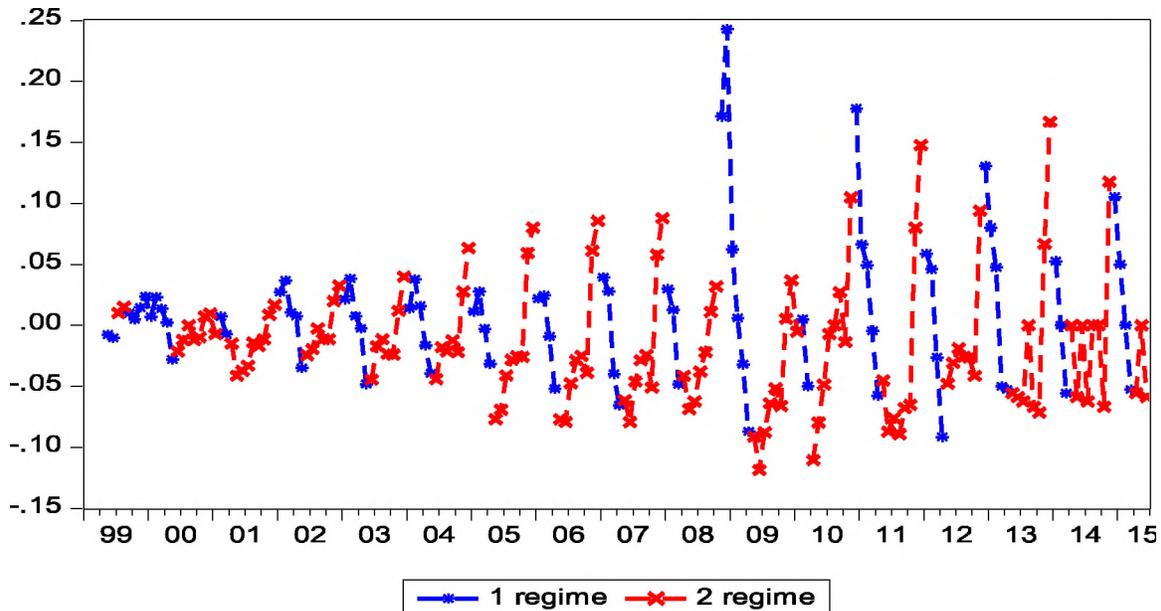


Figure 12: The estimated value of the registered unemployment grows rate $\Delta u_t/u_t$ based on the two-regime SETAR model using long differences

Source: author's evaluations

It should be noted that for the investigated sample periods of increasing of unemployment rate changes are characterized by auto regression with slope coefficients α_j . For periods of decline in unemployment rate are coefficients β_j .

CONCLUSIONS

The effectiveness of implementing social and economic policy measures requires an analysis, modeling and forecasting of processes in the labor market with modern flexible econometric tools, taking into account the asymmetry of responses to macroeconomic shocks that disturbed the economy. The results of the empirical study of a number of labor market indicators, namely nominal employment, labor productivity, real gross domestic product, the percentage of economic activity, registered unemployment rate and the level of unemployment, determined according to the ILO methodology, reveal differences in their reactions to positive and negative disturbances. We performed the comparison of the degree and duration impact of the shocks with different signs based on evaluation and analysis of asymmetric TDMA and TDAR time series models. It was found that negative shocks have stronger, more persistent and more affect the employment, economic activity and unemployment than positive ones. In addition, negative disturbances follow much more than positive to increase their volatility. The asymmetry of responses to the shocks of different signs that has been revealed show that it is necessary to take into account in the evaluation and forecasting of future tendencies in the development of processes in the social and labor sphere the asymmetry of the reaction of their indicators to different changes in market conditions.

The analysis substantiates the necessity of applying in the modeling of the dynamics of socio-economic processes the modern nonlinear time series models, in particular, threshold models and smooth transition regression models, which enable

to predict trends taking into account asymmetric effects. As a result of the empirical study of the registered unemployment rate in Ukraine, we estimated a number of nonlinear econometric specifications. The threshold auto regression models were justified for describing its nonlinear and asymmetric behavior. The investigation showed that the dynamics of the registered unemployment rate in the national economy was characterized by different regimes of behavior that changed one another depending on the threshold function value determined by the econometric analysis. Estimated based on real information the value of the threshold parameter revealed the branching of the TAR model in two directions, which in different ways characterized the dynamic growth and decline in unemployment. In particular, the first branch of the estimated nonlinear auto regression describes and allows predicting the dynamics of the growth of the registered unemployment rate in the short run, while the second set of estimations helps to characterize the dynamics of the decline in the percentage of registered unemployed in employment centers.

The developed nonlinear econometric models for the employment and unemployment dynamics complement the study of characteristic features that are inherent in the labor sector in Ukraine, and help to predict the effects of state measures in this area more accurately. The revealed nonlinear character of the dynamics of registered unemployment indicates the expediency of introducing various measures of the state employment policy in periods of growth and reduction. The estimated threshold value allows determining the moment of change of employment promotion programs in the short-term period.

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