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FOURIER DESCRIPTORS FOR SHAPE CHARACTERIZATION

This paper reports shape characterization methods based on Fourier descriptors. Fourier descriptors were implemented based on angular and complex shape representation and their ability to characterize the shapes of different objects were evaluated. A new shape descriptor using Fourier descriptors was proposed.

Keywords: **shap**e representation, shape description, Fourier descriptors.

Introduction

Objects can be characterized by certain features: grey levels, textures, edges, boundaries, shapes, locations, etc. An object or regions of interest consists of interior points or contents which are surrounded by a boundary often called the contour of an object. There is no universal definition of what shape of an object is. The shape of an object is the important visual feature for describing image content and is generally considered as the form of the object's boundary, consisting of a set of points, curves, surfaces, etc.

Here we consider shape boundary of the object as a closed planar curve that can be defined as function:

– in an explicit form as $y = (x)$;

– in an implicit form as $f(x, y) = 0$;

– in a parametric form by natural parameterization $c(t) = ((x(t), y(t))$. We will consider that the parameter t is given by the arc-length parameterization with $0 \le t \le L$, where L is the length of the shape boundary.

– a parametric form in the polar coordinates as $\tau(t) = ((d(t), \theta(t))$.

– a parametric form in the complex plane, $z(t) = x(t) + j \cdot y(t)$.

Usually a pre-segmented binary shape is represented by its external characteristics (shape representation) and then shape characterization or shape description is used as a post-processing technique. It generates descriptors of the shape. Descriptors of the shape are a set of numbers that describe specific characteristics or features of an object and are considered as shape parameters that allow comparing

and recognizing objects through matching and also to establish to which class object belongs. Descriptors should not depend on geometrical transformations such as translation, rotation and changes of scale [11, 2, 7]. It means that they should be invariant to these three transformations and should not change the shape of an object. Many different shape representation and description techniques have been proposed in the literature, a good review on techniques can be found in [11, 2, 14, 8].

Fourier descriptors have been applied for shape analysis of different biological objects [4], detection of defects for various plastic products [9], character recognition [6, 1], for shape representation [10], to model shapes in computer graphics [3].

Shape Signatures

Fourier descriptors are derived from Fourier transform of shape representation. In general it is one-dimensional function of an object boundary that often called shape signature. A signature describes shape in terms of a one-dimensional signal. A signature can be constructed in various ways from the given two-dimensional function of an object boundary. Chain coding, radius (angle), tangent angle, chord-length representations are all signatures and their many other forms of them have been described in literature [14, 5, 12]. Several possible forms of signatures are presented below.

Fourier descriptors

Fourier expansion in trigonometric form represents periodic functions $c(t)$ by summation of trigonometric functions that increase in frequency [5, 2]:

$$c(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos(k\omega t) + b_k \sin(k\omega t)). \quad (1)$$

Where $\omega = \frac{2\pi}{T}$ defines the fundamental frequency and T is the period of the function.

$$a_k = \frac{2}{T}\int_0^T c(t)\cos(k\omega t)dt \text{ and}$$
$$b_k = \frac{2}{T}\int_0^T c(t)\sin(k\omega t)dt. \quad (2)$$

The coefficients of this expansion, a_k and b_k, are known as the Fourier descriptors.

Properties of Fourier descriptors

In the following we examine how the change of curve $c(t)$ affects on Fourier descriptors.

1. A shifted curve can be expressed as $c'(t) = c(t + \alpha)$, α represents the shift value. Fourier descriptors for this function by considering the definition of Equation (2):

$$a'_k = \frac{2}{T}\int_0^T c(t'+\alpha)\cos(k\omega t')dt =$$
$$= \frac{2}{T}\int_0^T c(t)\cos(k\omega t - kw\alpha)dt = \quad (3)$$
$$= a_k \cos(k\omega\alpha) + b_k \sin(k\omega\alpha).$$

And $b'_k = \frac{2}{T}\int_0^T c(t'+\alpha)\sin(k\omega t')dt =$

$$= \frac{2}{T}\int_0^T c(t)\sin(k\omega t - k\omega\alpha)dt = \quad (4)$$
$$= b_k \cos(k\omega\alpha) - a_k \sin(k\omega\alpha).$$

From (3), (4) can be observed that $a_k^2 + b_k^2$ is independent of the shift α. That is:

$$a_k'^2 + b_k'^2 = (a_k \cos(k\omega\alpha) + b_k \sin(k\omega\alpha))^2 +$$
$$+ (b_k \cos(k\omega\alpha) + a_k \sin(k\omega\alpha))^2 = a_k^2 + b_k^2.$$

2. The scaling of shape can be expressed as $c'(t) = sc(t)$, s represents no zero value. We have $a'_k = sa_k$ and $b'_k = sb_k$.

3. The translation of a shape can be expressed as $c'(t) = c(t) + c_0$:

$$a'_k = \frac{2}{T}\int_0^T c'(t)\cos(k\omega t)dt =$$
$$= \frac{2}{T}\int_0^T [c(t)\cos(k\omega t) + c_0\cos(k\omega t)]dt =$$
$$= a_k + \frac{2}{T}c_0(\sin(k\omega T) - \sin(0)) = a_k,$$
$$a'_0 = \frac{2}{T}\int_0^T c'(t)\cos(k\omega t)dt = \frac{2}{T}\int_0^T [c(t) + c_0]dt = a_k + 2c_0,$$
$$a'_k = a_k, k > 1, b'_k = b_k.$$

This implicates that Fourier descriptors are invariant to translation except the first coefficient a'_0 and a_0.

Discrete computation

We assume that given shape defined by a closed curve $c(t)$with m number of points:

$$c_i = c(i\tau), i = 1..m, \tau = \frac{T}{m}.$$

According to the Nyquist theorem [2], the maximum frequency is obtained when $k = \frac{m}{2}$. The Fourier expansion can be redefined as

$$c(t) \approx \frac{a_0}{2} + \sum_{k=1}^{m/2}(a_k \cos(k\omega t) + b_k \sin(k\omega t)),$$
$$a_k = \frac{2}{T}\int_0^T c(t)\cos(k\omega t)dt \approx \frac{2}{m}\sum_{i=1}^{m} c_i \cos(k\omega i\tau),$$
$$b_k = \frac{2}{T}\int_0^T c(t)\sin(k\omega t)dt \approx \frac{2}{m}\sum_{i=1}^{m} c_i \sin(k\omega i\tau),$$
$$\tau = \frac{T}{m}.$$

The angular shape representation

A description of shapes boundary can be obtained by using the angular function. Assuming that shape is defined in the parametric form $c(t) = ((x(t), y(t))$ where t is arc length and $0 \le t \le L$. The angular function measures the angular direction of the tangent line $\varphi(t)$. However the angular function has discontinuities when the angular direction increases to a value of more than 2π or decreases to be less than zero. To overcome this problem [13] were proposed to use a normalized form of the cumulative angular function. The cumulative angular function $\psi(t)$ is the net amount of angular bend between the starting position $z(0)$ and position $z(t)$ on the shape boundary

$$\psi(t) = (\varphi(t) - \varphi(0)) \bmod (2\pi).$$

The normalised function where

$\psi^*(t) = \psi\left(\dfrac{L}{2\pi}t\right) + t$, t takes values from 0 to 2π.

The factor $\dfrac{L}{2\pi}$ normalises the angular function such that it is invariant under translation, rotation and scaling. For the simplest curve circle, will have simple representations, $\psi^*(t) \equiv 0$ and for other shapes $\psi^*(t) \ne 0$.

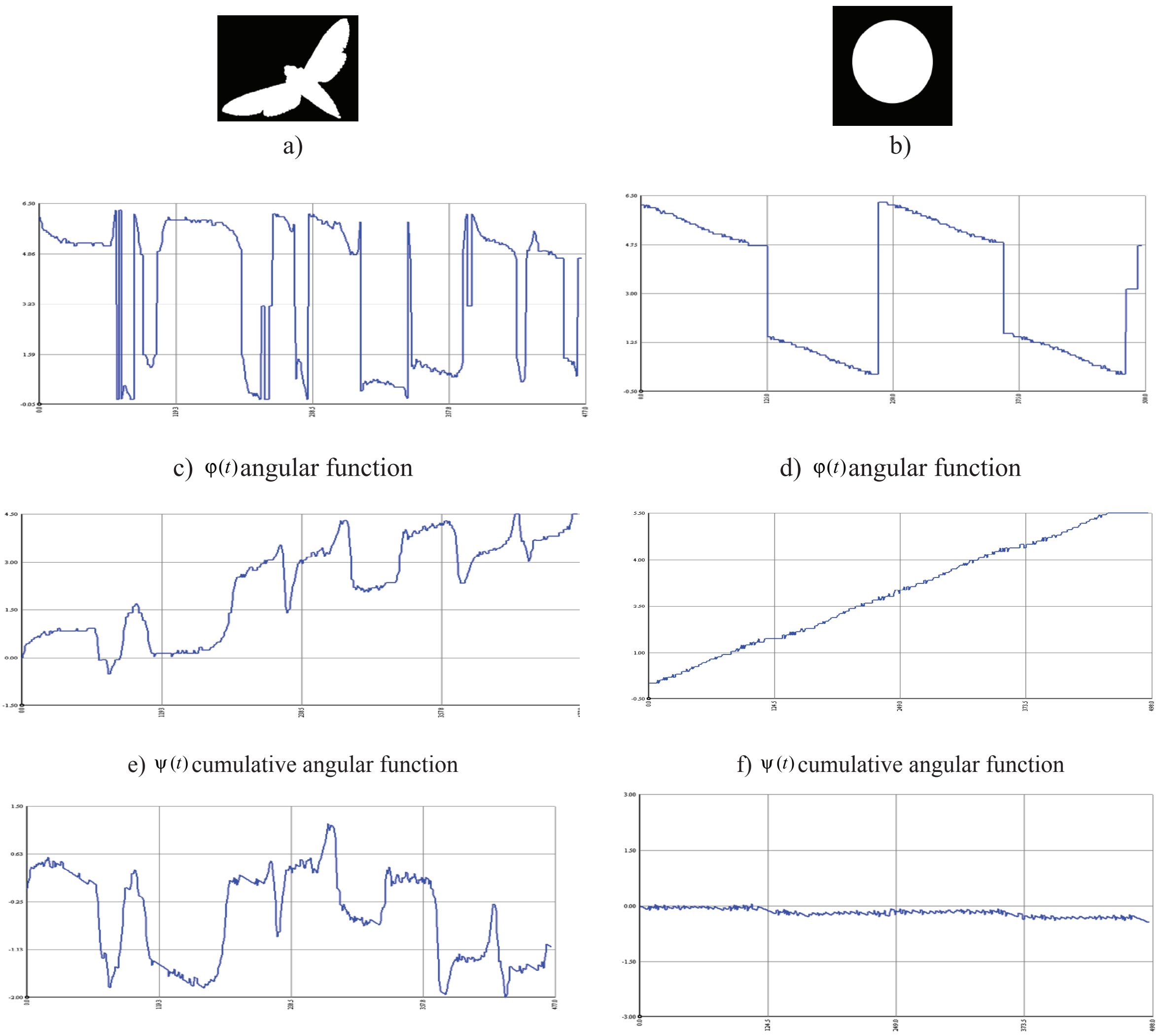
According to properties of Fourier descriptors coefficients d_{ka}, that is angular Fourier descriptors provide a rotation, scale and translation invariant description can be defined as:

$$d_a = \sqrt{a_k^2 + b_k^2}.$$

The feature vector of angular Fourier descriptors can be written as

$$AFD = \{d_2, \ldots, d_{m/2}\}.$$

The examples of angular function, cumulative angular function and normalized cumulative angular function are given in Fig. 1. Fig. 2 illustrates the angular Fourier descriptors, which were obtained from rotated objects. They were approximated as summation of values for each interval.



a) b)

c) $\varphi(t)$angular function d) $\varphi(t)$angular function

e) $\psi(t)$cumulative angular function f) $\psi(t)$cumulative angular function

g) $\psi^*(t)$ normalised cumulative angular function h) $\psi^*(t)$ normalised cumulative angular function

Fig. 1. Example of angular functions

Complex shape representation

Each point of the shape that is given parametrically $c(t) = ((x(t), y(t))$ can be represented by a complex number, the real and imaginary parts of which are the x and y coordinates of the points (x_0, y_0)... (x_{m-1}, y_{m-1}) in the real form $f_x(t) = x(t)$ and in the imaginary one $f_y(t) = y(t)$. This allows the shapes boundary to be expressed as the complex periodic function:

$$f(t) = f_x(t) + j \cdot f_y(t),\ j = \sqrt{-1}.$$

By considering the definition of Equation (1) the discrete Fourier transforms of coefficients $f_x(t)$ and $f_y(t)$ can be defined by a pair of Fourier descriptors:

$$c(t) = \frac{a_{x0}}{2} + \sum_{k=1}^{\infty}(a_{xk}\cos(k\omega t) + b_{xk}\sin(k\omega t)) +$$

$$+ j\left(\frac{a_{y0}}{2} + \sum_{k=1}^{\infty}(a_{yk}\cos(k\omega t) + b_{yk}\sin(k\omega t))\right)$$

$$a_{xk} = \frac{2}{T}\int_0^T f_x(t)\cos(k\omega t)dt \text{ and } b_{xk} = \frac{2}{T}\int_0^T f_x(t)\sin(k\omega t)dt$$

$$a_{yk} = \frac{2}{T}\int_0^T f_y(t)\cos(k\omega t)dt \text{ and } b_{yk} = \frac{2}{T}\int_0^T f_y(t)\sin(k\omega t)dt.$$

According to properties of Fourier descriptors complex Fourier descriptors d_{kc} invariant to rotation, scale and translation defined as:

$$d_c = \sqrt{\frac{a_{xk}^2 + a_{yk}^2}{a_{x1}^2 + a_{y1}^2}} + \sqrt{\frac{b_{xk}^2 + b_{yk}^2}{b_{x1}^2 + b_{y1}^2}},\ k \ge 2.$$

The feature vector of complex Fourier descriptors can be written as

$$CFD = \{d_{c2}, \ldots, d_{c(m/2)}\}.$$

The examples of Complex Fourier descriptors, which were approximated as summation of values for each interval, are given in Fig. 3.

A new shape descriptors *NCFD* and *NAFD* defined as

$$NCFD = \left[\sum_{k=2}^{m/2}\frac{d_{ck}}{k}\right] \Bigg/ \left[\sum_{k=2}^{m/2} d_{ck}\right] \text{ and}$$

$$NAFD = \left[\sum_{k=2}^{m/2}\frac{d_{ak}}{k}\right] \Bigg/ \left[\sum_{k=2}^{m/2} d_{ak}\right]$$

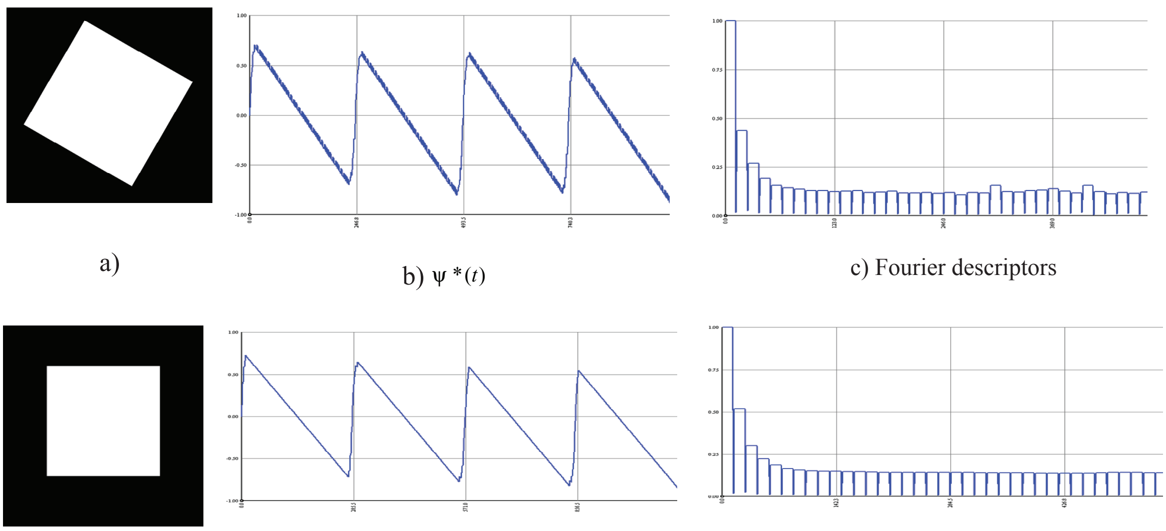
As an example, Euclidean norm of feature vectors of complex Fourier descriptors $\|CFD\|$, angular Fourier descriptors $\|AFD\|$ and shape descriptors *NCFD*, *NAFD* were obtained from objects, that were rotated, is shown in Fig. 4.

Conclusions

In this paper we are currently investigating shape characterization methods based on Fourier descriptors. To obtain Fourier descriptors the shape boundary should be represented as one-dimensional function. In our experiment Fourier descriptors were obtained by using the angular shape representation and complex shape representation. The proposed method is applicable to object with various shapes, which may be non-star-shaped.

The experiment showed that the complex Fourier descriptors and angular Fourier descriptors, which were approximated, are similar for the same objects. Euclidean norm of feature vectors of complex Fourier descriptors $\|CFD\|$ and angular Fourier descriptors $\|AFD\|$ show no clear ability to characterize the shape of different objects. A new shape descriptors *NCFD* and *NAFD* show a good ability of characterization. The most important observation is that *NCFD* and *NAFD* yield equal values.

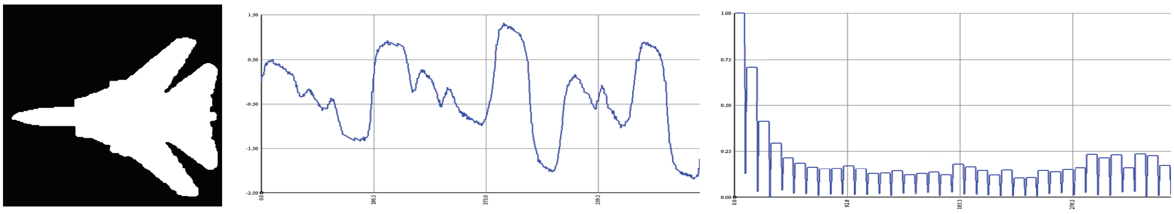
Fourier descriptors are in general have a good ability to characterize the shape of different objects.



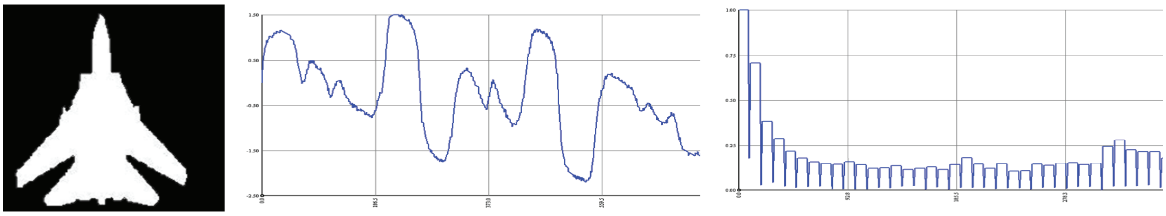
a) b) $\psi^*(t)$ c) Fourier descriptors

d) i) $\psi^*(t)$ f) Fourier descriptors

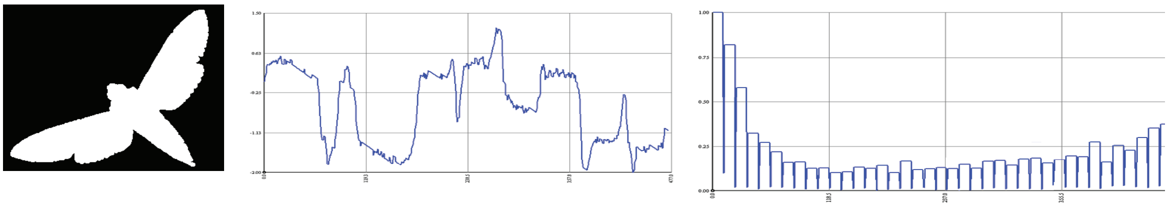
Fig 2. (Example of Angular Fourier descriptors, $\psi^*(l)$ is normalised cumulative angular function)



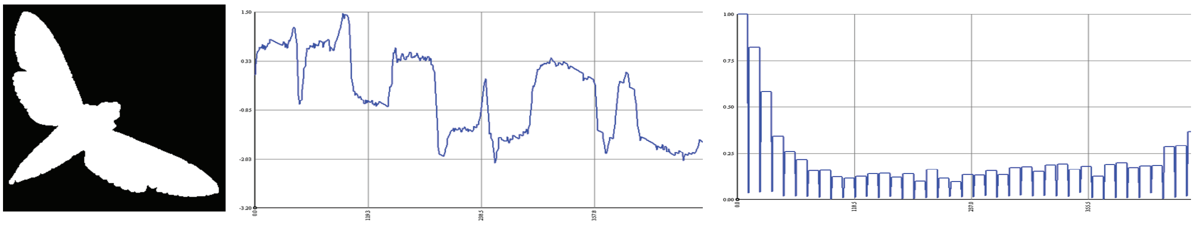
g) h) $\psi^*(t)$??) Fourier descriptors



j) k) $\psi^*(t)$ l) Fourier descriptors

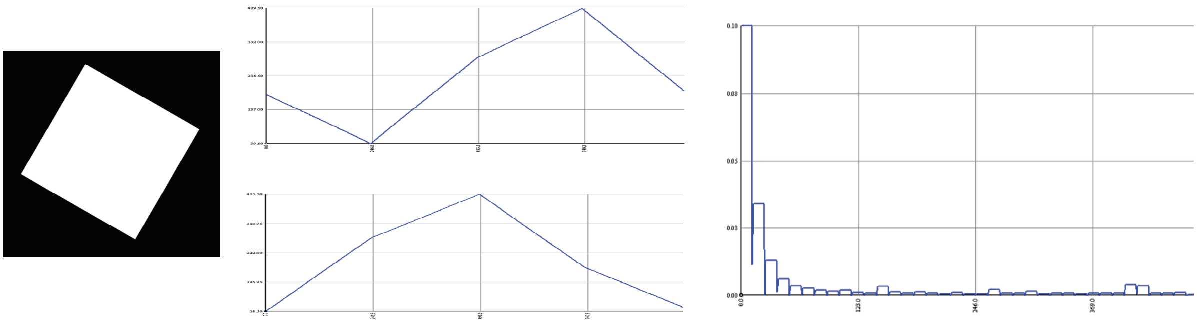


m) n) $\psi^*(t)$ o) Fourier descriptors



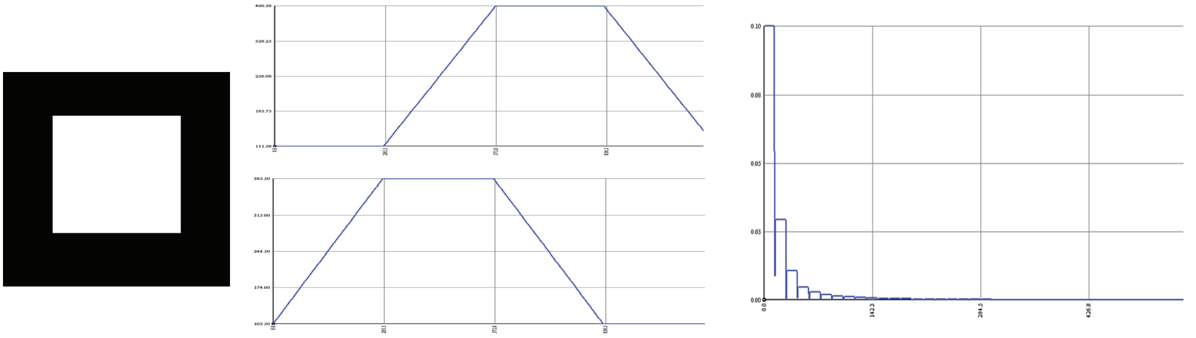
p) q) $\psi^*(t)$ r) Fourier descriptors

Fig 2. Example of Angular Fourier descriptors, $\psi^*(l)$ is normalised cumulative angular function

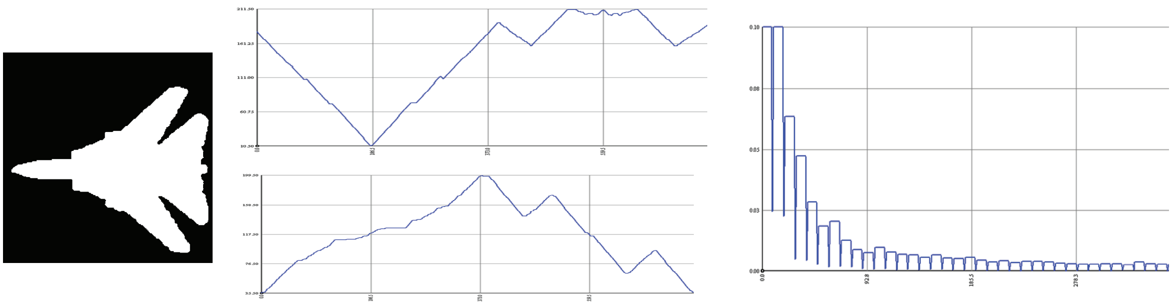


a) b) $x(t)$, $y(t)$ c) Fourier descriptors

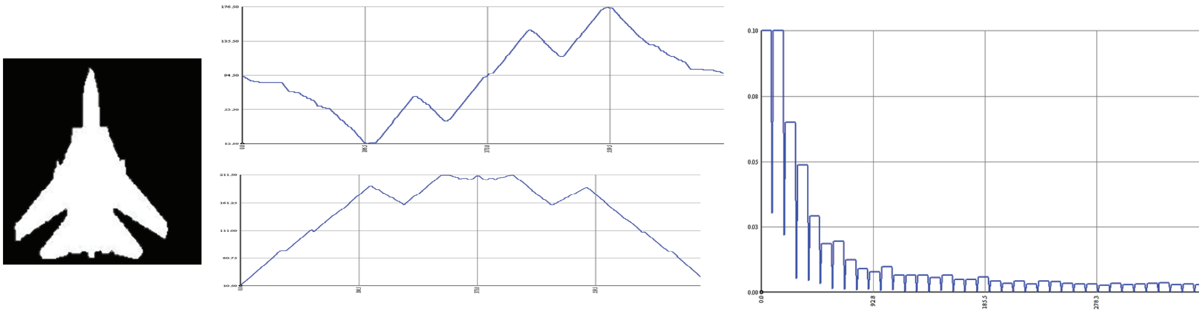
Fig 3. Example of Complex Fourier descriptors



d) e) $x(t)$, $y(t)$ f) Fourier descriptors

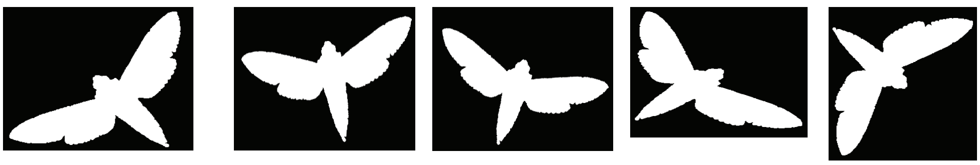


g) h) $x(t)$, $y(t)$ i) Fourier descriptors



j) k) $x(t)$, $y(t)$ l) Fourier descriptors

Fig 3. Example of Complex Fourier descriptors



$\|\|AFD\|\|$	14.9116556	15.0443492	14.6320218	13.9536189	14.8682044
$\|\|CFD\|\|$	4.0985746	4.1713933	4.1961798	4.1494936	4.2834784
NAFD	7.4320975	7.4884648745	7.48346985	7.43209750	7.4320975
NCFD	7.4320975	7.48846487	7.48346985	7.4320975005	7.4320975

$\|\|AFD\|\|$	13.9127367	14.3144764	14.8901750	12.5824716	13.9698909
$\|\|CFD\|\|$	4.3056931	4.2843708	4.4418956	4.2470277	4.34448694
NAFD	7.1885845	7.20327101	7.1777662	7.1885845	7.1885845
NCFD	7.1885845	7.20327101	7.1777662	7.1885845	7.1885845

Fig 4. Euclidean norm of feature vectors of complex Fourier descriptors ||*CFD*|| and angular Fourier descriptors||*AFD*||, new shape descriptors *NCFD* and *NAFD*

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ВИКОРИСТАННЯ ДЕСКРИПТОРІВ ФУР’Є ДЛЯ ОПИСУ ФОРМИ

У статті розглянуто методи опису форми на основі дескрипторів Фур’є. Побудовано дескриптори Фур’є на основі кутової і комплексної репрезентації форми, запропоновано новий дескриптор. Проаналізовано здібності дескрипторів характеризувати форми різних об’єктів.

Ключові слова: представлення форми, опис форми, дескриптори Фур’є.

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