

## On unicyclic graphs of metric dimension 2 with vertices of degree 4

Marharyta Dudenko and Bogdana Oliynyk

Communicated by A. P. Petravchuk

**ABSTRACT.** We show that if  $G$  is a unicyclic graph with metric dimension 2 and  $\{a, b\}$  is a metric basis of  $G$  then the degree of any vertex  $v$  of  $G$  is at most 4 and degrees of both  $a$  and  $b$  are at most 2. The constructions of unispider and semiunispider graphs and their knittings are introduced. Using these constructions all unicyclic graphs of metric dimension 2 with vertices of degree 4 are characterized.

### Introduction

The concept of metric dimension of a connected graph was introduced by Harary and Melter [1]. Slater [2] described the usefulness of this concept in connection to the long range aids to navigation.

A metric dimension as a graph parameter has numerous applications. In particular, to the representation of chemical compounds [3] in chemistry and to the problems of pattern recognition and image processing. Some of them involve hierarchical data structures [4]. Also, it is used in sonar [2] and combinatorial optimization [5].

The metric dimension decision problem is among the classical NP-hard problems [6], but for some families of graphs, for example, for trees [7], the polynomial algorithm can be built. It is easy to see, that if  $G$  has  $n$  vertices

---

*The authors thank to the International Charitable Foundation for Renaissance of the Kyiv-Mohyla Academy for the financial support of their research.*

**2010 MSC:** 05C12.

**Key words and phrases:** graph, distance, metric dimension, unicyclic graph.

then  $1 \leq \dim G \leq n - 1$ . There are results describing graphs having a given value of metric dimension. In particular, in [1] it is proved that a graph has metric dimension 1 if and only if it is a path. Moreover, metric dimensions of the cycle  $C_n$ , the complete graph  $K_n$  and the complete bipartite graph  $K_{s,t}$  are 2,  $n - 1$  and  $n - 2$ , respectively [1], [8].

S. Khuller, B. Raghavachari and A. Rosenfeld [7] considered some properties of graphs with metric dimension 2. They proved that if  $\{a, b\}$  is a metric basis of  $G$  with metric dimension 2 then degrees of both  $a$  and  $b$  are at most 3 and the maximum degree of any vertex  $u$  in the shortest path between  $a$  and  $b$  is 5. Later G. Sudhakara and A. R. Hemant Kumar [9] evolved these properties and showed that the maximum degree of any vertex in a graph  $G$  with metric dimension 2 is 8. For unicyclic graphs, i.e. graphs containing exactly one cycle, these estimates can be determined more precisely. We prove that if  $G$  is a unicyclic graph with metric dimension 2 and  $\{a, b\}$  is a metric basis of  $G$  then for any vertex  $v$  from  $G$  the following condition holds:  $\deg(v) \leq 4$  and degrees of both  $a$  and  $b$  are at most 2. Moreover, a vertex  $v$  such that  $\deg(v) = 4$  exists only for odd graphs.

In [10] and [11] all unicyclic graphs that have metric dimension 2 and vertices of degree at most 3 were widely studied.

In this paper we characterize all unicyclic graphs of metric dimension 2 with vertices of degree 4. For this purpose we introduce the constructions of unispider and semiunispider graphs and their knittings. We show that if a unicyclic graph  $G$  with  $\dim G = 2$  has a vertex of degree 4 then  $G$  is unispider or a semiunispider graph or a knitting of one of graphs of such type.

## 1. Preliminaries

Throughout this article, we consider only simple, finite, undirected, connected and nontrivial graphs.

For a graph  $G = (V, E)$  we define a *distance*  $d_G(u, v)$  between two vertices  $u$  and  $v$  as the length of the shortest path between  $u$  and  $v$ ; we put  $d_G(u, v) = 0$  when  $u = v$ .

A vertex  $u$  of  $G$  *resolves* vertices  $v_1$  and  $v_2$  of  $G$ , if  $d_G(u, v_1) \neq d_G(u, v_2)$ .

A *resolving* set is an ordered vertex subset  $S$  of  $V$  such that every two distinct vertices of  $G$  are resolved by some vertex of  $S$ . A resolving set also is called a *metric generator*. A resolving set of minimum cardinality is said to be a *metric basis* of  $G$ . A *metric dimension*,  $\dim G$ , of  $G$  is the cardinality of its metric basis.



to  $u$ , and  $u_1, u_2 \in \hat{V}$ ,  $u_3, u_4, u_5 \in V \setminus \hat{V}$  (see Figure 1). Suppose that a metric basis of  $G$  consists of vertices  $a, b$ . As  $\{a, b\}$  is a basis, vertices  $u_i$ ,  $1 \leq i \leq 5$ , are resolved by  $a, b$ . But  $u_1, u_2 \in \hat{V}$ , so one of vertices  $a, b$  is projected to some vertex  $w \in \hat{V}$ ,  $w \neq u$  (may be  $a = w$  or  $b = w$ ). We may assert that  $a \in \hat{V}$  or  $a$  is projected to  $w$  and resolves  $u_1$  and  $u_2$ . Hence,

$$d_G(a, u_3) = d_G(a, u_4) = d_G(a, u_5) = d_G(a, u) + 1.$$

In other words,  $a$  does not resolve  $u_3, u_4, u_5$ . Then  $b$  is projected to  $u$ . Without loss of generality we may assert that the shortest path, which connects vertices  $b$  and  $u$ , contains  $u_3$ . But in this case, we have

$$d_G(b, u_4) = d_G(b, u_5) = d_G(b, u) + 1.$$

So,  $b$  does not resolve  $u_4, u_5$ . Therefore, our assumption is incorrect.  $\square$

Note, that from Proposition 1 and Proposition 2 it follows that if a unicyclic graph has metric dimension 2, then the degree of any vertex from a cycle of  $G$  is less than 5 and the degree of any vertex out of the cycle of  $G$  is less than 4. In the next section, we describe in detail the structure of such graphs.

Recall that the inner vertices are the vertices of degree at least 3. An inner vertex  $v \in \hat{V}$  is called a *main vertex* if there is an inner vertex  $w \in V \setminus \hat{V}$  that is projected to  $v$ .

An inner vertex  $v$  is *close to a leaf*  $a$  (see [2]) if there is no other inner vertex  $w$  in the unique path between  $v$  and  $a$  in  $G$ , i.e. for every other inner vertex  $w$  of  $G$  the inequality

$$d_G(a, v) < d_G(a, w)$$

holds. If an inner vertex  $v$  is close to two different leaves, then we say that  $v$  is a *two-leaf* vertex.

We need the lemma from [13].

**Lemma 1** ([13]). *Let  $G = (V, E)$  be a unicyclic graph and  $\dim G = 2$ . Then there exist at most two main vertices in  $G$ .*

## 2. Odd and even unicyclic graphs with vertices of degree 4

Following [10], a unicyclic graph  $G$  is *even*, if  $|\hat{V}| = 2k$  for some positive integer  $k$ . If there is some positive integer  $k$ , such that  $|\hat{V}| = 2k + 1$ , then the unicyclic graph  $G$  is *odd*.

**Theorem 1.** *Let  $G = (V, E)$  be a unicyclic graph and  $\dim G = 2$ . If  $G$  is even, then the degree of any vertex of  $G$  is less than 4. If  $G$  is odd, then the maximum number of its vertices of degree 4 is 2.*

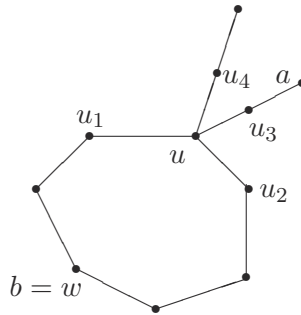


FIGURE 2.

*Proof.* Assume that a vertex  $u$  of  $G$  has the degree 4. Let  $u_1, u_2, u_3$  and  $u_4$  be the vertices adjacent to  $u$ , and  $u_1, u_2 \in \hat{V}$ ,  $u_3, u_4 \in V \setminus \hat{V}$  (see Figure 2). Denote by symbols  $\{a, b\}$  the metric basis of  $G$ . As  $\{a, b\}$  is a basis, the vertices  $a, b$  resolve  $u_i$ ,  $1 \leq i \leq 4$ . But  $u_3$  and  $u_4$  are projected to the single vertex  $u$ . Then one of the basis vertices is also projected to  $u$ . Without loss of generality we may assume that it is  $a$ . We may assert that the shortest path, which connects  $a$  and  $u$ , contains  $u_3$ . Then

$$d_G(a, u_1) = d_G(a, u_2) = d_G(a, u_4) = d_G(a, u) + 1.$$

Hence,  $a$  does not resolve  $u_1, u_2, u_4$ . So,  $u_1, u_2, u_4$  are resolved by  $b$ .

Let  $b$  is projected to  $w$ ,  $w \in \hat{V}$  (may be  $b = w$ ). As

$$d_G(b, u_i) = d_G(b, w) + d_G(w, u_i), \quad 1 \leq i \leq 4,$$

vertices  $u_1, u_2, u_4$  are resolved by  $b$  if and only if they are resolved by  $w$ . Hence,  $w$  resolves vertices  $u_1, u_2, u_4$ , and  $d_G(w, u_1) \neq d_G(w, u_2)$ . Assume that  $d_G(w, u_1) < d_G(w, u_2)$ . If the shortest path, which connects  $w$  and  $u_2$ , contains  $u_1$ , then its length equals the length of the shortest path between  $w$  and  $u_4$ . In other words, in this case  $w$  does not resolve  $u_2$  and  $u_4$ . Therefore, the shortest path between  $w$  and  $u_2$  does not contain  $u_1$ . So, if  $m$  is a number of vertices in the cycle of  $G$ , we obtain

$$d_G(w, u_2) + d_G(w, u_1) + 2 = m.$$

Moreover,  $d_G(w, u_2) < \frac{m}{2}$ ,  $d_G(w, u_1) < \frac{m}{2}$ . But,  $d_G(w, u_1) < d_G(w, u_2)$ , then from direct calculations we obtain that

$$\frac{m-2}{2} < d_G(w, u_2) < \frac{m}{2}.$$

If  $m = 2k$ , for some positive integer  $k$ , then

$$\frac{2k-2}{2} = k-1 < d_G(w, u_2) < k.$$

So, these inequalities never hold. Hence, if  $G$  is even,  $G$  does not have vertices of degree 4.

If  $m = 2k + 1$ , for some positive integer  $k$ , then

$$\frac{2k+1-2}{2} = k - \frac{1}{2} < d_G(w, u_2) < k + \frac{1}{2}.$$

Hence,

$$d_G(w, u_2) = k, \quad d_G(w, u_1) = k - 1, \quad d_G(w, u_4) = k + 1.$$

Similarly, we can show that in this case  $w$  can also be a vertex of degree 4. Note in addition, if  $G$  has a vertex of degree 4, then one of the vertices from the metric basis of  $G$  is projected to this vertex. Therefore, if  $G$  is odd, then the maximum number of its vertices of degree 4 is 2.  $\square$

### 3. Unispider and semiunispider graphs

**Definition 1.** An odd unicyclic graph  $G$  with  $|\hat{V}| = 2k + 1$  is said to be a *unispider graph* if the following conditions hold (see Figures 3–7):

- 1) for any vertex  $v$  from  $V \setminus \hat{V}$ ,  $\deg_G(v) \leq 3$ ;
- 2) for any main vertex  $w$  of  $G$  there exists exactly one two-leaf vertex, that is projected to  $w$ ;
- 3) in the cycle  $\hat{G}$  of  $G$  there are exactly two vertices  $w$  and  $u$  with the degree greater than 2, moreover, at least one of  $w$  and  $u$  has the degree 4, each of  $w$  and  $u$  is the main vertex or (and) a vertex of degree 4 and

$$d_G(w, u) = k.$$

Note, that from the definition of unispider graphs it follows that for any unispider graph  $G$  one of the following cases holds:

- 1) the vertices  $w$  and  $u$  are the main vertices of degree 4 (see Figure 3);

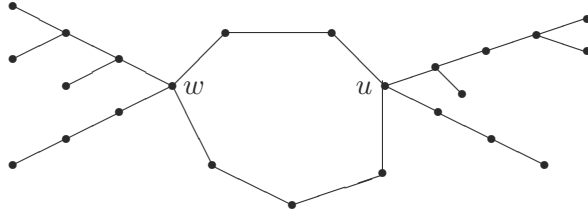


FIGURE 3. Two main vertices that are vertices of degree 4

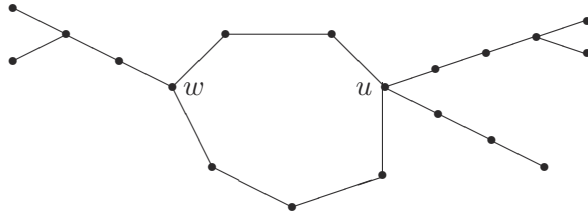


FIGURE 4. Two main vertices, one of them is vertex of degree 4

- 2) the vertices  $w$  and  $u$  are main vertices, but one of them is a vertex of degree 4 (see Figure 4);
- 3) the vertices  $w$  and  $u$  are vertices of degree 4, but only one of them is a main vertex (see Figure 5);
- 4) one of vertices  $w$  and  $u$  is a vertex of degree 4 and the other one is a main vertex (see Figure 6).
- 5) the vertices  $w$  and  $u$  are vertices of degree 4, but  $G$  does not have main vertices (see Figure 7);

**Proposition 3.** *Let  $G = (V, E)$  be a unispider graph. Then*

$$\dim G = 2.$$

*Proof.* Let  $G$  be unispider graph with two main vertices  $u$  and  $w$ , that are vertices of degree 4 (see Figure 8). For the other types of unispider graphs proof is very similar. Denote the leaves that are projected to  $u$  and  $w$  by  $a$  and  $b$ , respectively. Moreover, if there are inner vertices that are projected to  $u$ , then one of them is close to  $a$ . Similar, if there are inner vertices that are projected to  $w$ , then one of them is close to  $b$  (see Figure 8).

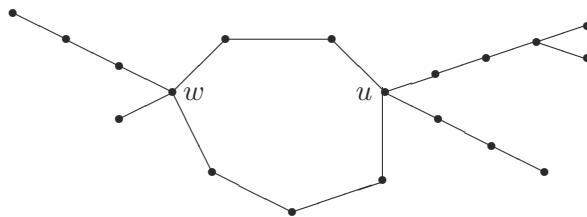


FIGURE 5. Two vertices of degree 4, one of them is the main vertex

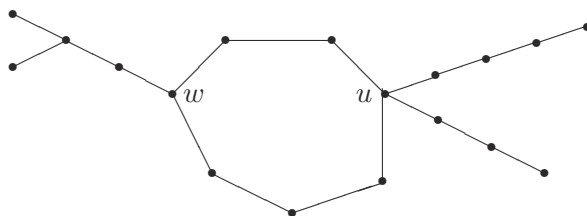


FIGURE 6. One main vertex and one vertex of degree 4

We need to show, that  $\{a, b\}$  is a metric basis of  $G$ .

Note, that  $a$  resolves all vertices that are adjacent to  $b$ , and vice versa.

In addition, if some pair of vertices from a subgraph  $\hat{G}$  is not resolved by the vertex  $b$ , say  $w_3$  and  $w_4$ , then this pair is resolved by  $a$ , because  $G$  is odd,  $|\hat{V}| = 2k + 1$ ,  $a$  and  $b$  are projected to  $u$  and  $w$ , respectively, and  $d_G(w, u) = k$  by the definition of unispider graphs. Similar, if some pair of vertices from  $\hat{G}$  is not resolved by  $a$ , say  $u_3$  and  $u_4$ , then this pair is resolved by  $b$ . Moreover, from the proof of Theorem 1 it follows that  $b$  resolves any pair of vertices from the set  $\{u_1, u_2, u_3, u_4\}$  and  $a$  resolves any pair of vertices from the set  $\{w_1, w_2, w_3, w_4\}$ . So, any pair of vertices from  $G$  is resolved by  $a$  or  $b$ . Therefore,  $\{a, b\}$  is a metric basis of  $G$ .  $\square$

**Definition 2.** An odd unicyclic graph  $G$  with  $|\hat{V}| = 2k + 1$  is said to be a *semiunispider graph* if the following conditions hold (see Figures 9 and 10):

- 1)  $\deg_G(v) \leq 3$  for any  $v$  from  $V \setminus \hat{V}$ ;



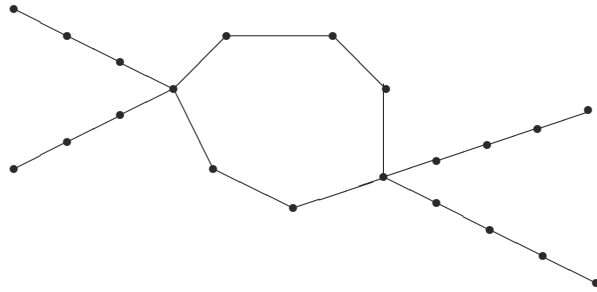


FIGURE 7. There are two vertices of degree 4, but the graph without main vertices

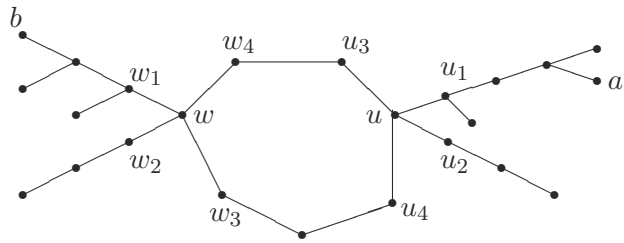


FIGURE 8.

- 2) in the cycle  $\hat{G}$  of  $G$  there is exactly one vertex  $w$  with the degree greater than two, moreover,  $\deg(w) = 4$ ;
- 3) the vertex  $w$  may be the main vertex, in this case there exists exactly one two-leaf vertex, that is projected to  $w$ .

**Proposition 4.** *Let  $G = (V, E)$  be a semiunispider graph. Then*

$$\dim G = 2.$$

*Proof.* Let  $G$  be a semiunispider graph with one main vertex  $w$  of degree 4. For the other type of semiunispider graphs the proof is similar. Denote the leaf that is projected to  $w$  by  $a$  (see Figure 9). Some inner vertex is close to  $a$ . Let  $b$  be a vertex of cycle  $\hat{V}$  such that  $d_G(w, b) = k$ .

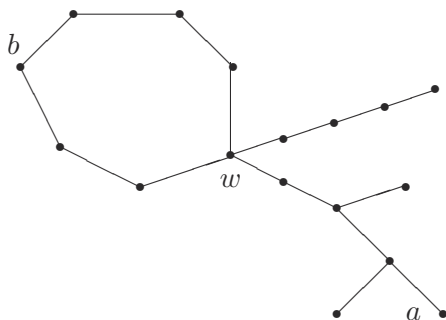


FIGURE 9. Semiunispider graphs with the main vertex

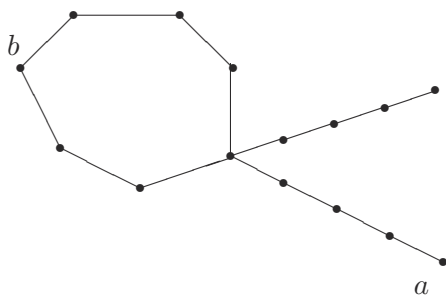


FIGURE 10. Semiunispider graphs without the main vertex

Similarly to the proof of Proposition 3 the set of vertices  $\{a, b\}$  is a metric basis of  $G$ .  $\square$

#### 4. The main result

The main result is based on the notion of a *knitting* of a graph ([10]). To introduce it we need the following definition.

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be simple graphs. Assume that  $v_1 \in V_1$  and  $v_2 \in V_2$ . Define the equivalence relation  $\sim$  on the set  $V_1 \cup V_2$  by the following way: the vertex  $u$  is equivalent to the vertex  $w$  if and only if  $v_1 = u$  and  $v_2 = w$  for all  $u \in V_1$  and  $w \in V_2$ .

A graph  $G = (V_1 \cup V_2 / \sim, E_1 \cup E_2)$  is built from  $G_1$  and  $G_2$  by *gluing* along vertices  $v_1$  and  $v_2$ . Roughly speaking, the vertex  $v_2$  is replaced by the vertex  $v_1$  for all edges of  $G_2$ .

**Definition 3.** Let  $G_1$  be a unispider graph. Denote by  $u$  and  $w$  the vertices of the degree greater than 2 of  $G_1$ . A unicyclic graph  $G$  is called a *knitting* of  $G_1$  by chains  $L_1, \dots, L_r$  if  $G$  is obtained from  $G_1$  by gluing vertices of degree 2 from the cycle of  $G_1$  and the leaves of chains  $L_1, \dots, L_r$  such that any vertex of degree 2 from the cycle  $G_1$  may be glued with one leaf of one chain  $L_j$ ,  $1 \leq j \leq r$  (see Figure 11).

Note that, if  $G_1$  is a unispider graph with one vertex  $u$  of degree 4, then we can glue the vertex  $w$  of degree 3 with the leaf of the chain  $L_0$ . In this case we have a knitting of the unispider graph  $G_2$  with two vertices of degree 4 ( $G_2$  is from Figures 3 or 5) by chains  $L_1, \dots, L_r$ .

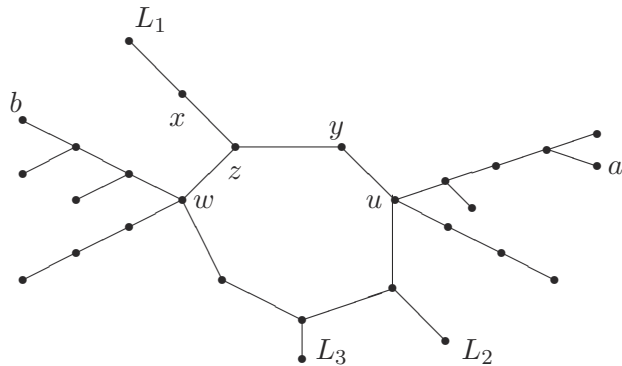


FIGURE 11. Knitting of a unispider graph

Let now  $G_1$  be a semiunispider graph with the vertex  $w$  of degree 4. Assume that  $b$  is a vertex from a cycle of  $G_1$  such that  $\deg(b) = 2$  and  $d_{G_1}(w, b) = k$ , where  $2k + 1$  is the number of vertices in the cycle of  $G_1$ .

**Definition 4.** A unicyclic graph  $G$  is called a *knitting* of  $G_1$  by chains  $L_1, \dots, L_r$  if  $G$  is obtained from  $G_1$  by gluing vertices with degree 2 from the cycle of  $G_1$  and leaves of the chains  $L_1, \dots, L_r$  such that any vertex with degree 2 of the cycle of  $G_1$  may be glued with the leaf of exactly one chain.

Note that, if the vertex  $b$  is glued with the leaves of two chains, then graph  $G$  is a knitting of the unispider graph  $G_1$  (see Figure 5 or Figure 7) by some chains.

The main result of this paper is the following.

**Theorem 2.** Let  $G = (V, E)$  be a unicyclic graph with vertices of degree 4. Then  $\dim G = 2$  if and only if one of the following conditions holds:

- (1)  $G$  is a unispider graph;
- (2)  $G$  is a knitting of some unispider graph;
- (3)  $G$  is a semiunispider graph;
- (4)  $G$  is a knitting of some semiunispider graph;

*Proof.* Let  $G = (V, E)$  be a unicyclic graph with vertices of degree 4. We need to show that if one of the conditions (1)–(4) holds then  $\dim G = 2$ . From Propositions 3 and 4 it follows that unispider and semiunispider graphs have metric dimension 2. So, if  $G$  satisfies (1) or (3) then  $\dim G = 2$ .

Let  $G$  be a knitting of some unispider graph. Denote the vertices of  $G$  with degree 4 by  $u$  and  $w$  and let  $a$  and  $b$  be the leaves that are projected to  $u$  and  $w$ , respectively (see Figure 11).

As  $d_G(w, u) = k$  and  $2k + 1$  is the number of all vertices in the cycle  $\hat{G}$  of  $G$ ,  $a$  and  $b$  resolve all pairs of vertices from  $\hat{G}$ . Moreover, from the proofs of Propositions 3 and 4 it follows that  $a$  resolves all the pairs vertices that are adjacent to  $b$ , and vice versa.

Let a chain  $L_j$ ,  $1 \leq j \leq n$  be glued with unicyclic graph  $G$  in point  $z$ . Without loss of generality, we may claim that this is  $L_1$  (see Figure 11). Assume that vertices  $x$ ,  $y$  and  $w$  are adjacent to  $z$ . Moreover,  $x$  belongs to  $L_1$ , and  $y$  is a vertex of  $\hat{G}$ . Then  $x$  and  $y$  are not resolved by the basis vertex  $b$ . But they are resolved by another basis vertex  $a$ . Hence, any pair of vertices from  $G$  is resolved by  $a$  or  $b$ . Therefore,  $\{a, b\}$  is a metric basis of  $G$ .

Let  $G$  be a knitting of some semiunispider graph, i. e.  $G$  satisfies condition (4). Assume that  $w$  is a vertex of  $G$  with  $\deg(w) = 4$ . Denote the leaf that is projected to  $w$  by  $a$ . Let  $b$  be a vertex of the cycle  $\hat{V}$  such that  $d_G(w, b) = k$ . Similarly to the case of a unispider graph,  $\{a, b\}$  is a metric basis of  $G$ .

Therefore, if  $G$  is a knitting of a unispider or semiunispider graph, then  $\dim G = 2$ .

Now let  $G = (V, E)$  be a unicyclic graph with vertices of degree 4 and  $\dim G = 2$ . From Theorem 1 it follows that  $G$  is odd and the maximum number of vertices of degree 4 is two. From Lemma 1 (see [13]) we have, that  $G$  can contain at most two main vertices. Moreover, from the proofs of Lemma 1 and Theorem 1 it follows that  $a$  and  $b$  are projected to main vertices and to vertices of degree 4, if they exist. Note that  $G$  has no less than one vertex of degree 4, then if basis vertices are projected to  $u$  and  $w$ , then  $u$  or  $w$  (may be  $u$  and  $w$ ) has degree 4 and

$$d_G(w, u) = k.$$

Hence,  $G$  is unispider, or semiunispider, or a knitting of some unispider graph, or a knitting of some semiunispider graph.

For example, if  $G$  has exactly two vertices  $u$  and  $w$  of degree 4 and two main vertices, then any vertex of degree 4 is the main vertex. Hence,  $G$  is a unispider graph or its knitting (see Figure 3, Figure 11).

For the other number of vertices of degree 4 and main vertices the proof is very similar.  $\square$

**Corollary 1.** *Let  $G = (V, E)$  be a unicyclic graph with vertices of degree 4, and let  $\{a, b\}$  be a metric basis of  $G$ . Then the degrees of  $a$  and  $b$  are no greater than 2.*

The proof of Corollary 1 directly follows from the proof of Theorem 2.

## References

- [1] Frank Harary, Robert A.Melter, *On the metric dimension of a graph*, Ars Comb., V.2, 1976, pp.191–195.
- [2] P. J. Slater, *Leaves of trees*, Congress. Numer., V.14, 1975, pp.549–559.
- [3] M. Johnson *Structure-activity maps for visualizing the graph variables arising in drug design*, J.Biopharm. Stat., **3** (2), 1993, pp 203-236.
- [4] Robert A. Melter, I. Tomescu, *Metric bases in digital geometry*, Comput.Vision, Graph. Image Process, **25** (1), 1984, pp 113-121.
- [5] A.Sebo, E.Tannier, *On metric generators of graphs*, Math.of Operations Research, **29**, 2004, pp.383-393.
- [6] Michael R. Garey, David S. Johnson, *Computers and intractability. A guide to the theory of NP-completeness*, San Francisco: W. H. Freeman and Company, 1979.
- [7] S.Khuller, B. Raghavachari and A. Rosenfeld, *Landmarks in graphs*, Disc. Appl.Math., **70**, 1996, pp. 217-229.
- [8] Gary Chartrand, Linda Eroh, Mark A. Johnson, Ortrud R. Oellermann, *Resolvability in graphs and the metric dimension of a graph*, Discrete Appl. Math., V. **105**, N. 1–3, 2000, pp. 99–113.
- [9] G. Sudhakara, A. R. Hemant Kumar, *Graphs with metric dimension two — a characterization*, Adv. and Appl. in Disc. Math., **4** (2), 2009, pp.169–186.
- [10] Marharyta Dudenko, Bogdana Oliynyk, *On unicyclic graphs of metric dimension 2*, Algebra Discrete Math. **23** (2), 2017, pp. 216–222.
- [11] Marharyta Dudenko, *Metric dimension of unicyclic graphs with at most one main vertex*, Visnyk of Lviv University, V. **83**, 2017, pp. 189–195.
- [12] Reinhard Diestel, *Graph Theory* (3rd ed.), Berlin, New York: Springer-Verlag, 2005.
- [13] Marharyta Dudenko, *Unicyclic graphs with metric dimension 2*, Nauk. Zapysky NaUKMA, V. **165**, 2015, pp. 7–10.

## CONTACT INFORMATION

- M. Dudenko** Department of Mathematics, National  
University of Kyiv-Mohyla Academy, Skovorody  
St. 2, Kyiv, 04070, Ukraine  
*E-Mail(s):* rita.dudenko@gmail.com
- B. Oliynyk** Department of Mathematics, National  
University of Kyiv-Mohyla Academy, Skovorody  
St. 2, Kyiv, 04070, Ukraine  
*E-Mail(s):* oliynyk@ukma.edu.ua

Received by the editors: 14.10.2018  
and in final form 18.12.2018.