

THE GOLDEN RULE FOR OVERLAPPING GENERATIONS

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The net profit of representative firm is equal to $\pi_t = Y_t - L_t w_t = L_t f(k_t) - L_t w_t - K_t r_t$, where: $Y_t = F(K_t, L_t)$ is its output function; K_t and L_t is its level of capital and labor supply, respectively, at the period t ; $w_t = \frac{W_t}{p_t} = x'_t + \frac{x_t^{t+1}}{1+r_{t+1}}$ is the real wage; $W_t = p_t(x'_t + s_t)$ is the nominal wage;

p_t is the price of consumption good; x'_t and x_t^{t+1} is the level of consumption by generation t during its first period t and during its second period $(t+1)$, respectively (each generation has two periods of life); $r_{t+1} = \frac{p_t - p_{t+1}}{p_{t+1}} = \frac{p_t}{p_{t+1}} - 1$ is the

interest rate at the period $(t+1)$; $k_t = \frac{K_t}{L_t}$; $s_t = k_{t+1}$ is the level of savings by

generation t ; $f(k_t) = \frac{Y_t}{L_t}$. If the labor market is competitive, then maximization

of the net profit with respect to L_t implies $w_t = f(k_t) - k_t f'(k_t)$, and maximization of the net profit with respect to K_t implies $r_t = f'(k_t)$.

Note $K_{t+1} = H_t(w_t - x'_t)$, where $L_t = H_t = (1+n)H_{t-1}$ is the population, n is the rate of population growth. Then in a steady state the values of x'_t , x_t^{t+1} , k_t , w_t , r_t are constant at any time period t and equal to certain quantities x^1 , x^2 , k , w , r , respectively: $x^1 = f(k) - k f'(k) - k(1+n)$; $x^2 = k(1+n)[1 + f'(k)]$.

The total consumption of both overlapping generations t and $(t-1)$ at the period t equals to $x'_t H_t + x_{t-1}^t H_{t-1} = H_t \left(x'_t + \frac{x_{t-1}^t}{1+n} \right)$. Then the economy in a steady state is characterized by the total consumption per capita $x^1 + (1+n)^{-1} x^2 = f(k) - n k$. Its maximization implies the Golden rule $f'(k^*) = n$ for the optimal capital k^* per labor.