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Akademika Hlushkova Ave, 4d, Kyiv, Ukraine, 03680
*E-mail address*: bogdov@gmail.com


# On sums of marginal subspaces


I.S. Feshchenko


Let $(\Omega, \mathcal{F}, \mu)$ be a probability space. Denote by $\mathbb{K}$ a base field of scalars, i.e., $\mathbb{R}$ or $\mathbb{C}$. For an $\mathcal{F}$-measurable function (random variable) $\xi : \Omega \to \mathbb{K}$ denote by $E\xi$ the expectation of $\xi$ (if it exists). Two random variables $\xi$ and $\eta$ are said to be equivalent if $\xi(\omega) = \eta(\omega)$ for $\mu$-almost all $\omega$. For $p \in [1, \infty) \cup \{\infty\}$ denote by $L^p(\mathcal{F}) = L^p(\Omega, \mathcal{F}, \mu)$ the set of equivalence classes of random variables $\xi : \Omega \to \mathbb{K}$ such that $E|\xi|^p < \infty$ if $p \in [1, \infty)$ and $\xi$ is $\mu$-essentially bounded if $p = \infty$. For $\xi \in L^p(\mathcal{F})$ set $\|\xi\|_p = (E|\xi|^p)^{1/p}$ if $p \in [1, \infty)$ and $\|\xi\|_\infty = \text{ess sup}|\xi|$ if $p = \infty$. Then $L^p(\mathcal{F})$ is a Banach space. For every sub-$\sigma$-algebra $\mathcal{A}$ of $\mathcal{F}$ we define the marginal subspace corresponding to $\mathcal{A}$, $L^p(\mathcal{A})$, as follows. $L^p(\mathcal{A})$ consists of elements (equivalence classes) of $L^p(\mathcal{F})$ which contain at least one $\mathcal{A}$-measurable random variable. Note that $L^p(\mathcal{A})$ is a complemented subspace in $L^p(\mathcal{F})$ (recall that a subspace of a Banach space is said to be complemented in the Banach space if there exists a bounded linear projection onto the subspace; the conditional expectation operator $\xi \mapsto E(\xi|\mathcal{A})$ is a norm one projection onto $L^p(\mathcal{A})$). Denote by $L^p_0(\mathcal{A})$ the subspace of all $\xi \in L^p(\mathcal{A})$ with $E\xi = 0$.

We study the following problem. Let $\mathcal{F}_1, ..., \mathcal{F}_n$ be sub-$\sigma$-algebras of $\mathcal{F}$. Question: when is the sum of the corresponding marginal subspaces, $L^p(\mathcal{F}_1) + ... + L^p(\mathcal{F}_n)$, complemented in $L^p(\mathcal{F})$? One can easily check that $L^p(\mathcal{F}_1) + ... + L^p(\mathcal{F}_n)$ is complemented in $L^p(\mathcal{F})$ if and only if $L^p_0(\mathcal{F}_1) + ... + L^p_0(\mathcal{F}_n)$ is.

We will provide a sufficient condition for subspaces $L^p_0(\mathcal{F}_1), ..., L^p_0(\mathcal{F}_n)$ to be linearly independent and their sum, $L^p_0(\mathcal{F}_1) + ... + L^p_0(\mathcal{F}_n)$, to be complemented in $L^p(\mathcal{F})$.


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Taras Shevchenko National University of Kyiv
*E-mail address*: ivanmath007@gmail.com



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# Interpolation problems for random fields from observations in areas with peculiarities


A.S. Florenko[1], N.Y. Shchestyuk[2]


Estimating of the unknown values of a random field in an area that represents a system of embedded rectangles is of interest in the study of random fields with singularities. The study of the dependence of the obtained formulas on the geometry and the number of embeds are the topical problems in the field of forecasting theory, in geology, geodesy and some other directions. Interpolation problems were investigated for the random stationary sequences by A.M. Kolmogorov [1]. Estimators of the functionals of random fields were created by M.P. Moklyachuk and N.Y. Shchetsyuk [2].

The problem under consideration is the optimal estimation of the linear functional

$$A_k\xi = \sum_{(l,k)\in K} a(k,l)\xi(k,l) =$$

$$= \sum_{t=0}^{s-1} \left( \sum_{k=2t\cdot l_x}^{m_x-2tl_x-1} \Big( a(k, 2tl_y)\xi(k, 2t\cdot l_y) + a(k, m_y - 2t\cdot l_y - 1)\xi(k, m_y - 2t\cdot l_y - 1) \Big) \right.$$

$$\left. + \sum_{j=2t\cdot l_y}^{m_y-2t\cdot l_y-1} \Big( a(2t\cdot l_x, j)\xi(2t\cdot l_x, j) + a(m_x - 2t\cdot l_x - 1, j)\xi(m_x - 2t\cdot l_x - 1, j) \Big) \right)$$

of unknown values of the field $\xi(k,j), (k,j) \in K$, which is observed with the noise $\xi(k,j) + \eta(k,j)$ for $(k,j) \in Z^2 \setminus K$, where $K$ is a domain that is the union of the edges of the rectangles $m_x \times m_y$, with the number of rectangles $s_x$, $l_x$ and $l_y$, spaced between the embeds on the $X$ and $Y$ axes, respectively. That is, we find a value $\tilde{A}_k\xi$ inside the class of linear functionals, which minimizes the value of the mean square error

$$\Delta = M \mid A_k\xi - \tilde{A}_k\xi \mid^2 .$$

To solve this problem, we use the classical method of projections in the Hilbert space. The spectral characteristic of the functional optimal estimator and the value of the mean-square error are found.

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National University "Kyiv-Mohyla Academy"
*E-mail address*: [1]florenko.anastasia@gmail.com, [2]Natalyshch@gmail.com


## Interpolation of periodically correlated stochastic sequences with missing observations


I.I. Golichenko[1], M.P. Moklyachuk[2]


We consider the problem of optimal estimation of the linear functional

$$A_s\zeta=\sum_{l=0}^{s-1}\sum_{j=M_l+1}^{M_l+N_{l+1}}a(j)\zeta(j),\ M_l=\sum_{k=0}^{l}(N_k+K_k),\ N_0=K_0=0,$$

which depends on the unknown values of a periodically correlated with period $T$ stochastic sequence $\zeta(j)$ from observations of the sequence $\zeta(j)+\theta(j)$ at points $j\in\mathbb{Z}\setminus S$, $S=\bigcup_{l=0}^{s-1}\{M_l+1,\dots,M_l+N_{l+1}\}$, where $\theta(j)$ is an uncorrelated with $\zeta(j)$ periodically correlated stochastic sequence. Assume that the number of missed observations at each of the intervals and the number of observations at each of the intervals are a multiple of $T$ ($K_l=T\cdot K_l^T$ and $N_{l+1}=T\cdot N_{l+1}^T$, $l=0,\dots,s-1$), and coefficients $a(j), j\in S$ are of the form

$$a(j)=a(j-[j/T]T)+[j/T]T)=a(\nu+\tilde{j}T)=a(\tilde{j})e^{2\pi i\tilde{j}\nu/T},$$

$$\nu=1,\dots,T,\ \tilde{j}\in\tilde{S},\ \tilde{S}=\bigcup_{l=0}^{s-1}\left\{M_l^T,\dots,M_l^T+N_{l+1}^T-1\right\},\ M_l=T\cdot M_l^T,\ l=0,\dots,s-1.$$

Formulas for calculation the spectral characteristic and the mean square error of the optimal estimate of the functional $A_s\zeta$ are obtained in the case where spectral densities of the sequences are exactly known. Formulas that determine the least favorable spectral densities and the minimax spectral characteristics are proposed for some classes of admissible spectral densities.


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[1]Department of Mathematical Analysis and Probability Theory, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, 03056, Ukraine; [2]Department of Probability Theory, Statistics and Actuarial Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, 01601, Ukraine
*E-mail address*: [1]idubovetska@gmail.com, [2]Moklyachuk@gmail.com


## Quantitive estimate of an expectation of the simultanious renewal for time-inhomogeneous Markov chain


V.V. Golomoziy


In this talk we consider two time-inhomogeneous Markov chains $X_t^{(l)}$, $l\in\{1,2\}$ with discrete time on a general state space. We assume existence of some renewal set $C$ and investigate time of the simultaneous renewal, that is, the first positive time when the chains hit the set $C$ simultaneously. The initial distributions for both chains could be arbitrary. Under the condition of stochastic domination and non-lattice condition for both renewal processes we derive an upper bound for the expectation of the simultaneous renewal time.

We proved that under the conditions described above an estimate for a moment of simultaneous renewal will satisfy an inequality:

$$\mathbb{E}[T]\le\mathbb{E}[\theta_0^{(1)}]+\mathbb{E}[\theta_0^{(2)}]+\frac{M}{\gamma},\tag{1}$$

where $\theta_0^{(1)}$ and $\theta_0^{(2)}$ are times to the first renewal for chains $X_t^{(1)}$ and $X_t^{(2)}$ respectively, $M$ and $\gamma$ are some constants.


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Taras Shevchenko National University of Kyiv, faculty of mechanics and mathematics, 03127, Kyiv, 4E, Academician Glushkov avenue
*E-mail address*: vitaliy.golomoziy@gmail.com