STUDENT-LIKE MODELS FOR RISKY ASSET AND CONCEPT OF FRACTAL MARKET

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A generation ago, the Efficient Market Hypothesis (EMH) was widely accepted by academic financial economists and still resides at the forefront of modern financial theory; for example, see Eugene Fama’s (Nobel Prize in 2013) influential survey article, “Efficient Capital Markets” (1970). The EMH states that asset prices fully reflect all available information, requires that agents have rational expectation and investors’ reactions be random and follow a normal distribution so that the net effect on market prices can not be reliably exploited to make abnormal profit. A direct implication is that it is impossible to "beat the market" consistently on a risk-adjusted basis. Thus there is a very close link between EMH and the random walk hypothesis (Louis Bachelier, 1900). Later financial economists began making heavy use of probability theory and statistics to model asset prices. In the Black-Scholes model (Nobel Prize in 1997) the instantaneous log return of stock price P_t is an infinitesimal random walk with drift μ; more precisely, it is a geometrical Brownian motion.

In the wake of the 2008 financial crisis, many have challenged the dominant economic theories and perspectives on markets. The EMH in particular has fallen short in explaining the crisis. Then alternative theories that provide a more accurate representation of markets have emerged. Fractal Market Hypothesis (FMH), for instance, focuses on the investment horizons and liquidity of markets and investors – factors limited in the framework of EMH. The theoretical framework of fractal markets can clearly explain investor behavior during periods of crisis and stability. Edgar

Peters formalized FMH in 1991 within the framework of chaos theory in 1991 to explain the heterogeneity of investors with respect to their investment horizons. The concept of fractals comes from mathematics (B. Mandelbrott) and refers to a fragmented geometric shape that can be broken into smaller parts that fully or nearly replicate the whole.

In this paper we present alternative to GBM that incorporates the Student-like distribution of the returns. This model gives asset prices as GBM but is driven by some nondecreasing stochastic "activity time" (or "fractal time") process.

We consider a model for stock price $\{P_t, t \geq 0\}$ driven by standard Brownian motion B, which has a "fractal clock" rather than a calendar clock:

$$\log P_t = \log P_0 + \mu t + \theta T_t + \sigma B(T_t), \qquad (1)$$

where $\mu \in R$, $\sigma > 0$ reflect drift and diffusion, $\theta \in R$ determined skewness, $\{T_t, t \geq 0\}$ is positive process, nondecreasing with stationary but not independent increments $\tau_t = T_t - T_{t-1}$. When random activity time process $T_t = t$, then equation (1) is Black-Sholes formula and $\log P_t$ is normal for any t. In this paper we construct the activity time $T_t^m = \sum_{i=1}^{t} \tau_i^m$, where τ_i has reciprocal (inverse) Gamma distribution $R\Gamma(\alpha, \beta)$. For this model we present distribution theory, find option price and consider delta-hedging problem.

References

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